

BULGARIAN ACADEMY OF SCIENCES INSTITUTE OF INFORMATION AND COMMUNICATION TECHNOLOGIES



ABSTRACT OF PhD THESIS

# INFLUENCE OF THE SUBJECTIVE FACTOR IN DECISION-MAKING SYSTEMS

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The dissertation has been discussed and approved for defense at an extended meeting of the "\_\_\_\_\_" section of the Institute of Information and Communication Technologies, Bulgarian Academy of Sciences (IICT-BAS), held on \_\_\_\_\_\_ (date).

The dissertation comprises \_\_\_\_\_ pages, including \_\_\_\_\_ figures, \_\_\_\_\_ tables, and \_\_\_\_\_ pages of literature, encompassing \_\_\_\_\_\_ titles.

The defense of the dissertation will take place on \_\_\_\_\_\_ (date) at \_\_\_\_\_ o'clock in the auditorium \_\_\_\_\_ at block \_\_\_\_\_ of the Institute of Information and Communication Technologies, Bulgarian Academy of Sciences, in an open session with the following members of the scientific jury:

 1.

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Materials for the defense are available to interested parties in room \_\_\_\_\_\_ at the Institute of Information and Communication Technologies, Bulgarian Academy of Sciences, 25A "Akad. G. Bonchev" Str.

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Title: "INFLUENCE OF SUBJECTIVE FACTOR IN DECISION-MAKING SYSTEMS"

# **Glossary of used terms and abbreviations**

# **ABBREVIATION/ TERM MEANING**

ERG Theory: Theory of Existence, Relatedness, and Growth, categorizing human needs into three groups: existence, relatedness, and growth.

SDT: Self-Determination Theory, exploring intrinsic motivation and self-determination.

SMART: Specific, Measurable, Achievable, Realistic, Timely; a framework for setting goals. DSS: Decision Support System.

ERP: Enterprise Resource Planning, involves the integration of business processes and functions.

CRM: Customer Relationship Management, managing interactions with customers.

SEMS: Environmental Management System.

AI: Artificial Intelligence.

ML: Machine Learning.

ARIMA: Autoregressive Integrated Moving Average, a time series forecasting method.

LP: Linear Programming.

IP: Integer Programming.

NLP: Nonlinear Programming.

PERT: Program Evaluation and Review Technique, a project management tool.

NoSQL: Non-relational database.

CPM: Cost Per Mille, a metric for advertising costs.

MCDA: Multi-Criteria Decision Analysis.

ER Model: Entity-Relationship Model, depicting entities and their relationships in a database.

ETL: Extract, Transform, Load; a process in data warehousing.

PERT: Program Evaluation and Review Technique, a project management tool.

DSS: Decision Support Systems.

CMP: Classical Network Flows.

MF: Network Flow.

ANALOG COMPUTERS: Analog Computers.

-CPA: Critical Path Analysis, a project management technique.

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#### INTRODUCTION

The present dissertation is dedicated to the exploration of motivation and its integration into decision-making systems. This study is highly relevant, especially in the context of modern information technologies. Throughout their development, it has become evident that the role of the subjective factor not only does not decrease as initially expected but, in certain application areas, it even begins to grow.

Simultaneously, the role of the science studying human behavior—psychology—has also increased. Another notable observation is the gradual convergence of precise mathematical models, created in an attempt to adequately describe real objects and processes, on one hand, and the non-formalized descriptions of these processes within the framework of psychology, on the other. This leads to qualitative changes in perspectives on how decision-making processes unfold.

#### **Relevance of the research**

There arises a necessity for the combination of psychological and mathematical models, aiming to enhance the effectiveness of integrated decision-making methods compared to their separate use. This, in turn, leads to the emergence of new research tasks in decision-making processes.

The topic of human motivation occupies an increasingly significant place in the modern complex and changing economic environment. Methods, techniques, and means are actively sought and applied to ensure a qualified, highly productive, and loyal workforce capable of achieving the organization's goals in the medium and long term. The emphasis is placed on the need for people to be stimulated or motivated, which, in turn, dictates specific directions in human behavior.

The dissertation pays special attention to decision-making systems. A formal description of discrete decision-making systems, taking motivation into account, has been provided. Concepts of sets, graphs, and network flows, which serve as tools for building mathematical models, have been discussed.

Based on the analysis conducted, it has been observed that discrete decision-making systems based on network flows allow for a relatively precise and adequate representation of motivational behavior in these systems.

# Study setting

Broadly, the main focus of this dissertation research is the influence of the subjective factor in decision-making systems, particularly the role of motivation in human-machine decisionmaking systems.

The subject of scientific inquiry is the possibility of constructing a human-machine decision-making system that takes motivation into account.

# **Goals and tasks**

The aim of the dissertation is to investigate the impact of the subjective factor, particularly motivation, in decision-making systems. To achieve the postulated goal, the following tasks have been defined:

- **4** To analyze the types of motivation and fundamental motivational models.
- **4** To analyze decision support systems.
- **4** To analyze discrete decision-making systems with consideration of motivation.
- To implement a numerical example of a discrete decision-making system considering motivation.

#### Hypothesis

The motivation as a crucial factor in the everyday necessity of making various decisions and contemporary decision-making software systems can be integrated into a computerized decision-making system or decision support system with consideration of motivation.

#### Methodology

The methodology used to achieve the research goal is structured around a review and analysis of motivation and decision-making systems, followed by the development of a practical-oriented mathematical model. This includes the creation, development, and evaluation of a specific computerized decision-making system.

The proposed dissertation consists of an introduction, four chapters, conclusion, bibliography, and author's reference. In the introduction, the relevance of the topic, the aim and objectives of the research work, the hypothesis, and the methodology used are discussed.

The first chapter provides a review of some theoretical elements of motivation, covering the essence of motivation, types of motivation, the motivational process, and factors influencing motivation. Content and process theories, as well as the role of aggression in motivation, are examined.

The second chapter formally describes discrete decision-making systems with consideration of motivation. It explores the types of processes in the context of decision-making, mathematical models, and the classification of mathematical models and structured systems. The problems of modeling different levels of decision-making are also addressed.

The third chapter presents a formal description of discrete decision-making systems with consideration of motivation. Concepts such as sets, graphs, and actions on them are discussed. Classical network flows and generalized network flows are described, along with the characteristics of decision-making systems based on network-flow models.

In the fourth chapter, a numerical example of a discrete decision-making system with consideration of motivation is developed. The results obtained in this chapter allow for the incorporation of motivation into a discrete decision-making system through a generalized network approach, demonstrating the usefulness of such an approach. It provides a bridge between psychological processes (particularly motivation) and models of discrete decision-making systems.

The conclusion presents a summary of the achieved results and the main contributions of the dissertation. A list of scientific publications on the topic and relevant citations is provided. The dissertation comprises 156 pages and includes references to 151 literary sources.

#### **CHAPTER 1: THEORETICAL ASPECT OF MOTIVATION**

Motivation is the primary driving force that propels individuals towards achieving their goals and plays a key role in decision-making systems. In addition to classical theories, contemporary perspectives such as self-determination theory underscore the significance of internal motivation, emphasizing autonomy, competence, and relatedness as crucial factors. Subjective factors such as emotions, cognitive biases, and personal values intertwine with motivation, creating a rich tapestry of influences on decision-making. For instance, emotions can impact choices, with fear leading to reluctance to take risks and excitement prompting adventurous decisions. Recognizing these subjective elements elucidates the complexity inherent in decision-making systems, highlighting the intricate dance between internal motivations and the choices individuals ultimately make.<sup>1</sup>

#### 1.1. The essence of motivation

The nature of motivation deepens into its fundamental mechanisms, emphasizing its complexity and the various theories attempting to explain its functioning. From an evolutionary perspective, motivation is rooted in survival instincts, such as the drive for food, shelter, and reproduction, shaping behavior that ensures species continuation. From a psychological standpoint, theories like the drive-reduction theory argue that motivation arises from the necessity to reduce internal tensions or drives, propelling individuals toward homeostasis.<sup>2</sup>

Furthermore, Bandura's Self-Efficacy Theory emphasizes individuals' beliefs in their abilities to achieve desired results, while the Expectancy Theory underscores the importance of expectations and valence in determining motivational levels - the belief that efforts lead to performance, and performance leads to desired outcomes.<sup>3</sup> Each of these theories contributes a different perspective through which the intricacies of motivation can be understood.

The nature of motivation is intertwined with emotions, cognition, and personality, weaving through these psychological aspects in decision-making processes. Emotions can serve as both motivators and factors influencing motivational states, stimulating actions or altering the perceived value of goals. Cognitive factors, such as perceptions, beliefs, and attitudes, shape the interpretation of motivational signals, guiding individuals' choices. Additionally, personal characteristics, like openness to experience or conscientiousness, interact with motivation, influencing goal-setting, persistence, and task performance.

The dynamic nature of motivation involves its susceptibility to changes, adaptation, and the impact of external factors. Social and cultural factors significantly contribute by shaping motivational frameworks through the reinforcement of specific values, norms, or expectations that shape individuals' aspirations and endeavors.

Understanding the multifaceted nature of motivation requires recognizing its integration into a network of psychological, social, and environmental factors, providing a comprehensive view to assess the driving forces behind human behavior and decision-making systems.<sup>4</sup>

<sup>&</sup>lt;sup>1</sup> Markovitz, H. Portfolioselection // The J. of Finance. 1952. Vol. VII, № 16. P. 60–91.

<sup>&</sup>lt;sup>2</sup> Locke, E. A., & Latham, G. P. (2002). Building a practically useful theory of goal setting and task motivation: A 35-year odyssey. American psychologist, 57(9), 705.

<sup>&</sup>lt;sup>3</sup> Axelrod, R. The Structure of Decision: Cognitive Maps of Political Elites. Princeton: University Press, 1976.

<sup>&</sup>lt;sup>4</sup> Ariely, D. (2008). "Predictably Irrational: The Hidden Forces That Shape Our Decisions." HarperCollins.

### 1.2. Types of motivation

Motivation encompasses a spectrum of types, each characterized by different driving forces and consequences for behavior. Intrinsic motivation arises from internal desires, personal enjoyment, or inherent satisfaction derived from the activity itself. This type of motivation fuels creativity, passion, and sustained commitment, as individuals perform tasks for the joy or satisfaction they experience.

On the other hand, extrinsic motivation stems from external factors such as rewards, recognition, or avoidance of punishment. It can take various forms, including external material rewards (such as money or prizes) and external non-material rewards (such as praise or social status).Both types play a crucial role in decision-making and goal pursuit, influencing the direction and intensity of actions.<sup>5</sup>

According to the self-determination theory, there are various levels of internalization of motivation. These levels range from external regulation (the least autonomous, driven by external rewards or pressure) to introjected regulation (where actions are somewhat driven by internal pressure, such as guilt or ego), identified regulation (when tasks align with personal values), and finally, intrinsic regulation (the most autonomous, when activities are inherently enjoyable and align with a person's identity).<sup>6</sup>

The complex interaction between these types of motivation shapes behavior, choices, and the persistence of individuals in pursuing their goals. Effectively balancing and utilizing these motivations can be crucial for cultivating sustainable commitment, fostering internal satisfaction, and achieving meaningful results.<sup>7</sup>

#### 1.3. Motivational processes

The motivational process is a dynamic sequence of events and cognitive-emotional states that drive individuals towards specific goals or outcomes. It involves several interconnected stages that elucidate the path from needs or desires to actions and eventual results.

**Identification of needs or desires**: The process often begins with the recognition or emergence of a need or desire. This can stem from internal signals (such as hunger or curiosity) or external cues (such as societal expectations or opportunities).

**Setting goals:** Once the need or desire is recognized, individuals often set goals to satisfy these needs. These goals can be short-term or long-term, realistic or ambitious, and play a crucial role in guiding behavior.

**Evaluation and selection of strategies**: Individuals assess different strategies or courses of action to achieve their goals. They evaluate feasibility, potential outcomes, and alignment with personal capabilities before choosing the most appropriate path.<sup>8</sup>

**Initiation and Persistence:** The motivational process involves initiating actions aligned with the chosen strategy and maintaining persistence despite obstacles or failures. This perseverance is influenced by factors such as perceived self-efficacy, internal interest, and the perceived value of the goal.<sup>9</sup>

Adaptation and Adjustment: As individuals progress towards their goals, they constantly monitor their progress and accordingly adjust their strategies. This phase involves adapting to changing circumstances, learning from experience, and modifying approaches as needed.

Achievement or Modification of Goals: The process concludes with the attainment of goals,

<sup>&</sup>lt;sup>5</sup> Boulding, W., Kalra, A., Staelin, R., & Zeithaml, V. A. (1993). A dynamic process model of service quality: From expectations to behavioral intentions. Journal of marketing re-search, 30(1), 7-27.

<sup>&</sup>lt;sup>6</sup> Baumeister, R. F., & Vohs, K. D. (Eds.). (2004). "Drive: Psychology of Human Motivation." Psychology Press.

