REVIEW

for defense of PhD thesis of topic:

Design methods of wavelet and multiwavelet filter banks

for obtaining an educational and science degree „Doctor“

by

Vasil Stefanov Kolev,

Science field: **4. Natural science, mathematics and informatics,**

Professional field: **4.6. Informatics and Computer Science,**

Doctoral program: **Informatics,**

The review was prepared by: **Prof. DSc. Ognyan Kounchev, Institute of Mathematics and Informatics at Bulgarian Academy of Sciences**, in my capacity a member of the science jury, by Order № 111/26.04.2024 of the director of Institute of Information and Commutation Technologies at Bulgarian Academy of Sciences.

1. **Structure of the dissertation work, general characteristics and presented materials**

The main research in the PhD Thesis is devoted to development of new methods for building new classes of multiwavelet filter banks from polynomials and splines. The PhD Thesis contains 150 pages and consists of an introduction, 5 chapters presenting the author's contributions. The bibliography consists of 153 titles, including 6 publications by the author. A 54-page abstract is also presented, which adequately reflects the content of the dissertation.

2. **Data and personal impressions about the candidate**

I personally know the PhD student Vasil Kolev since 2003, as an actively working in an interesting new field of the wavelet theory, multirate systems and their applications. I have attended his presentations at various conferences where he has reported results related to the topic of the dissertation. He was also a participant in the scientific project "Astroinformatics" 2008-2012 with the National Science Fund of Bulgaria, of which I was head. Within this project, Vasil Kolev was actively involved in the core activities, in particular, in the work package on the application of new signal analysis methods to astronomical data/images.

3. Analysis of the candidate's scientific and applied scientific achievements contained in the PhD thesis

Vasil Kolev's PhD thesis is devoted to the theory of discrete wavelet transforms, to the construction of new families of wavelets and multiwavelets, and the corresponding filter banks, and also to some of their applications; in particular, methods and algorithms for construction of orthogonal multiscaling functions were investigated. This is a topic that is very relevant in various areas of mathematics applications in Signal and Image Analysis, Control theory, and others. The dissertation presents new original results published in six papers.

Let us first mention that in the dissertation the results are motivated by the general concepts of Applied Harmonic Analysis of Signals established at the end of the 20^{th} century, namely, the analysis of signals is focused not only on the characterization of classes of signals, but also in finding new classes of basis functions to use for signal decomposition, and also corresponding dual basis sets for subsequent signal synthesis, preferably without loss of accuracy, or with minimal loss of accuracy. In doing so, the main conceptual component is to obtain signal decompositions with a small number of large coefficients, i.e. the so-called sparse representations, which today play an exceptional role in Applied Mathematics.

Thus, wavelet and multiwavelet theory is a modern alternative to classical Fourier analysis, where trigonometric functions are replaced by wavelet functions. The epoch-making discovery made 35 years ago in the wavelet theory was that, apart from the property of orthogonality, there are also wavelets that have a compact carrier and also high smoothness. The wavelet theory relies on a rich algebraic structure that underlies the beautiful algebraic and geometric properties of the wavelets, and allows information to be extracted from signals at different scales. It is the coefficients connecting the different levels of scaling in the equations satisfied by the basis functions that are the main constituents of the concept of Filter Banks, which has long been popular among Signal Analysis specialists.

One of the main trends of the wavelets is the concept of the multiwavelet. The multiwavelets are obtained by a natural generalization of Multiresolution Analysis, where the underlying space V_0 is allowed to be generated by a Riesz basis composed of more than one scaling function. Most of the results known for ordinary wavelets are also generalized for multiwavelets, incl. the smoothness properties. Like regular wavelets, most examples of the multiwavelet filter banks are based on splines.

According to the aforementioned perspective, the PhD thesis offers new methods for finding scaling and multiscaling functions and extends the spectral factorization theory by solving the complex problem of obtaining scaling and multiscaling filter functions from singular scalar or matrix polynomials by the spectral factorization. These are new results that have not been published before.

