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Models and Methods for Optimizing and Managing Portfolio using Time Series

ABSTRACT OF PhD THESIS

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The PhD thesis was discussed and allowed to be defended during an extended session of the Department of information processes and decision support systems at IICT-BAS, which had been held on 12.03.2020.

The full volume of the dissertation is 130 pages. It consists of an introduction and three chapters. The list of references contains 253 items. The work includes three Appendices. The text of the dissertation includes 22 tables and 17 figures.

The defense of the PhD thesis had been held on .... 2020 at .... in Room 507, Block 2, IICT-BAS.

Approved by Supervising Committee:

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Title: **Models and methods for optimizing and managing portfolio using time series**

Introduction

Mathematical modeling in the financial field is of great scientific and practical interest, since the study of economic systems such as stock exchanges, banks, insurance and investment companies is important both to the participants themselves in these systems and to the state in terms of the functioning of the financial system. Regarding mathematical modeling, in particular, portfolio optimization, in our country has contributed to the works of Rachev [14, 15], Stoyanov [21], Stoilov [20], and others.

One of the key problems of the decision-maker is known about the distribution of the future price of financial instruments. Knowledge of the principle of distribution of future value as a random variable is necessary to evaluate the mathematical expectation, as well as the variance and other risk measures used in optimal portfolio models. These facts make it necessary for the decision-maker to construct complex models of optimal portfolio with different optimization criteria. For these reasons, the mathematical modeling of a financial decision maker in the dissertation is up-to-date.

The dissertation is structured in an introduction, 3 chapters, conclusion, contributions, list of publications, declaration of originality and bibliography.

The first chapter analyzes the existing mathematical models of the decision maker, outlining their advantages and disadvantages. The basis of this analysis justifies the need for new, intelligent, efficient, yet sufficiently accurate methods and techniques, such as evolutionary algorithms. Perspective research areas are identified, and the Aim and specific tasks of the dissertation research are formulated.

The second chapter presents the proposed bicriteria optimization model for the formation of a portfolio of different assets. To solve this model, was proposed a Hybrid Algorithm for portfolio selection. This hybrid algorithm for portfolio selection is based on the combination of both methods Firefly (FFA) and the Pattern search (PS). This combines the advantages of both methods to find a global optimal solution (in case the objective function is multimodal) and the precise localization of the optimum using the technique of shrinking the network size to a predetermined tolerance in Pattern search. Using the proposed bicriteria model and hybrid algorithm, a methodology for portfolio selection is presented.

The third chapter describes the results of the tests performed for the formulated two models - the first with 3 assets, based on historical data from 2008 to 2018, and the second with 6 assets with historical data from 2007 to 2018, for the problem of portfolio selection. Through two optimization approaches: *Interior Point* in Matlab and the proposed hybrid algorithm for portfolio selection have solved the relevant optimization problems, and the obtained results were compared by three criteria - the number of iterations, the number of calculations of the objective function, and the time for which the calculations were performed. The obtained numerical results showed that the proposed bicriteria model and a hybrid algorithm are applicable and effective in solving the portfolio optimization problem with constraints for multi-periods. It has been found that the proposed hybrid algorithm finds a precise solution in the optimization of portfolios in a relatively short time, which proves its effectiveness to solve real problems of portfolio selection. It is shown that this hybrid algorithm overcomes one of the disadvantages of accurate mathematical methods for time series, due to their considerable computational difficulty.

The conclusion summarizes and analyzes the obtained results and gives a possible further direction for research.

Chapter 1. Analysis of models and methods for portfolio optimization and management

The first chapter presents the mathematical modeling of the decision maker's actions as conditioned by an adequate description of uncertainty, the need to measure statistical regularities, to account for and to measure a huge volume of dynamically changing information. Investment portfolio management is a key fundamental process in managing investments in the financial field. The choice of investment policy involves defining the investor's goals and the volume of investments, assessing the types of assets and selecting the most favorable ones, taking into account the factors of profitability and risk. From the point of view of psychology for studying the causes and peculiarities of human behavior, classical models for the formation of an optimal investment portfolio and models considering the dynamics in the value of different financial instruments, the estimation of parameters based on statistical data is examined. The basis of this analysis justifies the need for new, intelligent, efficient, yet sufficiently accurate methods and techniques, such as evolutionary algorithms.