<sup>&</sup>lt;sup>7</sup> Marakas, G. M. (1999). "Decision Support Systems: Concepts and Resources for Managers." Pearson

<sup>&</sup>lt;sup>8</sup> Monden, Y. (1983). Toyota Production System: Practical Approach to Production Management. Industrial Engineering and Management Press.

<sup>&</sup>lt;sup>9</sup> Carayon, P., Hoonakker, P., & Wetterneck, T. B. (2015). "Motivational Decision Support: Tailoring Interventions to Improve Decision-Making Outcomes." Ergonomics, 58(4), 568–583.

redefining goals, or recognizing that the initial goals need to be adjusted based on changing needs or circumstances.

Throughout this process, various factors - both internal and external - influence motivation. Internal factors encompass emotions, beliefs, perceptions, and personality traits, while external factors include social influences, signals from the environment, and the availability of resources or support.

Feedback Loops and Iterations: As individuals advance through the stages, they encounter feedback from their actions and the environment. This feedback loops back into the motivational process, influencing subsequent decisions and behavior. Positive feedback, such as goal achievement or encouragement, often reinforces motivation. Conversely, negative feedback, such as failures or criticism, may prompt individuals to reassess their strategies or goals, leading to adjustments in their motivational approach.<sup>10</sup>

**Influence of Emotions and Cognition:** Emotions play a decisive role at every stage of the motivational process. They can act as powerful motivators, shaping preferences and decisions. Cognitive factors such as perception, attention, and memory also influence how people interpret motivational signals, guiding their choices and strategies.<sup>11</sup>

**Social and Cultural Context:** The motivational process is significantly influenced by social interactions and cultural norms. Social support, peer influence, and societal expectations can either strengthen or diminish individual motivation. Cultural values and beliefs shape individuals' aspirations, determining what is considered valuable or desirable, subsequently influencing the setting of goals and the strategies chosen to achieve them.

**Personal Development and Self-regulation**: Over time, individuals may undergo changes in their motivational orientations and regulatory styles. Through self-reflection and selfregulation, individuals can adapt their motivational strategies, develop new skills, and refine goal-setting processes. This ongoing development contributes to the complexity and flexibility of the motivational process.

# 1.4. Factors influencing motivation

Numerous factors intricately influence an individual's motivation, shaping their aspirations, persistence, and commitment in pursuing goals. These factors can be categorized as internal, external, and contextual influences:

# **1. Internal Factors:**

Needs and Desires: Basic physiological needs (such as hunger or thirst) and higherorder needs (such as autonomy, competence, and relatedness) identified in theories like Maslow's hierarchy or self-determination theory deeply impact motivation.

Interests and Passions: Personal interests, hobbies, and passions often stimulate internal motivation, driving individuals to engage in activities for the inherent satisfaction they provide.

Beliefs and Values: Individual beliefs, values, and personal aspirations significantly influence motivation, directing individuals towards goals aligned with their principles.

#### **2. External Factors:**

**Rewards and Incentives**: External rewards like money, recognition, or praise can influence motivation, either reinforcing or diminishing internal drive depending on their perceived impact.

**Punishments or Threats**: Fear of negative consequences or punishments can act as motivators, albeit often with limited and short-term effectiveness.

**Social Influence**: Peer pressure, societal norms, and societal expectations can exert a strong influence on motivation, shaping goals and behavior to align with accepted

<sup>&</sup>lt;sup>10</sup> Csikszentmihalyi, M. (1990). "Flow: The Psychology of Optimal Experience." Harper & Row.

<sup>&</sup>lt;sup>11</sup> Dweck, C. S. (2006). "Mindset: The New Psychology of Success." Random House.

standards.12

### 3. Contextual factors:

**Environment and Resources**: Access to resources, a supportive environment, and appropriate tools can significantly influence motivation by facilitating goal pursuit and reducing obstacles.

**Culture and Society**: Cultural values, societal norms, and cultural expectations shape people's aspirations, influencing the nature and direction of motivation. The cultural context plays a crucial role in determining what individuals find motivating.

**Feedback and Support:** Constructive feedback, encouragement, and social support can enhance motivation by providing validation and guidance for achieving goals. Positive interactions and a supportive social network contribute to a motivational environment.

## 4. Psychological factors:

**Self-efficacy**: Believing in one's own abilities to perform tasks strongly influences motivation. Higher self-efficacy often leads to increased motivation and persistence.

**Perception**: How individuals perceive situations and attribute success or failure affects motivation. Positive attributions tend to enhance motivation, while negative attributions can hinder it.

The interplay of these multifaceted factors results in a complex network of influences that impact motivation differently for each individual and in various contexts. Recognizing and understanding these factors can guide efforts to enhance motivation, facilitate goal setting, and create an environment conducive to sustained engagement and achievement.<sup>13</sup>

# 2.1. Content theories of motivation

The content theories of motivation, also known as need-based motivational theories, focus on both internal and external factors that drive individuals toward achieving specific goals and satisfying particular needs, motivating them to take action.

These theories concentrate on identifying specific factors that motivate individuals, while procedural theories delve into the mechanisms and structures that stimulate motivation.

**Maslow's Hierarchy of Needs** remains influential, outlining a structured framework that assumes people prioritize satisfying lower-level needs (physiological and safety) before those at higher levels (such as social belonging, esteem, and self-actualization). However, its hierarchical nature has been criticized for being too rigid and not accounting for individual or cultural differences. In practice, individuals may simultaneously pursue needs at different levels, rather than strictly adhering to a step-by-step progression.

Contemporary research focuses on additional factors shaping motivation, including contextual influences and individual preferences, aiming to more intricately and accurately capture the dynamics of the motivational process..<sup>14</sup>

Alderfer's theory is directed towards addressing certain limitations of Maslow's hierarchy by allowing for the simultaneous satisfaction of multiple needs. The theory acknowledges that dissatisfaction in achieving higher-level needs may lead individuals to revert to lower-level

<sup>&</sup>lt;sup>12</sup> Olson, D. L., & Wu, D. (2017). "Data-Driven Decision Making and Dynamic Systems." Springer

<sup>&</sup>lt;sup>13</sup> Oliver, R. L. (1980). A cognitive model of the antecedents and consequences of satisfaction decisions. Journal of marketing research, 17(4), 460-469.

<sup>&</sup>lt;sup>14</sup> Ryan, R., Deci, E.: Self-determination theory and the facilitation of intrinsic motivation, social development, and wellbeing. American Psychologist, 55(1), 68-78 (2000).

needs. However, it has also faced criticism for lacking empirical support and ambiguity in categorizing needs.

**Herzberg's Two-Factor Theory** provides insights into factors contributing to job satisfaction and dissatisfaction. By distinguishing motivators (leading to satisfaction) and hygiene factors (preventing dissatisfaction but not motivating), Herzberg emphasizes the importance of internal aspects such as recognition and growth opportunities in fostering motivation. Critics argue, however, that the theory oversimplifies the complex work-related attitudes and fails to account for individual differences in the impact of these factors.<sup>15</sup>

**McClelland's Needs Theory** emphasizes the role of specific needs, such as the need for achievement, affiliation, and power, in stimulating behavior. This theory has found applications in areas such as employee motivation and leadership development. However, it has been contested due to its dependency on situational factors and cultural differences in the significance and prioritization of these needs.

**Self-Determination Theory (SDT)** focuses on internal motivation and the importance of satisfying fundamental psychological needs for autonomy, competence, and relatedness. SDT underscores the significance of internal motivation in promoting personal growth and wellbeing, assuming that an environment supporting these needs enhances internal motivation and overall satisfaction.

Each theory contributes a unique perspective to understanding motivation, emphasizing different aspects of human needs and aspirations. However, contemporary research often integrates elements from multiple theories to provide a more comprehensive understanding of motivation, considering the complexity of human behavior and the interplay between internal and external motivators.

#### 2.2. Process motivational theories

Procedural motivation theories focus on how the process of setting and achieving goals, as well as the procedures or systems used, influences motivation. Unlike content theories (such as Maslow's hierarchy or Herzberg's two-factor theory) that concentrate on identifying specific factors that motivate individuals, procedural theories delve into the mechanisms and structures that stimulate motivation. Some key procedural motivation theories include:<sup>16</sup>

**Goal-setting theory**: Popularized by Edwin Locke and Gary Latham, this theory emphasizes the importance of setting clear and challenging goals for motivating individuals. It assumes that specific, measurable, achievable, relevant, and time-bound (SMART) goals enhance motivation and effectiveness by providing a clear direction and a sense of purpose. Additionally, feedback on goal progress is crucial for maintaining motivation.

**Equity theory**: Developed by Stacy Adams, the equity theory focuses on workplace justice. It argues that people evaluate their outcomes (rewards, recognition) in relation to the inputs they invest (effort, contributions) and compare this ratio with that of others. If individuals perceive an imbalance or injustice, whether they are under-rewarded or over-rewarded compared to others, it can lead to changes in motivation levels and behavior to restore perceived fairness.<sup>17</sup>

**Expectancy theory:** Victor Vroom's expectancy theory focuses on the belief that people are motivated by their expectations of the outcomes of their actions. It assumes that three factors influence motivation: expectancy (the belief that efforts will lead to results), instrumentality (the belief that the results will lead to rewards), and valence (the value attached to the expected rewards). Individuals are motivated when they believe that their efforts will lead to desired

<sup>&</sup>lt;sup>15</sup> Pink, D. H. (2009). Drive: The Surprising Truth About What Motivates Us. Penguin.

<sup>&</sup>lt;sup>16</sup> Bargh, J. A., & Gollwitzer, P. M. (1994). "Integrating Motivation into Decision Support Systems: A Theoretical Framework." In J. D. Gould & M. J. Atkinson (Eds.), "Motivation and Cognition: Interactions in Social Behavior" (pp. 263–285). American Psychological Association.

<sup>&</sup>lt;sup>17</sup> Bommel, P., Bruskiewich, R. M., & Gascuel-Odoux, C. (Eds.). (2011). "Decision Support Systems in Agriculture, Food and the Environment: Trends, Applications and Advances." Springer.

outcomes.

**Expectancy theory of motivation**: Developed by Lyman Porter and Edward Lawler, this theory expands and refines Victor Vroom's expectancy theory. Their theory, as an extension of Vroom's model, emphasizes the role of expectancy, instrumentality, and valence in determining an individual's motivation. Porter and Lawler's theory underscores the importance of taking into account individual perceptions, beliefs, and satisfaction levels in understanding and enhancing motivation within organizations.<sup>18</sup>

The reinforcement theory, developed by B.F. Skinner, is based on the principle that behavior is influenced by the consequences that follow it. This theory posits that behavior followed by desirable consequences is more likely to be repeated, whereas behavior followed by undesirable consequences is less likely to be repeated.<sup>19</sup>

The reinforcement theory provides valuable insights into the impact of consequences on behavior and is particularly relevant in the context of personnel management and motivation in organizations.

Self-determination theory is a psychological theory developed by Edward Deci and Richard Ryan. The theory examines the internal motivational forces that influence human motivation and well-being. According to the self-determination theory, satisfying these three psychological needs is essential for internal motivation, optimal development, and psychological well-being. The self-determination theory offers a useful framework for creating innovative and supportive environments, both in educational institutions and organizations. By focusing on internal motivational factors, it emphasizes the importance of meeting individuals' psychological needs to achieve high levels of motivation and well-being.<sup>20</sup>

Procedural motivational theories represent a valuable approach to understanding the influence of processes and systems on individuals' motivation. These theories provide insights into how goal-setting, perceptions of fairness, and expectations for outcomes shape motivation in the work environment.

#### 3.1. Types of motivational models

There are many different models to explain motivation, with each highlighting various aspects and factors that influence how people are motivated. Here are some of the main types of motivational models:

1. Needs Theories: These models, including Maslow's Hierarchy of Needs, focus on the fundamental needs that motivate people. They suggest the idea that individuals strive to satisfy specific levels of needs, such as physiological, safety, belongingness, esteem, and self-actualization.

2. **Procedural Models of Motivation**: These models, such as Vroom's Expectancy Theory and Locke and Latham's Goal-Setting Theory, focus on the processes and mechanisms that lead to motivation and goal achievement. They typically concentrate on the perceived connection between effort, performance, and outcomes, as well as internal expectations and incentives.

**3.** Self-Determination Theories: These models, based on Deci and Ryan's Self-Determination Theory, focus on internal motivational factors such as autonomy, competence, and relatedness. They explore how supporting these internal needs influences motivation, engagement, and well-being.

<sup>&</sup>lt;sup>18</sup> Duckworth, A. (2016). "Grit: The Power of Passion and Perseverance." Scribner.

<sup>&</sup>lt;sup>19</sup> Kahneman, D. (2011). "Thinking, Fast and Slow." Farrar, Straus and Giroux.