A more detailed analysis of the obtained results follows:

Chapter 1 is devoted to an overview of the existing methods for developing wavelet and multiwavelet filter banks, a brief presentation of elements of the theory of basis functions for polynomials and splines is made, and the scaling and multiscaling filter functions are defined for the case of linear B-splines, and for cubic Hermitian splines. The orthogonal Legendre polynomials and related orthogonal Alpert multiscaling functions are considered, which are the first of their kind.

The basic concepts of filter banks are introduced, as well as the important concept of para-Hermitian scalar and matrix polynomials; the latter are the basis for constructing (by using spectral factorization) scaling or multiscaling functions.

The methods for building the filter banks, which allow solving the problems arising due to the presence of the decimation and interpolation operators, are considered.

Considered are:

In paragraph 1.3.1, the main properties of MRA (Multiresolution analysis);

In paragraph 1.3.2, design of multiplierless modules for a scalar filter bank;

In paragraph 1.3.3, multifilter (vector filter) banks, in particular, the main properties of these banks are given;

In paragraph 1.4., the concept of spectral factorization was introduced, as the factorization of a para-Hermitian matrix polynomial on the unit circle. This is a key point for building multiscaling functions and multiwavelets. Bauer's method, which is a well-known spectral factorization method, is explained;

In paragraph 1.5., basis functions from splines, cubic and quintic Hermitian splines are considered.

The material presented in Chapter 1 shows the rich general culture of the candidate Vasil Kolev in the area.

In ***Chapter 2*** methods are developed for constructing scaling and multiscaling functions from polynomials or spline functions, as well as their complementary wavelet (multiwavelet) functions from polynomials and splines. In particular, the basis change method, the direct method, and the scalar product method have been developed.

Let us mention a very important fact: unlike the spectral factorization methods (presented in *Chapters 3 and 4*), these methods lead directly to obtaining scaling or multiscaling functions – for ordinary and also Hermitian splines as well as Alpert splines.

In ***Chapter 3*** overview of existing spectral factorization methods, the matrix filter product smoothness condition, and then the Alpert multifilter product are developed. In particular, spectral factorization methods are developed: polynomial roots method, quadratic equation method, Cepstral method, and Bauer method. Unlike other methods, Bauer's method requires the product of a scalar (matrix) filter to be constructed in advance, i.e. scalar (matrix) para–Hermitian polynomial. To obtain a matrix spectral factor (multiscaling function) with desired properties, it is necessary for the determinant to satisfy certain conditions. One such example is finding an Alpert matrix product filter.

The details follow reference [89] (Kolev V., Cooklev T., Keinert F.) from the Bibliography.

In short, the main novelty here is that a new Alpert matrix filter has been constructed (covered in detail in section 3.1 of the thesis).

Chapter 4 is devoted to the development of fast Bauer method algorithms and their solution by three numerical methods. Two Bauer spectral factorization algorithms are developed and orthogonal Alpert multifilter banks are constructed. Also shown are the differences between the classical and fast Bauer methods.

The spectral decomposition of a product filter with a basis greater than [0,1] increases the computational complexity. For this purpose, it is suggested that the coefficients of the product filter with basis [0, N] be converted to basis [0,1].

Let us list the main applied scientific results in Chapter 4:

1. An algorithm has been developed for reducing the degree of the product of matrix filters (in section 4.1).

2. Two new fast Bauer method algorithms are developed:

(a) *Algorithm 1* - to calculate a fast Bauer method (in section 4.2).

(b) *Algorithm 2* - to calculate an exact Bauer method (in section 4.2).

3. Two new numerical methods for solving the NME (a nonlinear matrix equation) for FBM (fast Bauer method) are developed and their computational complexity is shown (in section 4.3.1 and 4.3.2).

4. Bauer's classical and fast method for spectral factorization of the scalar product filter of a filter (in section 4.4) and the matrix product filter of an Alpert multifilter (in section 4.5.1) is applied.

5. Two new variants for finding symmetric Alpert wavelet functions are developed. (in section 4.5.2).

The details follow the papers [89] and [90] (Kolev V., Cooklev T., Keinert F.) from the Bibliography.