<sup>&</sup>lt;sup>20</sup> Deci, E., Koestner, R., Ryan, R.: A meta-analytic review of experiments examining the effects of extrinsic rewards on intrinsic motivation. Psychological Bulletin, 125(6), 627–668 (1999).

4. Reinforcement and Punishment Theories: These models, including Skinner's Reinforcement Theory, examine how various types of reinforcements (positive, negative) and punishments can influence desired or undesired behavioral patterns.

5. Socio-Cultural Models: These models focus on how the social environment, culture, and social interactions influence motivation. They explore how society, social expectations, and interpersonal interactions shape motivational forces.

Each of these models presents a unique perspective on motivation and can be applied in various domains such as education, work, sports, and personal development. Combining and utilizing these diverse models can provide a more comprehensive understanding of how people are motivated.<sup>21</sup>

The model of the economically rational individual is part of economic theory, assuming that people always act in a way that maximizes their personal interest or benefit. This model assumes that individuals are rational and make decisions by comparing different options, choosing the one that provides the greatest benefit or satisfaction.

The model of the socially oriented individual relates to the idea that people are strongly influenced by social factors, values, ideas, and norms in the decision-making process. In this model, individuals are not only seen as striving to maximize their personal benefit but also as social beings embedded in society and oriented towards social influences.

**The self-actualization model** is a theoretical approach that focuses on the continuous process of personal development and growth of the individual. This model is based on the idea that people have an internal inclination towards development, improvement, and achieving their full potential.<sup>22</sup>

When we talk about **a complex model** in the field of motivation, it often refers to the integration of various theories and approaches to explain motivation. This approach utilizes a combination of different theories and models to elucidate the diversity and complexity of human motivation. The complex model may include elements from various motivation theories such as Maslow's Hierarchy of Needs, Vroom's Expectancy Theory, Deci's Self-Determination Theory, and others. The goal is to offer a broader and more detailed understanding of how motivation is formed, sustained, and controlled in individuals.

The Japanese model of motivation is often associated with specific aspects of Japanese corporate culture and management. This model has unique characteristics that differ from Western models and are based on traditional Japanese values and principles.

The Japanese model of motivation combines traditional Japanese values with contemporary management and motivation strategies. One of its key features is the emphasis on collectivism and group responsibility. Instead of individual achievement, Japanese workers are encouraged to think and act in the interest of the entire group or organization. This aspect of motivation underscores interdependence and the importance of successful teamwork.<sup>23</sup>

The long-term approach to motivation is also a key feature of Japanese corporate culture. Companies invest in the training and development of their employees, supporting them in building a long-term career within the organization. This creates greater stability and loyalty among the personnel.

**Aggression** plays a complex and multi-faceted role in the motivational process of an individual. Although aggression is often associated with negative connotations, it can be an important factor in various contexts and scenarios.

It is important to note that aggression is not always a constructive motivational factor, and managing and understanding it are key to maintaining healthy and productive relationships both personally and professionally.

Aggression, despite its negative connotations, can be seen as a complex mechanism that

<sup>&</sup>lt;sup>21</sup> Deckers, L. (2014). "Motivation: Biological, Psychological, and Environmental." Routledge.

<sup>&</sup>lt;sup>22</sup> Heckhausen, J., & Heckhausen, H. (2010). "Motivation and action." Cambridge University Press.

<sup>&</sup>lt;sup>23</sup> Porter, L., Lawler, E.: Managerial Attitudes and Performance. Homewood, IL: Dorsey Press (1968).

acts as a response to various internal and external factors. Understanding its role in the motivational process provides an opportunity to analyze human behavior in different contexts.

In the realm of self-defense, aggression may emerge as a result of instinctive reactions when sensing a threat. This is a natural response of the body to maintain physical and emotional safety. Recognizing this aspect of aggression allows for a better understanding of what motivates people to defend themselves in different situations.<sup>24</sup>

In the context of social interactions, aggression can be used as a means of asserting social position. In social groups, conflicts over resources, power, and recognition can lead to aggressive reactions, serving as a mechanism to regulate social dynamics.

Additionally, aggression can be employed as an expression of the desire to achieve goals. In a competitive environment where resources are limited, aggressive behavior may be stimulated by the desire to overcome competitors and achieve success. In this context, aggression can be seen as a means of highlighting personal competence.

However, it is important to note that uncontrolled and destructive aggression can have serious negative consequences for both the individual and society as a whole. Managing aggression, understanding the context, and identifying the reasons behind it are crucial elements in building healthy and productive interpersonal relationships.

# 3.2. Conclusions of the first chapter

The summary of the role of motivation in human behavior reveals a multitude of complex and interconnected aspects. From theoretical models of motivation, such as Maslow's hierarchy of needs, through social and economic models, to the Japanese model and the role of aggression, all contribute diversity to understanding what drives and influences people.

The overall picture emphasizes that motivation is a dynamic phenomenon, interrelated with internal and external factors, cultural influences, and social contexts. Understanding this diverse nature can serve as a foundation for creating more comprehensive and integrated strategies in various areas of life - from personnel management to education and interpersonal interactions.

<sup>&</sup>lt;sup>24</sup> Locke, E. A., & Latham, G. P. (2004). Motivated cognition: Effects of reward and emotion on cognition and action. Handbook of motivation and cognition, 1, 509-549

# CHAPTER 2: THEORETICAL ASPECT OF THE THEORY FOR DECISION MAKING An overview of decision theory

In essence, decision theory deals with the intricacies of human cognition and behavior, as well as the dynamics of interactions among rational agents. It acknowledges the inherent challenges faced by individuals and organizations when navigating decision spaces filled with uncertainty, incomplete information, and conflicting goals.

The theory of expected utility, a foundational concept in decision theory, posits that decision-makers seek to maximize the expected satisfaction or gain, considering both the probabilities and the utility associated with different outcomes. Bayesian decision theory extends this framework by incorporating iterative updating of beliefs in response to new information, acknowledging the evolving nature of decision contexts. Game theory, on the other hand, explores strategic interactions where the choice of one decision-maker affects the outcomes for others, emphasizing the interdependence inherent in many decision scenarios.

In this context, the theoretical exploration of decision theory not only provides analytical tools but also contributes to a deeper understanding of the cognitive, psychological, and social dimensions that shape our choices.<sup>25</sup>

The decision-making theory is also closely linked to the concept of rational decisionmaking, where individuals or organizations strive to make optimal decisions based on available information and the goals they pursue.<sup>26</sup> The rational approach involves using logical steps, assessing risks, and predicting the consequences of the decisions made. Additionally, decision theory contributes to the study of aspects such as ethical dilemmas, social impacts, and the influence of cultural factors on the decision-making process. This integrative approach further emphasizes the importance of the context in which decision-making occurs and encourages a broader understanding of the interaction between individuals and their decisions in various areas of life.<sup>27</sup>

The decision-making theory also explores concepts such as "systematic efficient decisions" and "bounded rational decision-making." In the first case, the theory examines how decisions can be structured and optimized within certain systematic methods, while in the second case, it acknowledges the limitations of resources and information available to decision-makers. This approach recognizes that people often resort to optimization strategies that are applicable under conditions of constraints and uncertainty.

#### 2.1. Concepts related to decision making

In this chapter, we will explore fundamental concepts related to decision-making that serve as building blocks for further examination of the theoretical aspects of decision theory.

First and foremost, it is important to understand the concept of the "principle of expected utility," which assumes that decision-makers seek to maximize their expected satisfaction or gain, taking into account the probabilities and benefits of different outcomes. Additionally, we will acquaint ourselves with the "theory of probabilities," providing a formal model for measuring uncertainty and the likelihood of various events. Complementarily, the "Bayesian decision theory" provides a framework for updating beliefs in light of new information, which

<sup>&</sup>lt;sup>25</sup> Burchard, B. (2014). "The Motivation Manifesto." Hay House.

<sup>&</sup>lt;sup>26</sup>. Checland P. B. Models Validation in Soft Systems Practice // System Research. 1995. Vol. 12, № 1. P. 47–54.

<sup>&</sup>lt;sup>27</sup> Burchard, B. (2014). "The Motivation Manifesto." Hay House.

is crucial for dealing with changing situations. With these concepts as a starting point, we delve into a deeper exploration of the theoretical aspects of decision theory.<sup>28</sup>

Additionally, we will also become acquainted with the "game theory," which represents a powerful tool for analyzing strategic interactions between different agents.

In addition to analyzing these theoretical concepts, an overview of their applications in various fields such as economics, business, and social sciences has been provided. Alongside the presented concepts, attention has been given to the importance of "learning from experience" in the decision-making process. This aspect underscores the need for continuous development and adaptation to the dynamics of circumstances, aiming for more effective decision-making in different situations.

The relationship between decision theory and innovation is also emphasized. In the face of rapid technological changes and the constant development of society, making innovative and successful decisions requires the integration of theoretical knowledge with creativity and adaptability.

# 2.2. Certainty of the results of the decisions made

Security of the outcomes resulting from decision-making is a crucial aspect in the field of decision theory. In this chapter, we will familiarize ourselves with concepts and methods aimed at risk management and ensuring reliable results in decision-making. We will explore risk assessment models that involve probability distributions and statistical methods, with the aim of predicting and evaluating the likelihood of various outcomes.<sup>29</sup>

Another important aspect of result security is related to "information perception." The main question here is how decision systems gather, process, and interpret information to form the basis for decision-making. Techniques for information management will be explored, including data filtering, statistical analysis, and machine learning, aiming to improve the quality and reliability of the information foundation.<sup>30</sup>

Finally, the "moral and ethical aspects" on result security will be discussed. Decisionmaking is not only a technical process but also a matter of values and ethical principles. Emphasizing the importance of result security, we also need to examine "uncertainty management." Decision-making often occurs in contexts where uncertainty and unpredictability are inevitable. We will explore methods and models for managing this kind of uncertainty, focusing on how decision systems can adapt their strategies and processes to cope with changing conditions and factors.

Additionally, we will touch upon the "psychological aspects" of decision-making and their role in result security. The human factor, including emotions, cognitive biases, and personal preferences, can influence the decision-making process and lead to distortions in risk assessment and opportunities.

In the context of information security, we will discuss the "technological aspects" of decision-making. With the crucial role of technology in modern decision-making dynamics, the introduction of innovations such as automation, artificial intelligence, and big data analytics can play a key role in enhancing the security and efficiency of the process.<sup>31</sup>

In the conceptual scope of "information perception," we emphasize the importance of techniques such as data filtering, statistical analysis, and machine learning. These methods not only improve the quality and reliability of information but also create a foundation for more secure and well-founded decisions. Finally, we focus on the "moral and ethical aspects"

<sup>&</sup>lt;sup>28</sup> Carver, C. S., & Scheier, M. F. (2001). "On the Self-Regulation of Behavior." Cambridge University Press.

<sup>&</sup>lt;sup>29</sup> Checland P. B. Models Validation in Soft Systems Practice // System

Research. 1995. Vol. 12, № 1. P. 47–54.

<sup>&</sup>lt;sup>30</sup> Cialdini, R. B. (1984). "Influence: The Psychology of Persuasion." HarperCollins.

<sup>&</sup>lt;sup>31</sup> Churchman C. W. The system approach and its enemies. N. Y. : Basic books, 1979.

concerning the security of outcomes. The ethical principles of decision-makers and organizations can significantly contribute to forming more secure and sustainable decisions. We will explore how ethical considerations can be integrated into decision-making strategies and how they can serve as a foundation for ensuring the security of results.<sup>32</sup>

Within the discussion of result security, it is important to consider "communication and information exchange" between different agents in the decision-making process. Effective communication plays a key role in preventing misunderstandings and ensuring the correct understanding of the context and goals of decision-making.

Additionally, we explore "creativity and innovation" in the context of result security. Innovations and a creative approach can contribute to the formation of safer and more successful decisions.

# 2.3. Decision evaluation criteria

In the realm of decision-making, establishing reliable "evaluation criteria" is of paramount importance for ensuring effective and informed choices. Economic criteria serve as a fundamental pillar, encompassing the assessment of costs, benefits, and overall financial implications associated with a given decision.

Moving to "strategic criteria," decisions are evaluated based on their alignment with broader organizational goals and objectives.

Social criteria take into account the ethical and societal dimensions of decisions. In an era where corporate social responsibility is increasingly emphasized, decision-makers assess the potential social impact, ethical consequences, and overall responsibility of the decision to the broader community. Balancing economic goals with social and ethical considerations is imperative for sustainable and responsible decision-making.<sup>33</sup>

Environmental criteria have gained significance due to the growing ecological awareness. Evaluating decisions through the lens of the environment involves considering their ecological footprint, resource usage, and potential impact on the environment.