In ***Chapter 5*** a comparative analysis of the four methods for constructing scaling and multiscaling functions, Bauer's spectral factorization methods for Haar and Daubechies scaling functions, and Alpert's multiscaling function is made. Experimental studies of Bauer's fast and accurate spectral factorization method for seven examples of scalar and matrix polynomials using *Algorithm 2* as well as using built-in software functions for 14 different versions of Matlab ('*dare*','*idare*') and Maple 17 ('*dare*').

The lifting scheme of the Alpert multifilter was developed and investigated for image denoising of the gray-levels digital images. Applications of orthogonal scalar and vector filters for image compression from scanned astronomical photographic plates are investigated.

A few words need to be said essentially about the skills of the candidate Vasil Kolev in achieving the profound scientific results contained in the articles and dissertation.

One significant problem that he has tackled is the unsolved problem of obtaining a multiscaling function as the multiplier of the spectral factorization of a singular matrix polynomial with primes or multiple zeros. To appreciate the difficulty of the problem, it should be noted that Bauer's method does not work for one of the most classical cases – when factoring the product of the Daubechies 4-filter (see formula (5.13), page 96).

4. Evaluation of the dissertation's publications

The results of the dissertation were published in six papers. The scientometric indicators of these articles are significantly higher than the minimum requirements for obtaining the educational and scientific degree "doctor" in the scientific field and professional direction (as defined in Decree No. 26 of 02/13/2019). The scientific publications fall into Group G7 and collect a total of 104 credits, with a minimum requirement of 30 credits for the respective field. The first and second papers are worth 12 credits, while the fifth and sixth paper are classified in Q3 and Q1, and are rated 30 and 50 credits respectively. In short, the obtained 104 credits significantly exceed the minimum requirements for a Ph.D.

I declare that:

а) the scientific works satisfied the minimum national requirements (under article 2b, paragraphs 2 and 3 of the LDASRB) and, accordingly, the additional requirements of IICT-BAS, for the acquisition of an educational and scientific degree "doctor" in the scientific field and professional direction of the procedure;

б) the presented results of Vasil Kolev in the PhD thesis and related scientific works do not repeat those from previous procedures for acquiring a scientific title and academic position;

в) no plagiarism has been proven, in accordance with the law, in the submitted dissertation and scientific works under this procedure.

5. Qualities of the abstract

The abstract contains 54 pages and meets all technical requirements. The presentation of the results in the Abstract is adequate to the content of the PhD thesis.

6. Critical notes and recommendations

I have no essential critics. However, there are questions and problems with terminology in Bulgarian concerning the term "spectral factorization" of a polynomial, which in Bulgarian may be translated in the same way as “spectral decomposition”, which is very widely used and established in another sense, in areas as Linear Algebra, Operator Theory, and also in Mathematical Analysis.

However, as an excuse of the various linguistic mistakes made, one must take into account the large volume of the work and also the fact that almost all related papers are written in English, while the Bulgarian terminology remains rather undeveloped and poorly established. The fact that there is a conflict of terminology in engineering and mathematical literature is not to be ignored.

7. Conclusion

After familiarized myself with the dissertation work presented in the procedure and the accompanying scientific works, and based on the analysis of their significance and the scientific and applied scientific contributions contained in them, I confirm that the presented dissertation work and the scientific publications to it, as well as the quality and the originality of the results and achievements presented in them, meet the requirements of the LDASRB, the regulations for its application and the relevant regulations for **obtaining an educational and science degree „doctor“** in science field: **4. Natural science, mathematics and informatics,** in professional field: **4.6 Informatics and Computer Science, IICT-BAS, Bulgaria.**

In particular, the candidate satisfies the minimum national requirements in the professional direction and no plagiarism has been found in the scientific works submitted for the competition.

As a consequence of the above, I have all grounds for a positive assessment and I propose to the esteemed Scientific Jury to award the educational and scientific degree „doctor“ in science field: **4. Natural science, mathematics and informatics,** in professional field: **4.6 Informatics and Computer Science to Vasil Stefanov Kolev.**

17.6. 2024 г. **Prepared the**

НА ОСНОВАНИЕ ЗЗЛД