Time-related criteria focus on the temporal aspects of decisions. This includes assessing the time required to implement a given decision and its adaptability to changing circumstances over time. In a dynamic and rapidly evolving environment, decisions that can be efficiently executed and flexibly adapted are often more sustainable and effective.<sup>34</sup>

Finally, technological considerations play a crucial role in decision-making. Evaluating technological feasibility, compatibility, and the potential for leveraging innovations ensures that decisions are not only contemporary but also capable of harnessing the advantages of emerging technologies.<sup>35</sup>

The comprehensive evaluation of decisions involves a detailed analysis of economic, strategic, social, environmental, temporal, and technological criteria. This multifaceted approach enables decision-makers to navigate the complex landscape of contemporary decision-making, encouraging choices that are not only effective in the short term but also sustainable and adaptive in the long run.<sup>36</sup>

<sup>&</sup>lt;sup>32</sup> Hackman, J., Oldham, G.: Motivation through the design of work: Test of a theory. Organizational behavior and human performance, 16(2), 250-279 (1976).

<sup>&</sup>lt;sup>33</sup> Grant, A. M. (2008). The significance of task significance: Job performance effects, relational mechanisms, and boundary conditions. Journal of applied psychology, 93(1), 108.

<sup>&</sup>lt;sup>34</sup> Maehr, M. L., & Zusho, A. (2009). Achievement goal theory: The past, present, and future.

<sup>&</sup>lt;sup>35</sup> Hackman, J., Oldham, G.: Motivation through the design of work: Test of a theory. Organizational behavior and human performance, 16(2), 250-279 (1976).

<sup>&</sup>lt;sup>36</sup> Ouchi, W. (1981). Theory Z: How American Business Can Meet the Japanese Challenge. Addison-Wesley.

### 2.4. Decision support systems

Decision Support Systems (DSS) represent a crucial component of the toolkit for contemporary decision-makers, providing sophisticated tools and frameworks to enhance the decision-making process. The Decision Support System integrates data, analytical models, and user-friendly interfaces to facilitate well-informed and effective decision-making across various domains.

One of the key aspects of DSS is its ability to aggregate and process vast amounts of data from diverse sources. The integration of artificial intelligence and machine learning algorithms further enhances the system's capability to recognize patterns, trends, and anomalies within complex datasets.

In addition to data processing, DSS often includes simulation and modeling capabilities. Decision-makers can simulate various scenarios, exploring potential outcomes of different decisions in a risk-free environment.

Furthermore, DSS frequently incorporates collaboration features, allowing multiple stakeholders to contribute to the decision-making process. This encourages transparency, collective intelligence, and the alignment of diverse perspectives.<sup>37</sup>

The evolution of Decision Support Systems (DSS) continues to be shaped by emerging technologies, with a particular focus on the integration of Artificial Intelligence (AI) and Machine Learning (ML). Advanced algorithms enable DSS not only to analyze historical data but also to predict future trends and outcomes. This predictive capability allows decision-makers to proactively respond to potential challenges and opportunities, providing a future-oriented dimension to the decision-making process.<sup>38</sup>

Ethical considerations become an integral part of designing and implementing decision support systems. Decision support systems continue to evolve in response to technological advancements, user needs, and ethical considerations. The integration of artificial intelligence, big data, user personalization, and ethical practices in the field of artificial intelligence transforms decision support systems into indispensable tools for navigating the complexity of decision-making in an increasingly dynamic, data-driven world. <sup>39</sup>

# 2.5. Математическо моделиране при вземане на решения

Mathematical modeling is a challenge in the field of decision-making, providing a systematic and quantitative framework for analyzing complex situations, predicting outcomes, and optimizing choices. This approach uses mathematical structures and techniques to represent real-world scenarios, aiding decision-makers in understanding, evaluating, and ultimately making informed decisions.<sup>40</sup>

Probability and statistics play a crucial role in mathematical modeling, especially when dealing with uncertainty and risk. Decision-makers can use probabilistic models to quantitatively determine uncertainty, assess risks, and make decisions that account for the probability of different outcomes. Bayesian statistics and Monte Carlo simulations are examples of probability modeling techniques widely used in decision analysis.

Decision trees and game theory represent approaches to mathematical modeling that are particularly effective in making strategic decisions. Decision trees provide a graphical

<sup>&</sup>lt;sup>37</sup> Ryan, R. M., & Deci, E. L. (2000). Intrinsic and extrinsic motivations: Classic definitions and new directions. Contemporary educational psychology, 25(1), 54-67.

<sup>&</sup>lt;sup>38</sup> Roeder, T. M., Sniezek, J. A., & Tomaka, P. J. (2002). "Incorporating Motivation into Decision Models." Organizational Behavior and Human Decision Processes, 88(2), 554–571.

<sup>&</sup>lt;sup>39</sup> Ryan, R. M., & Deci, E. L. (2000). Intrinsic and extrinsic motivations: Classic definitions and new directions. Contemporary educational psychology, 25(1), 54-67.

<sup>&</sup>lt;sup>40</sup> Roeder, T. M., Sniezek, J. A., & Tomaka, P. J. (2002). "Incorporating Motivation into Decision Models." Organizational Behavior and Human Decision Processes, 88(2), 554–571

representation of decision-making options and potential outcomes, aiding decision-makers in visualizing different scenarios. <sup>41</sup>

The game theory, on the other hand, is used when decisions involve interactions with other decision-makers, helping analyze strategic interactions and make optimal decisions in competitive or cooperative situations.

Time series analysis and forecasting models are crucial in situations where decisions need to account for trends and patterns over time. These models, including methods like Autoregressive Integrated Moving Average (ARIMA) and exponential smoothing, help decision-makers forecast future values based on historical data. Such forecasting is essential for decision-making that considers the development of variables over time.

Machine learning algorithms, a subset of mathematical modeling, have gained significance in decision-making processes, especially with the increasing availability of large datasets.<sup>42</sup>

Mathematical modeling provides decision-makers with a structured approach to scenario analysis. An important aspect of mathematical modeling in decision-making is its adaptability to different domains. This flexibility allows decision-makers to use mathematical techniques in various contexts by tailoring models to the specific characteristics and requirements of the decision-making problem. <sup>43</sup>

With the advancement of technologies, the integration of artificial intelligence (AI) and machine learning (ML) further enhances the capabilities of mathematical models. Mathematical modeling remains a fundamental tool in decision-making, offering a systematic and quantitative approach to addressing complex challenges. Its ability to handle different decision-making contexts, facilitate scenario analysis, integrate with technological systems, and adapt to ethical considerations presents mathematical modeling as a challenge for informed and strategic decision-making in various fields.<sup>44</sup>

In summary, mathematical modeling in decision-making provides a systematic and rigorous framework for analyzing complex problems related to decision-making, incorporating various elements such as uncertainty, preferences, and constraints. These models enable decision-makers to make informed choices based on quantitative analysis and optimization techniques.

#### 2.5.1. Static and dynamic models

Static models, also known as steady-state models, reflect the relationships between variables at a specific point in time, assuming that the system does not change over time. These models are particularly useful when decision-making involves an instantaneous snapshot, and the relationships between variables remain constant.

On the other hand, dynamic models account for the development of the system over time, capturing changes and trends in variables. These models incorporate the time element, allowing decision-makers to analyze how variables interact and influence each other over different periods. Dynamic models are crucial in decision-making situations where the temporal aspect plays a significant role, such as forecasting future trends, understanding the impact of policy changes over time, or optimizing resource allocation in a changing environment.<sup>45</sup>

<sup>&</sup>lt;sup>41</sup> Shingo, S. (1988). Non-Stock Production: The Shingo System of Continuous Improvement. Productivity Press.

<sup>&</sup>lt;sup>42</sup> Vallerand, R. J. (1997). Toward a hierarchical model of intrinsic and extrinsic mo-tivation. Advances in experimental social psychology, 29, 271-360.

<sup>&</sup>lt;sup>43</sup> Янкулов Я, Забунов. Мениджмънт С. 1997 г.

<sup>&</sup>lt;sup>44</sup> Барталев С. А. Информационная система дистанционного монито-

ринга лесных пожаров Федерального агентства лесного хозяйства РФ (состояние и перспективы развития) // Современные проблемы дистанционного зондирования Земли из космоса. М.: Ин-т косм. исслед. РАН, 2008.Т. 5, № 11. С. 419–429.

<sup>&</sup>lt;sup>45</sup> Vallerand, R. J. (1997). Toward a hierarchical model of intrinsic and extrinsic mo-tivation. Advances in experimental social psychology, 29, 271-360.

Static and dynamic models each have their strengths and limitations. Static models are often simpler to apply and analyze, providing a quick overview of optimal solutions for well-defined problems. The choice between static and dynamic models depends on the nature of the decision-making problem. In situations where decision variables and relationships are relatively stable, a static model may be sufficient for effective analysis and decision-making.

On the other hand, when the decision context involves dynamic changes, trends, or feedback loops, dynamic models offer a more realistic representation of the system's dynamics. They allow decision-makers to account for temporal dependencies and make more informed choices over time.<sup>46</sup>

The interaction between static and dynamic models provides decision-makers with a versatile toolkit for solving a wide range of decision-making problems. The choice between these models depends on the nature of the decision context, emphasizing the need for a thoughtful and context-specific approach to mathematical modeling in decision-making.<sup>47</sup>

 $\Box$  Static Models: Static models represent decision-making problems where the decision is made at a single moment, and its consequences are immediately considered.

 $\Box$  Dynamic Models: Dynamic models involve decisions made over several time periods, and the effects of these decisions are considered over time.

 $\Box$  Hybrid Models: In some cases, a combination of static and dynamic modeling is used to solve decision-making problems that have both immediate and long-term consequences.

Understanding the difference between static and dynamic models is crucial for choosing an appropriate modeling approach based on the nature and requirements of the decision-making problem.

# 2.5.2. Structured models

Structured models represent a category of mathematical models characterized by a welldefined and organized framework for presenting decision-making problems. These models are particularly valuable when the decision-making process involves clear and standardized relationships between variables, allowing decision-makers to systematically analyze and optimize solutions.

One well-known type of structured model is the deterministic model, where the relationships between variables are precisely defined, and there is no randomness or uncertainty. Deterministic models are useful when decision-makers seek to identify optimal solutions under precisely defined conditions.

Another subtype of structured models includes probabilistic or stochastic models, which acknowledge the presence of uncertainty in decision problems. In these models, variables are influenced by probability distributions, allowing decision-makers to account for variability and risk. Probabilistic models are valuable when decisions are made in an environment where outcomes are subject to randomness, such as in financial forecasting or project management under uncertain conditions.

<sup>&</sup>lt;sup>46</sup> Беляев А. И., Коровин Г. Н., Лупян Е. А. Использование спутниковых данных в системе дистанционного мониторинга лесных пожаров МПР РФ // Современные проблемы дистанционного зондирования Земли из космоса: Физические основы, методы и технологии мониторинга окружающей среды, потенциально опасных объектов и явлений: сб. науч. ст. М.: GRANP polygraph, 2005. Т. 1. С. 20–29.

<sup>&</sup>lt;sup>47</sup> Блюмин С. Л., Шуйкова И. А. Модели и методы принятия решений в условиях неопределенности. Липецк: ЛЭГИ, 2001. 138 с.

Structured models also encompass optimization models, which aim to find the best possible solution from a set of feasible alternatives.

Decision trees represent another approach to structured modeling, especially useful in scenarios involving sequential decision-making and uncertainty. Structured models often use mathematical programming languages and software tools to facilitate their formulation and solution. These languages, such as AMPL (A Mathematical Programming Language) or GAMS (General Algebraic Modeling System), provide standardized syntax for expressing mathematical models and algorithms, optimizing the execution and analysis of structured models.<sup>48</sup>

One notable advantage of structured models lies in their ability to distill complex decisionmaking problems into well-defined mathematical formulations. This not only facilitates a clearer understanding of the relationships between variables but also streamlines the process of finding optimal solutions.<sup>49</sup>

Optimization models, a subset of structured models, offer a powerful tool for decisionmakers to determine the most favorable course of action from a set of feasible alternatives. Structural modeling approaches often leverage specialized programming languages and software tools. The use of mathematical programming languages improves the reproducibility and scalability of structured models, making them applicable to a variety of decision-making scenarios.

The adaptability of structured models across different industries and their ability to distill complexity into valuable insights make them invaluable tools in the pursuit of optimal decision-making outcomes. The choice of structured models compared to other modeling approaches depends on the specific characteristics and requirements of the decision-making problem. <sup>50</sup>

Structured models are mathematical representations of decision-making problems characterized by a well-defined and organized format. These models feature a clear and systematic arrangement of components, making them easier to analyze and solve.Видове структурирани модели:

Linear Programming (LP): A type of structured model where the objective function and constraints are linear.

Integer Programming (IP): Extends linear programming by allowing some or all decision variables to take integer values, suitable for discrete decision-making problems.

Nonlinear Programming (NLP): Deals with objective functions or constraints that involve nonlinear relationships between variables.

Network Models: Represent decision-making problems involving interconnected elements, such as project planning (PERT/CPM).

Queueing Models: Used for the analysis of systems where entities (such as customers or tasks) wait in queues.

### **Integration with Other Models:**

Structured models can be part of larger decision support systems: They can be integrated with other models, such as statistical models or machine learning algorithms, to enhance decision-making capabilities. Understanding and effectively using structured models are crucial for decision-makers and analysts seeking to optimize decision outcomes across a wide range of applications.

<sup>&</sup>lt;sup>48</sup> Вероятностные методы в вычислительной технике / под ред. А. Н. Лебедева и Е. А. Чернявского. М.: Высш. шк., 1986. 312 с.

<sup>&</sup>lt;sup>49</sup> Вероятностные методы в вычислительной технике / под ред. А. Н. Лебедева и Е. А. Чернявского. М.: Высш. шк., 1986. 312 с.

<sup>&</sup>lt;sup>50</sup> Доррер Г. А., Попов А. А., Сысенко К. В. Исследование жизненного цикла электронных информационных ресурсов // Вестн. СибГАУ.

<sup>2009. № 2.</sup> C. 128–132.

### 2.5.3. Semi-structured models

Semi-structured models represent a flexible and adaptive category within the spectrum of mathematical modeling, combining elements of both structured and unstructured modeling approaches. These models are particularly useful when decision-making problems exhibit characteristics that fall between well-defined, predictable scenarios and entirely unpredictable, complex situations.

In semi-structured models, decision-makers have the flexibility to incorporate both quantitative and qualitative elements into the modeling process. Unlike fully structured models, which rely on precise mathematical relationships, semi-structured models acknowledge the degree of uncertainty and ambiguity inherent in many real decision-making contexts.

One common type of semi-structured modeling involves the use of decision matrices or decision tables. Decision matrices provide a structured framework for considering a range of factors, making them applicable in areas such as project selection, supplier evaluation, or risk assessment.

Fuzzy logic modeling is another semi-structured approach that addresses uncertainty by allowing degrees of truth or membership to decision criteria. In situations where decision variables are not easily measurable or may have imprecise boundaries, fuzzy logic models enable decision-makers to express and analyze subjective or ambiguous information. This makes fuzzy logic applicable in areas such as decision support systems, management systems, and pattern recognition.<sup>51</sup>

Incorporating optimization models with qualitative considerations is another characteristic of semi-structured modeling. Multi-Criteria Decision Analysis (MCDA) is an approach that integrates both quantitative optimization and qualitative criteria.

Semi-structured models are particularly suitable in dynamic decision-making environments, where variables can evolve, and decision criteria may change over time.

One key advantage of semi-structured models lies in their ability to capture subjective assessments and expert opinions, which are often integral to decision-making processes but may not be easily measurable.

Semi-structured models occupy a valuable niche in the decision-making process, providing a flexible and adaptive approach that addresses the complexity of uncertain and dynamic decision-making environments. By combining structured and unstructured elements, these models empower decision-makers to make well-informed choices in scenarios where both quantitative and qualitative considerations play a significant role.<sup>52</sup>

#### 2.5.4. Official models

In the realm of decision-making within an official or governmental context, the use of official models plays a decisive role in shaping policies, guiding regulatory actions, and informing strategic decisions. These official models are often developed and approved by government agencies, regulatory bodies, or authoritative institutions. The application of official models contributes to evidence-based decision-making and provides a systematic framework for understanding complex issues. Several types of official models are commonly used in decision-making processes within government administration:<sup>53</sup>

<sup>53</sup> Шибанов А. П. Обобщенные GERT-сети для моделирования протоколов, алгоритмов и программ телекоммуникационных систем: дис.

д-ра техн. наук. Рязань: РГРА, 2003. 307 с.

<sup>&</sup>lt;sup>51</sup> Кемени Дж., Снелл Дж. Конечные цепи Маркова. М.: Наука,

<sup>1970. 450</sup> c.

<sup>&</sup>lt;sup>52</sup> Кофман А., Фор Р. Займемся исследованием операций: пер. с фр.М.: Мир, 1966. 280 с.

Economic Models: Economic models are widely used by governments to analyze and forecast economic trends, assess the impact of policy changes, and formulate strategies for economic growth.

Public Policy Models: Public policy models are designed to assess the potential outcomes and consequences of various policy options. These models consider a range of factors, including social, economic, and environmental impacts.

Healthcare Models: In the healthcare sector, governments use models to analyze and plan initiatives in the field of public health, resource allocation, and strategies for responding to health crises.

Environmental Models: Governments utilize environmental models to assess the impact of policies on ecosystems, air and water quality, and climate change.

Risk Assessment Models: Official models are used for the assessment and management of risks associated with various aspects, including finance, public safety, and national security.

Regulatory Models: Regulatory authorities often use models to evaluate the potential impact of proposed regulations on industries, consumers, and the overall economy.<sup>54</sup>

Official models are characterized by their reliance on data, empirical evidence, and a structured approach to decision-making. These models provide a systematic and transparent way for governments to navigate complex issues, anticipate consequences, and make decisions that align with their overarching goals.

The continuous integration of different perspectives, data-driven insights, and advanced modeling techniques ensures that official models remain reliable tools for informed and effective governance.<sup>55</sup>

#### 2.5.5. Unstructured models

B decision theory, unstructured models represent a category of modeling approaches that deviate from the systematic and well-defined frameworks characteristic of structured models. Unstructured models are particularly applicable in decision-making scenarios where the complexity of the problem, lack of clear data, or involvement of subjective and qualitative factors make it challenging to formulate precise mathematical dependencies. This category encompasses various methods that provide decision-makers with flexibility and adaptability when dealing with complex and dynamic decision-making environments.

Qualitative Models: Unstructured modeling often involves qualitative methods that capture subjective insights, expert opinions, and non-quantitative aspects of decision-making problems.<sup>56</sup>

Scenario Planning: Unstructured models find application in scenario planning, a strategic decision-making tool that involves forecasting and analyzing multiple possible future scenarios.

Soft Systems Methodology (SSM): SSM is an unstructured modeling approach that takes into account the social and subjective aspects of decision-making problems.

Narrative Models: Unstructured models can take the form of stories or narratives that describe the context of the decision, challenges, and potential outcomes. This approach is particularly suitable for cultural, social, or ethical considerations. <sup>57</sup>

<sup>&</sup>lt;sup>54</sup> Советов Б. Я., Яковлев С. А. Моделирование систем. М.: Высш.шк., 1985. 350 с.

<sup>&</sup>lt;sup>55</sup> Боев В. Д., Кирик Д. И., Сыпченко Р. П. Компьютерное моделирование: пособие для курсового и дипломного проектирования. СПб.: ВАС, 2011. 348 с.

<sup>56</sup> Доррер Г. А. Методы моделирования дискретных систем: учеб. пособие. Красноярск: ИПЦ КГТУ, 2005. 171 с.

<sup>&</sup>lt;sup>57</sup> Доррер Г. А., Коморовсий В. С. Оценка и прогнозирование дина-мики крупных лесных пожаров [Электронный ресурс] // Технологии техносферной безопасности: интернет-журн. МЧС России, Акад. ГПС. 2011. Вып. 2. URL: http://www.ipb.mos.ru/ttb

Cause-and-Effect Diagrams: Cause-and-effect chain diagrams are graphical representations that illustrate feedback loops and causal relationships between variables in a given system. <sup>58</sup>

Swarm Intelligence Models: Inspired by collective behavior observed in nature, swarm intelligence models use algorithms that mimic the behavior of swarms or groups.

The choice between structured and unstructured models depends on the specific characteristics of the decision problem and the depth of understanding required for effective decision-making.<sup>59</sup>

#### 2.5.6. Data models

Data models represent an important component in the decision-making process by providing a structured framework for organizing, storing, and analyzing information. These models facilitate the transformation of raw data into meaningful insights, enabling informed and data-driven decision-making processes.

Data models serve as the foundation for data-driven decision-making, offering decision-makers a structured way to interact with and extract insights from vast amounts of data. Each type of data model brings unique advantages to the decision-making process, allowing decision-makers to extract valuable insights, make informed choices, and adapt to the dynamic nature of the respective domains.<sup>60</sup>

Data models refer to mathematical representations that include data-driven elements in decision-making processes. These models leverage available data to enhance the accuracy and reliability of decision outcomes.

In summary, data models play a crucial role in decision-making, incorporating empirical evidence and trends from available data, thereby enhancing the accuracy and reliability of decision outcomes.

### 2.6. Classification of mathematical models of structured systems

Mathematical models in structured systems play a crucial role in decision-making by offering a systematic approach for analyzing and optimizing complex scenarios. The classification of these models is of paramount importance for understanding their diverse applications and functionalities. In the field of mathematical models in structured systems, there are several key classifications:

Deterministic Models: Deterministic models assume that the relationships between variables are precisely defined and do not exhibit randomness.

Probabilistic Models: In contrast to deterministic models, probabilistic models acknowledge the presence of uncertainty and randomness in decision-making problems.

Optimization Models: Optimization models focus on finding the best possible solution from a set of feasible options. Linear programming, nonlinear programming, and integer programming are commonly used optimization techniques applied in structured decision-making.<sup>61</sup>

<sup>&</sup>lt;sup>58</sup> Котов В. Е. Сети Петри. М.: Наука, 1984. 158 с.

<sup>&</sup>lt;sup>59</sup> Ломазова И. А. Вложенные сети Петри: моделирование и анализ распределенных систем с объектной структурой. М.: Науч. мир, 2004. 208 с.

<sup>&</sup>lt;sup>60</sup> Ноженкова Л. Ф., Исаев С. В., Ничепорчук В. В. Применение

экспертной ГИС для анализа пожарной обстановки в Красноярском крае //Пробл. безопасности и чрезвычайных ситуаций. 2009. № 2. С. 75–85.

<sup>&</sup>lt;sup>61</sup> Основы теории вычислительных систем / под ред. проф.

С. А. Майорова. М.: Высш. шк., 1978. 408 с.

Simulation Models: Simulation models replicate real-world processes through mathematical representation, allowing decision-makers to observe the system's behavior over time.

Game Theory Models: Game theory provides a framework for analyzing strategic interactions among a set of decision-makers known as players.

Queueing Theory Models: Queueing theory models concentrate on studying queues of waiting entities and the flow of subjects through systems.

Network Models: Network models represent relationships and connections between entities in a given system.

Markov Models: Markov models are probabilistic models that represent systems with sequential states and transitions between them.

Dynamic Models: Dynamic models reflect the temporal aspects of decision-making problems, considering how variables evolve over time.

These models are valuable in scenarios where outcomes are entirely predictable based on specific input data. Linear programming, as a deterministic model, is widely used to optimize resource allocation, production schedules, and other decision variables in various industries. <sup>62</sup>

In conclusion, the classification of mathematical models in structured systems provides decision-makers with a diverse toolkit to address different decision-making scenarios. Each type of model brings its own set of strengths and considerations, allowing for an adaptable approach to the complexity and dynamics inherent in different decision-making environments. The choice of a specific modeling approach depends on the specific characteristics of the decision problem, the nature of the studied system, and the goals of the analysis.

# 2.7. Problems of Multilevel Modeling in Decision Making:

Multilevel decision-making modeling involves considering factors acting at different levels - individual, group, organizational, or even societal. While this approach offers a more comprehensive understanding of decision-making processes, it comes with its own set of challenges:

Integration of Levels: Coordinating and integrating information from different levels can be complex. Decision-makers need to navigate the intricate interaction between individual choices and broader organizational or societal influences.

Gathering and analyzing data at different levels requires sophisticated methodologies. Integrating diverse datasets, accounting for different measurement scales, and addressing potential deviations are crucial considerations.

Decision-making often depends on context, and multilevel models must account for nuances in different contexts. Differences in organizational culture, leadership styles, or societal norms can significantly influence decision-making processes.

Decision-making is dynamic, and factors at one level can impact or interact with those at other levels. Understanding these dynamic interactions is crucial for developing accurate models.

Individuals in a group may have different decision-making styles, preferences, and motivations. Incorporating these individual differences into multilevel models requires careful consideration to avoid oversimplification.

Multilevel decision-making models must adhere to ethical considerations, ensuring fairness and equality across all levels. The potential for power imbalances or unforeseen consequences necessitates ethical control in decision-making processes.

<sup>&</sup>lt;sup>62</sup> Черноруцкий И. Г. Методы оптимизации в теории управления: учеб. пособие. СПб.: Питер, 2004. 256 с

Effective communication between different levels is crucial. Improper communication or lack of transparency can hinder the success of multilevel decision-making models, impacting the implementation of decisions.

# 2.8. Conclusions

The challenges and nuances associated with multilevel decision-making modeling emphasize the need for an integrated approach that considers factors at the individual, group, and organizational levels. Questions such as data complexity, dynamic interactions, and ethical considerations are key aspects that need to be addressed when applying multilevel models.

In conclusion, understanding motivation is of crucial importance for comprehending human behavior and decision-making processes. Content theories offer valuable frameworks, but their practical application requires a nuanced approach that considers individual differences, contextual factors, and the dynamic nature of motivation. Furthermore, integrating multilevel models into the decision-making process provides a more comprehensive perspective, albeit with inherent challenges that require careful consideration and refinement of the methodology.

# CHAPTER 3. FORMAL DESCRIPTION OF DISCRETE DECISION-MAKING SYSTEMS WITH MOTIVATION ASSESSMENT

#### SETS, GRAPHS, NETWORK FLOWS

# 3.1. Concepts of sets

The following notations will be introduced: :63 64 65 66

 $a \in B$  indicates that the element *a* belongs to the set *B*, where  $\in$  is the symbol for membership.

 $a \notin B$  means that *a* does not belong to the *B*.

 $A \subset B$  indicates that the set A is a subset of the set B.

If  $A \subset B$  and  $B \subset A$ , then  $A \equiv B$ .

The empty set with the symbol  $\emptyset$  indicates that it contains no elements.

 $\begin{vmatrix} A \end{vmatrix}$  is equal to the cardinality of the set A. It represents the number of elements in the set A.

If A and B are arbitrary sets, their union  $A \cup B$  is called the set, all elements of which belong to A or B, or to A and B simultaneously.

If A and B ca are arbitrary sets, their intersection  $A \cap B$  contains only those elements that belong simultaneously to both A, and B.

Subtraction  $C = A \setminus B$  of the two sets A and B is the set of those elements from A, that are not contained in B, where simultaneously assumes  $B \subset A$ .

Symmetric difference  $A\Delta B$  is called the set obtained by taking the union of the differences  $A \setminus B$  and  $B \setminus A$ . It can also be denoted by  $A\Delta B = (A \setminus B) \cup (B \setminus A)$ .

The set *C* is divided into subsets *A* and *B*, if  $A \cap B = \emptyset$  and  $A \cup B = C$ .

If all the considered subsets are subsets of some set U, then U it is called a universal set. For arbitrary sets A and B the following three dependencies are mutually equivalent:  $A \subseteq B$ ;  $A \cap B = A$ ;  $A \cup B = B$ .

<sup>&</sup>lt;sup>63</sup> Ford, L. R,D.R. Fulkerson. Maximal flow through a network. - Canadian Journal of Mathematics, 1956, 8, pp. 399-404.

<sup>&</sup>lt;sup>64</sup> Don Phillips, Garcia-Diaz. Fundamentals of Network Analysis. Prentice Hall, Englewood Cliffs, New York, 1981. 474 pp., DOI: 10.1002/net.3230120210

<sup>&</sup>lt;sup>65</sup> Jensen, P. A., J.W.Barnes. Network flow programming. New York, John Wiley and Sons, Inc., 1980.

<sup>&</sup>lt;sup>66</sup> Christofides, N. Graph theory: An Algorithmic Approach. London [etc.]. Academic Press, 1986.

In terms of cardinality m(A) of the set A refers to the number of elements |A| in that set, i.e., m(A) = |A|.

The absolute complement of the set A is called the set  $\overline{A}$  of those elements that do not belong to the set A, i.e.  $\overline{A} = U \setminus A$ . From this, it follows that  $X \setminus A = X \cap \overline{A}$ .

The set  $\{i/i \in A; P(i)\}$  includes all elements of the set A for which the predicate is true P(i).

Parentheses  $\{\dots\}$  will be used solely for denoting sets.

If A and B ca are arbitrary sets, then it is claimed that on A the function is defined  $\Gamma^1$ , taking values from B, if for every element of A is assigned exactly one element from B. For arbitrary sets, instead of the term "function," the term "mapping" is used"

For the image of the function from *A* in *B* is expressed as  $\Gamma^1 : A \to B$ .

If *a* is an element of *A*, then the corresponding element from *B* is  $b = \Gamma^{-1}(a)$ ;  $b \in B$ .

In a similar manner, the inverse image is defined  $\Gamma^{-1}(b)$ . The dependencies exist:

$$\Gamma^{-1}(A \cup B) = \Gamma^{-1}(A) \cup \Gamma^{-1}(B);$$
  

$$\Gamma^{-1}(A \cap B) = \Gamma^{-1}(A) \cap \Gamma^{-1}(B);$$
  

$$\Gamma^{1}(A \cup B) = \Gamma^{1}(A) \cup \Gamma^{1}(B).$$

The notations are introduced:

$$J = \left\{ \left( i, j \right) / x_{ij} \in U \right\};$$
$$\sum_{(i, j \in J)} f_{ij} = F;$$

where  $\{f_{ij}\}\$  is a flow function whose flow function on the arc  $(x_i, x_j) = x_{ij}$  is equal to  $f_{ij}$ . "Reflexivity"  $\varphi$  of the set *N* means that x = x.

Symmetricality  $\varphi$  On the same set, it shows that for  $x, y \in N$  from x = y follows y = x. Transitivity shows that from y = x and x = z follows y = z.

In the set of real numbers, the relation  $\leq$  is reflexive, symmetric, and transitive, i.e., both elements x and y are equivalent to each other. The relation < is transitive, but not reflexive and

symmetric, i.e., both elements." x and y are not equivalent to each other. The symbol  $i \in I$  means that the union of elements from *I*.

#### 3.2. Graphs and actions on them

The graph G is defined by a set of elements (or vertices)  $N = \{x_1, x_2, ..., x_n\}$  and a set of edges (or arcs)  $\{u_1, u_2, ..., u_m\}$ , which are denoted by U and encompass all or part of the elements of U. This allows the graph, as a mathematical structure, to be labeled as G(N, U). 67

<sup>&</sup>lt;sup>67</sup> Christofides, N. Graph theory: An Algorithmic Approach. London [etc.]. Academic Press, 1986.

If all elements of U are directed arcs, which means that in each of the elements  $u_k \in U$ ;  $u_k = (x_i, x_j)$  vertex  $x_i$  is the starting, and  $x_j$  - last. Then the graph G(N, U) is a directed graph.

Figure 1 shows a directed graph with five vertices and six edges. Arrows indicate the directions of the edges - from the initial vertices to the terminal ones. It is customary to note that the vertices of the edge  $u_k$  or the edge – from  $x_i$  to  $x_j$  – are incident to the edge, and vice versa." It includes the vertices  $\{x_1, x_2, x_3, x_4, x_5\}$  and the edges  $\{u_1, u_2, u_3, u_4, u_5, u_6\}$ .

In the second way of defining the graph, the right plays an essential role  $\Gamma^1$  and vice versa  $\Gamma^{-1}$  image. Formally, they can be written as follows:

$$\Gamma_{i}^{1} = \{ j/(i, j) \in J; j \in I \}; \Gamma_{i}^{-1} = \{ j/(j, i) \in J; j \in I \}.$$

For the graph in Figure 1, one can write:



#### FIGURE 1.

$$\Gamma(x_1) = \{x_2, x_3, x_4\};$$
  

$$\Gamma(x_2) = \{x_1, x_3\};$$
  

$$\Gamma(x_3) = \emptyset;$$
  

$$\Gamma(x_4) = \{x_3, x_5\};$$
  

$$\Gamma(x_5) = \emptyset,$$

where  $\emptyset$  is an empty set.

In the context of the inverse image  $\Gamma^{-1}$  can be expressed as:

$$\Gamma^{-1} (x_1) = \{ x_2 \};$$
  

$$\Gamma^{-1} (x_2) = \{ x_1 \};$$
  

$$\Gamma^{-1} (x_3) = \{ x_1, x_2, x_4 \};$$
  

$$\Gamma^{-1} (x_4) = \{ x_1 \};$$
  

$$\Gamma^{-1} (x_5) = \{ x_4 \}.$$

It is possible to define a double image  $\Gamma^2(x)$  in the following way:

$$\Gamma^{2}(x) = \left\{ z/\Gamma(x) = y; \Gamma(y) = z; (x, y) \in U; (y, z) = U \right\}.$$

The edge can  $(x_1, x_2)$  to be replaced by two equivalent arcs  $\{(x_1, x_2) \cup (x_2, x_1)\}$ .

#### 3.3. Network Flows

Network flows, or flows on graphs, are widely used mathematical structures that enable theoretical investigations and practical applications. In recent years, they have undergone significant evolutionary development, leading to substantial results and applications in decision-making systems.

In this study, three variations of these network flows are employed to explore motivational and emotional mental processes. Two of them, namely classical network flows (CNF) and generalized network flows (GNF), are well-established.

#### **3.4.**Classical Network Flows

In defining network flows, the following arc functions play a crucial role: <sup>68</sup>

—  $c_{ij}$  – non-negative capacity function on the arc" или "non-negative capacity function along the edge  $x_{ij}$  for which, for every  $(i, j) \in J$ , is satisfied:

$$0 \le c_{ij} \le x_{ij}; \tag{3.1}$$

—  $a_{ij}$  – non-negative value function on the arc  $x_{ij}$ :

$$0 \le a_{ij} \le m; \ (i, j) \in J \tag{3.2}$$

where m is a finite rational number A.

The network flow, or equivalently, the flow on the graph, is defined as follows for each  $i \in I$ and  $(i, j) \in J$ :

$$\sum_{j\in\Gamma_i^{l}} f_{ij} - \sum_{j\in\Gamma_i^{-l}} f_{ji} = \begin{cases} V, \text{ ako } x_i = S; \\ 0, \text{ ako } x_i \neq s_i t; \\ -V, \text{ ako } x_i = t; \end{cases}$$
(3.3)

$$f_{ij} \le c_{ij} \quad \text{for each} (i, j) \in J; \tag{3.4}$$

$$f_{ij} \ge 0$$
 for each  $(i, j) \in J$ . (3.5)

Vertex  $S \in N$  is a source of the flow  $U \ge 0$ , and the vertex  $t \in N$  is a consumer of the same flow.

The objective function for network flow management is

<sup>&</sup>lt;sup>68</sup> Jensen, P. A., J.W.Barnes. Network flow programming. New York, John Wiley and Sons, Inc., 1980.

$$\sum_{\substack{(i,j)\in J}} a_{ij} f_{ij} \to \min(\max).$$
(3.6)

It is possible to have multiple sources  $S \in N$  and multiple sinks  $T \in N$  where

$$\left| S \right| \ge 1; \left| T \right| \ge 1 \tag{3.7}$$

and the equality must be satisfied:

$$\sum_{x_i \in S} V_i = -\sum_{x_j \in T} V_j = 0;$$
(3.8)

$$S \cap T = \emptyset. \tag{3.9}$$

If the optimization problem of flow optimization (3.6) is solved while satisfying the requirements from (3.1) to (3.5), an optimal distribution of the flow over the network will be obtained, meeting the requirements for capacity (3.4) and non-negativity of the flow (3.5).

Equation (3.3) is called the conservation equation. It shows that for the vertices  $x_i \neq s, t$  it is always necessary that the sum of the incoming flow into the respective vertex equals the flow outgoing from that vertex. The dependency (3.3) is called the conservation equation and holds fundamental significance in network flow.

When increasing the value of the flow V on the network, saturation of certain edges will be reached  $x_{ij}$ ,  $f_{ij} = c_{ij}$ . In these cases is in force, the so-called mincut – max flow theorem by Ford and Fulkerson is valid, according to which the maximum flow  $V_{\text{max}}$  is equal to the minimum cut.  $(N_0, \overline{N}_0)_{:69}$ 

$$V_{max} = f(N_0, \bar{N}_0) - f(\bar{N}_0, N_0) \le c(N_0, \bar{N}_0); \qquad (3.10)$$
  
$$f(N_0, \bar{N}_0) = 0, \qquad (3.11)$$

where the cut  $(N_0, \overline{N}_0)$  is equal to

$$\left(N_{0}, \overline{N}_{0}\right) = \left\{x_{ij} \middle| x_{i} \in N; x_{j} \in \overline{N}; (i, j) \in J\right\}.$$
(3.12)

Upon reaching the maximum value of the flow  $V_{\text{max}}$  e the following dependency is true:

$$V_{\max} = f\left(N_0, \overline{N}_0\right) = c\left(N_0, \overline{N}_0\right) \tag{3.13}$$

when the flow value on the cut is zero  $(N_0, N)$ .

# 3.5. Generalized Network Flow

An important variation of classical network flow is the generalized network flow or the flow with profits and losses. It boils down to the fact that if in the initial vertex  $x_i$  the flow function has a value  $f_j$ , then at the destination vertex  $x_j$  on the same edge, the flow value is now  $g_{ij}f_j$ , where  $g_{ij}$  a positive rational number – a coefficient of amplification or attenuation of the flow.<sup>70</sup> It is assumed that for each  $(i, j) \in J$  is valid:

<sup>&</sup>lt;sup>69</sup> Ford, L. R,D.R. Fulkerson. Maximal flow through a network. - Canadian Journal of Mathematics, 1956, 8, pp. 399-404.

<sup>&</sup>lt;sup>70</sup> Don Phillips, Garcia-Diaz. Fundamentals of Network Analysis. Prentice Hall, Englewood Cliffs, New York, 1981. 474 pp., DOI: 10.1002/net.3230120210

$$0 \le g_{ij} \le p_{ij}, \tag{3.14}$$

where  $p_{ij}$  is a finite positive rational number.

Since depending on  $\{g_{ij}\}\$  the flow is either amplified or attenuated, the flow *V* at the source *S* in general is not equal to the flow -V at the sink *t*, which is different from *V*, i.e.  $V \neq -V$ . In the generalized network flow, the conservation equation takes the following form: for each  $i \in I$  and  $(i, j) \in J$ ,

$$\sum_{j\in\Gamma_i^1} f_{ij} - \sum_{j\in\Gamma_i^{-1}} g_{ji} f_{ji} = \begin{cases} \leq V, \text{ ако } x_i = S; \\ 0, \text{ ако } x_i \neq s_i t; \\ -V, \text{ ако } x_i = t. \end{cases}$$
(3.15)

The objective function in the generalized network flow coincides with the objective function in (3.6). The other two equations of the generalized network flow are the same as in (3.4) and (3.5).

The coefficients  $\{g_{ij}/(i, j) \in J\}$  The coefficients provide the opportunity to take into account the external influence on the flow implementation, which has important practical implications for the network implementation of a series of real processes. If  $g_{ij} = 1$  for each  $(i, j) \in J$ , the generalized network flow coincides with the classical network flow.<sup>71</sup>

# 3.6. Features of Decision-Making Systems Based on Network Flow Models

Network flow methods and tools often provide the opportunity to develop systems for decision-making or decision support that are suitable for a wide range of real-world objects and processes. Most commonly, tasks addressed through network flow models are related to logistics problems in transporting and storing various resources. In such cases, it is necessary not only to create a comprehensive, preferably optimal plan but also to monitor the processes of movement from one point to another. This implies readiness to make real-time decisions over different periods. Suitable stationary and mobile sensors and sensor systems are necessary to provide sufficiently accurate information about the location and condition of transported and stored resources.

The strategy for determining and implementing control actions in the deterministic case and considering stochasticity is different:

1. If the decision-making system has negligible stochasticity, the process is treated as deterministic, and the optimal plan is determined using the dependencies from (3.1) to (3.5) – in the case of classical network flow (CNF), or from (3.4) to (3.6) and (3.15) – in the case of generalized network flow (GNF). In each of these cases, the objective function (3.6) remains the same. Thus, a single-stage decision-making process is implemented. The computed optimum starts to be executed only if there are force majeure events, reconsidering with regard to the emerging new circumstances.

<sup>&</sup>lt;sup>71</sup> Don Phillips, Garcia-Diaz. Fundamentals of Network Analysis. Prentice Hall, Englewood Cliffs, New York, 1981. 474 pp., DOI: 10.1002/net.3230120210

When creating the optimal plan, it is crucial to determine the network and network flow parameters as accurately as possible.

$$\left\{ c_{ij} / (i, j) \in J \right\}, \left\{ g_{ij} / (i, j) \in J \right\}, \left\{ a_{ij} / (i, j) \in J \right\},$$

Too much depends on the quality of the decisions made. Before the next management cycle, it is necessary to refine the values of the four parameters described above. The formulated method implements a one-time, single-stage decision-making in a deterministic environment.

2. In the presence of greater stochasticity, a transition is made to multi-stage decisionmaking. It is necessary for the zone of transportation and storage of resources to be equipped with corresponding sensors to implement feedback control.

Then, it is clarified which of the two network flow models – CNF and GNF – should be used.

The first step is determined similarly to single-stage control with deterministic parameters. Through the sensor system u and feedback, the real state of the controlled process is determined. Then the second step is carried out. It is similar to the first but with a different location of the resource on the network and with refined values of some of the four parameters.  $\{c_{ii}, g_{ii}, a_{ii}, p_i\}$ .

The third and subsequent steps are carried out similarly to the first two. This enables the implementation of multi-stage decision-making and the management of objects and processes in the presence of significant stochasticity.

The described network flow models provide the opportunity to account for certain psychological aspects in decision-making, such as motivation and emotions. This will be demonstrated in the following chapters of this work.

# CHAPTER 4: NUMERICAL EXAMPLE OF A DISCRETE DECISION-MAKING SYSTEM WITH CONSIDERATION OF MOTIVATION

To illustrate the results obtained from the previous chapters, an example of decisionmaking in the transfer of resources across a network will be used, taking motivation into account.

Such an effective model can be constructed more generally based on network flow (graph). For this purpose, the generalized network flow is most suitable<sup>72</sup> – sometimes referred to as profit and loss flow, where motivation is taken into account through amplification or attenuation coefficients.  $\{g_{ii}/(i, j) \in J\}$ .

Let the transposition of the resource be carried out on the following graph G(X, V)::



Фиг.4.1

In it, the number of nodes is 5, which implies that

 $X = \{ x_1, x_2, x_3, x_4, x_5 \}; I = \{ 1, 2, 3, 4, 5 \};$ (4.1)

The same graph has six pairs of indices connecting 5 vertices, namely,

$$U = \left\{ x_{1,2}, x_{1,3}, x_{2,5}, x_{3,4}, x_{3,5}, x_{4,5} \right\};$$
(4.2)

"In the same graph, there are six pairs of indices that belong to the following set J:

$$J = \{ (1, 2); (1, 3); (2, 5); (3, 4); (3, 5); (4, 5) \};$$
(4.3)

The set V can be succinctly described using J, namely,

$$V = \left\{ x_{i,j} / (i, j) \in J \right\};$$
(4.4)

Similarly, the set of vertices can be concisely described using

$$X = \left\{ i/i \in I \right\}; \tag{4.5}$$

<sup>&</sup>lt;sup>72</sup>Sgurev, V., Doukovska, L., Drangajov, St., Intelligent Network-flow Solutions with Risks at Transportation of Products. Sgurev V., Jotsov V., Kacprzyk J. (Eds.), Chapter of Book: Advances in Intelligent Systems Research and Innovation, Series: Studies in Systems, Decision and Control, 379, Springer International Publishing, Switzerland, 2021, ISBN:978-3-030-78123-1, DOI:10.1007/978-3-030-78124-8\_19, pp. 417-439.

The resource source S is located at vertex  $x_1$ , and the resource consumer T is at vertex  $x_5$ .

The quantity of the resource from point  $x_i$  to point  $x_j$  is denoted by  $f_{ij}$ . This quantity is always non-negative.<sup>73</sup>

$$0 \le f_{ii}$$
 за всяко  $(i, j) \in J;$  (4.6)

There exists an upper limit constraint for the resource quantity  $f_{ij}$  on the arc (edge) from point  $x_{ij}$ , called the capacity  $C_{ij}$  for the arc (edge)  $x_{ij}$ .<sup>74</sup>.It can be expressed as:

$$f_{ij} \le C_{ij}$$
 for each  $(i, j) \in J;$  (4.7)

The capacity is a non-negative function for which:

 $0 \le C_{ij} \text{ for each } (i, j) \in J; \tag{4.8}$ 

This follows from the previous two inequalities (4.6) and (4.7).

The value of the flow function at the source S is denoted  $v_0$ , and at the sink – by V.

In the generalized network model, the consideration of motivation can be accomplished through the coefficients for amplification or attenuation of the arc flow functions  $g_{ij} \ge 0$ ;  $(i, j) \in J$ .<sup>75</sup> These coefficients will be referred to as motivation coefficients.

At this,

If  $0 \le g_{ij} < 1$ , the motivation decreases the value of the stream function that has already been acquired.

$$g_{ij}f_{ij} < f_{ij}; 0 \le g_{ij} < 1;$$
(4.9)

a) If  $g_{ii}$  it has value  $g_{ii}$ 

$$g_{ij}f_{ij} > f_{ij}; g_{ij} > 1;$$
 (4.10)

Motivation amplifies the effect of resource transfer from point  $x_i$  to point  $x_i$ ;

b) If  $g_{ii} = 1$ , then motivation does not affect the transfer of resources, i.e.

$$g_{ij}f_{ij} = f_{ij}; g_{ij} = 1; (4.11)$$

What value will the coefficient take  $g_{ij} = 1$ , depends on how expertly the external and internal motivations of the decision-maker for the segment (curve) will be evaluated.  $x_{ij}$ . If his motivation is significant, then the amount of resource movement along this segment will be larger, and therefore.  $g_{ij} > 1$ ,

If the entity interested in expediting the process (for example, the owner of the system)

<sup>&</sup>lt;sup>73</sup> Sgurev, V., S. Drangajov. Resources' Allocation with Minimization of Accom- panying Risks, Information Technologies and Control, John Atanasoff Society of Automatics and Informatics, Sofia, Bulgaria, Print: ISSN 1312-2622; On- line: ISSN 2367-5357, No 1, 2017.

<sup>&</sup>lt;sup>74</sup> Sgurev, V., S. Drangajov. Two Stage Method for Network Flow Control of Resources and the Risks Related to Them. - In: Proc of the International Conference of Automatics and Informatics 2016, Bulgaria, Sofia, Oct. 4-5, 2016, John Atanasoff Society of Automatics and Informatics, Proc.: ISSN 1313-1850, CD: ISSN 1313\_1869 O UAI, pp. 143-149.

<sup>&</sup>lt;sup>75</sup> Sgurev, V., Doukovska, L., Drangajov, St., Intelligent Network-flow Solutions with Risks at Transportation of Products. In: Sgurev V., Jotsov V., Kacprzyk J. (Eds.), Chapter of Book: Advances in Intelligent Systems Research and Innovation, Series: Studies in Systems, Decision and Control, 379, Springer International Publishing, Switzerland, 2022, ISBN:978-3-030-78123-1, DOI:10.1007/978-3-030-78124-8\_19, pp. 417-439.

takes appropriate measures and increases motivation in specific segments, then the overall amount of transferred resources will be greater. Determining the distribution of resources across individual segments (curves) of the network is the subject of a corresponding optimization task. Defining it requires the determination of costs  $\{a_{ij}/(i, j) \in J\}$  for transporting one unit of resource along the segment (curve)."  $x_{ii}$ .<sup>76</sup>

These costs, also referred to as arc weights, always have a non-negative value:

$$0 \le a_{ij}$$
 for each $(i, j) \in J$ ; (4.12)

The generalized network flow can be defined most generally through the following dependencies:"за всяко  $i \in I$  и  $(i, j) \in J$ 

$$\sum_{j \in \mathbf{\Gamma}_{i}^{1}} f_{ij} - \sum_{j \in \mathbf{\Gamma}_{i}^{-1}} g_{ji} f_{ji} = \begin{cases} v_{0}, \text{ ако } x_{i} = S; \\ 0, \text{ ако } x_{i} \neq S, T; \\ -v, \text{ ако } x_{i} = T; \end{cases}$$
(4.13)

$$f_{ii} \le C_{ii} \text{ for each } (i, j) \in J; \tag{4.14}$$

$$\begin{aligned} f_{ij} &\leq C_{ij} \text{ for each } (i, j) \in J; \end{aligned} \tag{4.14} \\ 0 &\leq f_{ij} \text{ for each } (i, j) \in J; \end{aligned}$$

"Upon the thus defined generalized network flow with motivations, at least three optimization problems can be formulated—A, B, and C, with different objective functions:

Problem A: Maximum generalized network flow with motivations and the following objective function:"

$$L = u = u_{\max} \to \max; \tag{4.16}$$

where  $v_0$  is the quantity of initial resource at vertex  $S = x_1$  in the network.

Problem B: Maximum generalized network flow with motivations and with a minimum or maximum value in the following objective function:"

$$L = \sum_{(i, j) \in J} a_{ij} f_{ij} \to \min\left(\max\right);$$
(4.17)

"where  $v_0$  is the quantity of initial resource, and  $v = v_{max}$  is obtained when solving problem A.

Problem C: Generalized network flow with motivation and with a minimum (maximum) value of the objective function L from (4.17), where  $v_0$  is with a fixed value.

In the considered numerical example, the coefficients  $\{g_{ij}/(i, j)\}$  have the following values:

<sup>&</sup>lt;sup>76</sup> Sgurev, V., Doukovska, L., Multivalued Network Logic with One Real and Two Imaginary Logic Structures. Proceedings of the IEEE International Conference Automatics and Informatics - ICAI'23, 5-7 October 2023, Varna, Bulgaria, IEEE Xplore, 2023, DOI:10.1109/ICAI58806.2023.10339033, pp. 395-398.

$$g_{1,2} = 1,2; g_{1,3} = 1,4; g_{2,5} = 0,8; g_{3,4} = 1,6; g_{3,5} = 1,1; g_{4,5} = 0,5;$$
 (4.18)

The arc capacities are equal to:

$$C_{1,2} = 5; C_{1,3} = 6; C_{2,5} = 4,8; C_{3,4} = 4; C_{3,5} = 6; C_{4,5} = 8;$$
(4.19)

The provided data allows solving each of the described optimization problems A, B, or C for a generalized network flow, taking motivations into account.<sup>77</sup>

#### **Optimization Problem A**

In this problem for maximum generalized network flow with motivations, the objective function is defined by (4.16), and the constraints (4.13) take the following form:

$$a_{1}: f_{1,2} + f_{1,3} = 10; \qquad (4.20)$$

$$a_{2}: f_{2,5} - 1,2f_{1,2} = 0;$$

$$a_{3}: f_{3,4} + f_{3,5} - 1,4f_{1,3} = 0;$$

$$a_{4}: f_{4,5} - 1,6f_{3,4} = 0;$$

$$a_{5}: -0,8f_{2,5} - 1,1f_{3,5} - 0,5f_{4,5} + v = 0; \qquad (4.21)$$

The constraints from (4.14) and (4.15) are described by the dependencies:

$$\begin{array}{l} a_{6}:f_{1,2}\leq 5;\\ a_{7}:f_{1,3}\leq 6;\\ a_{8}:f_{2,5}\leq 4,8;\\ a_{9}:f_{3,4}\leq 4;\\ a_{10}:f_{3,5}\leq 6;\\ a_{11}:f_{4,5}\leq 8;\\ a_{12}:f_{1,2}\geq 5;\\ a_{13}:f_{1,3}\geq 0;\\ a_{14}:f_{2,5}\geq 0;\\ a_{15}:f_{3,4}\geq 0;\\ a_{16}:f_{3,5}\geq 0;\\ a_{17}:f_{4,5}\geq 0; \end{array}$$

In the discussed numerical example, the constraints consist of a total of 17 equations and inequalities. Solving the above optimization maxflow problem through a standard linear programming package shows that the maximum possible flow of resources from the source  $x_1$  to  $x_5$  the sink is equal to v = 11,36.<sup>78</sup> This means that,  $\begin{cases} g_{ii}/(i, j) \in J \end{cases}$  with the motivation

<sup>&</sup>lt;sup>77</sup> Sgurev, V., Doukovska, L., Implication and Inference Rules in Multivalued Logic with Network Configuration. Proceedings of the 8th IEEE International Conference on Big Data, Knowledge and Control Systems Engineering - BdKCSE'23, 2–3 November 2023, Sofia, Bulgaria, IEEE Xplore, 2023, DOI:10.1109/BdKCSE59280.2023.10339696, pp. 1-4.

<sup>&</sup>lt;sup>78</sup> Sgurev, V., Drangajov, St., An Approach for Analysis of Decisions, Risks, and Losses at Antagonistic Conflicts. IFAC-PapersOnLine, 52, 25, Elsevier, 2020, DOI:10.1016/j.ifacol.2019.12.479, pp. 236-239.

used, the transported resource from the source to the consumer will increase by 11.36%.

The coefficients  $\{g_{ij}/(i, j) \in J\}$  on the arcs entering the vertices  $\{x_2, x_3, x_4\}$ , take an additional resource due to the motivation from those at the vertex in order to increase the final transported resource from  $v_0 = 10$  to v = 11.36%.

This visually demonstrates how motivation can influence decision-making and increase (or decrease) the transportation resource.

In the considered next optimization problem B, the arc weights (4.12) have the following values:

$$a_{1,2} = 3; a_{1,3} = 4; a_{2,5} = 3; a_{3,4} = 6; a_{3,5} = 3; a_{4,5} = 7;$$
 (4.22)

# **Optimization Problem B**

In this problem, the maximum generalized flow with motivations is determined, where the objective function L is defined in (4.17). This is known as the maincost-maxflow problem. The first step in it is solving optimization problem A to determine the maximum generalized network flow with motivations, where  $u_0 = 10$  is assumed. After solving this problem,  $v = v_{max} = 11,36$  is obtained.

Following the determination of  $v = v_{max}$ , the second step is taken, in which the equation  $a_5$  takes the form.

$$a_5 = -0.8f_{2,5} - 1.1f_{3,5} - 0.5f_{4,5} = -11.36;$$
 (4.23)

and the objective function L is equal to

$$L = 3f_{1,2} + 4f_{1,3} + 3f_{2,5} + 6f_{3,4} + 3f_{3,5} + 7f_{4,5} \rightarrow \min;$$
(4.24)

A new optimization problem is solved with the constraints from  $a_1$  to  $a_{17}$ , where instead of using  $a_5$ ,  $a_5$  from (4.23) is employed, and the objective function L is equal to (4.24). The linear programming software package used leads to the following arc flow functions: <sup>79</sup>

$$f_{1,2} = 4; f_{1,3} = 6; f_{2,5} = 4,8; f_{3,4} = 2,4; f_{3,5} = 6; f_{4,5} = 3,84;$$
 (4.25)

This is the optimal distribution of transported resources in a network across the specified segments (arcs) of the network, taking into account the influence of motivation. It has the same overall effect on increasing quantities and decreasing the total value of resource transportation. In optimization problem B – as well as in problem A – there is an increase in transported resources by 11.36%.

The total value of transported maximum possible resources with minimal cost is:

<sup>&</sup>lt;sup>79</sup> Sgurev, V. Artificial Neural Networks as a Network Flow with Capacities. Comptes Rendus de l'Academie Bulgare des Sciences. T. 71, No 9, 2018, pp. 1245-1252, ISSN 1310-1331.

$$L = \sum_{(i, j) \in J} a_{ij} f_{ij} = 3 \cdot 4 + 4 \cdot 6 + 3 \cdot 4, 8 + 6 \cdot 2, 4 + 3 \cdot 6 + 7 \cdot 3, 84 = 109,68 \text{ ed.}; \quad (4.26)$$

The maximum possible flow (resource) from point "B" in the amount of 10 cannot be transported, taking into account the motivations, to point  $x_5$  in quantity  $v = v_{max}$ , It cannot be transported for a value less than 109.68 units.

From the obtained solutions, it follows that the arcs  $\{x_{1,3}; x_{2,5}\}$  are saturated, i.e

$$f_{1,3} = C_{1,3} = 6$$
 и  $f_{2,5} = C_{2,5} = 4,8;$  (4.27)

They form a cut

$$(x_0, \bar{x}_0) = \{x_{1,3}; x_{2,5}\}; (\bar{x}_0, x_0) = \emptyset;$$
 (4.28)

where  $\emptyset$  is an empty set.

On this cut, the flow function and the capacity of the cut can be defined as follows:

 $f(x_0, \bar{x}_0) = f_{1,3} + f_{2,5} = 6 + 4,8 = 10,8; \qquad (4.29)$ 

$$C(x_0, \bar{x}_0) = C_{1,3} + C_{2,5} = 6 + 4,8 = 10,8; \qquad (4.30)$$

$$f(\bar{x}_0, x_0) = 0; \ C(\bar{x}_0, x_0) = 0;$$
 (4.31)

Then, for this cut, the well-known mincut-maxflow theorem by Ford and Fulkerson holds, according to which, for the given numerical example, it is true: <sup>80 81</sup>

$$u \le f(x_0, \bar{x}_0) - f(\bar{x}_0, x_0) = C(x_0, \bar{x}_0) = 10.8; \qquad (4.32)$$

From the obtained dependencies, it follows that if the owner of the transportation system and its management wants to increase the total amount of transported resources from  $x_1$  to  $x_5$ , then they can do so by increasing only the capacities  $C_{1,3}$  and  $C_{2,5}$ . The other capacities require expenses and increments.

# **Optimization problem C**

Through this problem, the minimum (maximum) value of a generalized network flow is determined with motivations for a fixed initial value of the transported resource. Moreover, it is not necessary for the resource to be maximal, i.e., it is not required to be equal to  $v_{\text{max}}$ . This implies that instead of  $a_5$  from (4.23) should be used  $a_5$  from (4.21). For this purpose, an optimization network flow problem with coefficients is solved.  $\{g_{ij}/(i, j) \in J\}$  and with a fixed initial amount of resources  $v_0$ .<sup>82</sup>

Let  $v_0 = 10$ , where the parameters  $\{g_{ij}/(i, j) \in J\}, \{C_{ij}/(i, j) \in J\} \bowtie \{a_{ij}/(i, j) \in J\}$  and

<sup>80</sup> Сгурев, В. Мрежови потоци с общи ограничения. София, Издателство на БАН, 1991

<sup>&</sup>lt;sup>81</sup> Sgurev, V., Drangajov, St., Network Risks in Markov Decision Processes. Proc. of the 21st International Conference on Computer Systems and Technologies - CompSysTech'20, Association for Computing Machinery, New York, United States, 2020, ISBN:978-1-4503-7768-3, DOI:10.1145/3407982.3408015, pp. 7-10

<sup>&</sup>lt;sup>82</sup> Sgurev, V., St, Drangajov. Risk estimation and stochastic control of innovation processes, Cybernetics and Information Technologies (CIT), Print ISSN 1311- 9702; Online ISSN 1314-4081, DOI 10.2478/cait-2014-0012, Vol. 14,No 1, 2014, pp. 3-10

coincide with those (4.18), from (4.19) and (4.22) except for the requirement

$$C_{1,2} = 4;$$
 (4.33)

After solving the linear programming problem with the linear form L from (4.17) under the constraints from  $a_1$  to  $a_{17}$ , taking into account (4.23), optimal arc flow functions from (4.25) will be obtained.

This solution coincides with the optimal solution to the previous problem B. The main reason for this is that six arcs –  $\{x_{1,2}; x_{2,5}; x_{1,3}; x_{3,5}\}$  from the graph in Fig. 4.1 – receive saturated flows, and there are no opportunities to maneuver the flows along the individual arcs.

- 1. The results obtained in this chapter provide the opportunity to incorporate motivation into a discrete decision-making system through a generalized network approach, visually demonstrating the usefulness of such an approach. It enables bridging the gap between psychological processes, especially motivation, and rigorous models of discrete decision-making systems.
- 2. The positive outcomes open up new avenues for research and the development of novel decision-making systems that consider other psychological processes.
- **3.** It is of interest to explore the behavior of systems within the described class of discrete decision-making systems using a generalized network flow when motivation dynamically changes and has a partially stochastic nature.

#### CONCLUSION

The present dissertation is dedicated to the exploration of motivation as a subjective factor in the development of a decision-making system.

Motivation is a complex set of psychological processes that determine the strength and direction of human behavior. It plays a crucial role in the daily necessity of making various decisions. Human motivation is associated with social orientation, diversity, adaptability, and the influence of intellect, speech, and consciousness.

When creating computer decision-making systems or decision support systems, it is necessary to consider the motivation of the various subjects involved in the process. It is also essential to be familiar with modern decision-making software systems.

# Validation of the Results

The results achieved during the research for the dissertation work are as follows:

- An extensive analysis of motivation and its role in decision-making systems has been conducted. The overall analysis contributes to a broader understanding of the multifaceted nature and complexity of motivation and its role in understanding the decision-making process – Chapter 1.
- A comparative analysis of different types of motivation and motivational theories and models has been carried out, emphasizing their characteristics, driving forces, and impact on individuals' behavior – Chapter 1.
- A comprehensive, multi-layered overview, providing a multidisciplinary and systematic description of concepts from the theory of decision-making, as well as decision support systems, has been conducted Chapter 2.
- A formal description of discrete decision-making systems with consideration of motivation has been developed. Concepts related to sets, graphs, and network flows have been explored – Chapter 3.
- A numerical example of a discrete decision-making system with consideration of motivation has been implemented Chapter 4.
- A classification of motivational theories has been proposed based on a comprehensive review, taking into account their influence on decision-making systems or the support of these decisions. Preference has been given to motivations that are related to the work of operators in real-time control systems.
- It has been observed that in most cases, motivation aligns well with discrete decisionmaking systems.
- It has been determined that discrete decision-making systems based on network flows provide the opportunity for a relatively accurate and adequate modeling of discrete decision-making systems when considering motivation.
- It has been noted that the most suitable are generalized network flows with coefficients for increasing or decreasing flows on individual arcs. Through them, models for decision-making can be created, incorporating elements from motivation theory, graphs, and flows over them. These arc coefficients reflect the influence of motivation on decision-making positively (if KIJ > 1) or negatively (if 0 < KIJ < 1).</li>
- The functionality of the proposed discrete generalized network flow with coefficients for amplification and reduction of their influence has been suggested and demonstrated for use in decision-making systems with motivation, based on a numerical example.

• The capabilities of the proposed generalized network flow have been outlined for modeling psychological processes with a broader scope than motivation.

# List on the publications on the dissertation work

1. **Tsopanova, E.,** Motivation in Decision-Making Systems. Problems of Engineering Cybernetics and Robotics, 79, Prof. Marin Drinov Academic Publishing House, 2023, ISSN:2738-7356, DOI:10.7546/PECR.79.23.04, pp. 67-74.

2. **Tsopanova, E.,** The Role of Emotions in Decision-Making Systems. Problems of Engineering Cybernetics and Robotics, 80, Prof. Marin Drinov Academic Publishing House, 2023, ISSN:2738-7356, DOI:10.7546/PECR. 80.23.04, pp. 33-40.

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