As a result of the review, the Aim of the dissertation research was formulated, namely: to propose models and methods / algorithms for portfolio optimization using time series in the financial field. To accomplish this, the following specific tasks need to be accomplished:

- to review the existing evolutionary single and multi-criteria algorithms for portfolio optimization;

- to propose a portfolio optimization model that provide individual qualities of the portfolio;
- to propose a portfolio optimization approach / methodology using time series.
- to propose a portfolio optimization and management algorithm for a given criteria that is sufficiently accurate and fast.
- to conduct numerical experiments to test the performance of the proposed models and algorithms.
- to develop a toolbox - a set of software modules of Matlab to enable the above tasks to be accomplished.

Chapter 2. Optimization by a combination of FFA-PS. Hybrid evolutionary algorithm and methodology for portfolio selection

This chapter describes the proposed bicriteria optimization model and the proposed hybrid algorithm то solve the problem of portfolio optimization and management under given constraints. A summary methodology for portfolio selection is also presented.

2.1. A portfolio model for the multi-period

Studies in behavioral portfolio theory have shown that investors may have multiple cognitive distortions (e.g. mental accounting, loss aversion, etc.) that play an important role in the decision-making process [16, 22].

2.1.1 Output conditions of the portfolio problem for the multi-period

Let there are one riskless asset a_0 and n risky assets $\{a_1, \dots, a_n\}$ in security market for trading. An investor wants to make a multi-period investment strategy, where the investment duration is divided into T periods. Suppose that the investor holds *a* portfolio $X(t) = [x_{0,t}, x_{1,t}, \dots, x_{n,t}]^T$ at time t, where $x_{0,t}$ denotes the wealth of riskless asset a_0 at time t, and $x_{i,t}$ denotes the wealth of risky asset a_i at time t, $i = 1, \dots, n$, $t = 0, \dots, T$.

Let $r_{0,t}$ and $r_{i,t}$ be the return of riskless asset a_0 and risky asset a_i at period t respectively, $i = 1, \dots, n$, $t = 1, \dots, T$. Then the wealth of asset a_i at time t is:

$$x_{i,t} = (r_{i,t} + 1)^{x+}{}_{i,t-1},\ i = 0, 1, \dots, n,\ t = 1, \dots, T \tag{2.1}$$

Using the recursive relationship in the multi-period investment, Eq. (1) can be rewritten as:

$$x_{i,t} = g_i(1,t)x_{i,0} + \sum_{j=1}^{t} g_i(j,t)\Delta x_{i,j-1},\ i = 0,1,\dots,n,\ t = 1,\dots,T; \tag{2.2}$$

where $g_i(j,t)$ denotes the cumulative return of asset a_i from period j to period t, $g_i(j,t) = (r_{i,t}+1)(r_{i,t-1}+1)\ldots(r_{i,j}+1)$, $g_i(t,t) = r_{i,t}+1$.

From Eq. (2.2), the multi-period portfolio wealth at time t is given by

$$W_t = \sum_{i=0}^{n} x_{i,t} = \sum_{i=0}^{n} \sum_{j=1}^{t} g_i(j,t)\xi_{i,j-1}, \qquad t = 1,\ldots,T \tag{2.3}$$

where $\xi_{i,0} = x_{i,0} + \Delta x_{i,0}$; $\xi_{i,j} = \Delta x_{i,j}$; $i = 0,1,\ldots,n$; $j = 1,\ldots, T-1$

2.1.2 Robust optimization approach

The goal of robust optimization is to find a solution which is feasible for all possible data realizations and optimal subject to a certain level of conservatism. Following the notation in [4]. In [16] define a parameter $\Gamma_t \in \Re^+$ and a subset S_t to control the level of conservatism in W_t, where $\Gamma_t \in [0,|J_t|]$, $S_t \subseteq J_t$, $|S_t| = \lfloor \Gamma_t \rfloor$, and $J_t = \{(i,j) \mid i = 1,\ldots,n,\ j = 1,\ldots,t\}$.

2.2 Dynamic prospect theory value function

The prospect theory value function introduced in [12] is expressed by

$$PV(W) = \begin{cases} (W-\hat{y})^{\alpha}, W \geq \hat{y} \\ -\lambda(\hat{y}-W)^{\beta}, W < \hat{y} \end{cases} \tag{2.9}$$

where PV denotes the prospect theory value (PT value) function, W denotes the portfolio wealth; λ denote the loss aversion ratio; $\hat{y}$ denotes the given reference wealth; α and β denote the curvature parameters for gains and losses respectively. Tversky and Kahneman [21] experimentally determined the values of α = β = 0,88; λ = 2,25, which are considered as appropriate for describing most decision maker's behavior and used to make optimal decision-makings.

2.3 Bicriteria model for portfolio selection

In multi-criteria optimization problems, several criteria are optimized simultaneously, and in general there is no single alternative that optimizes all criteria. Unlike single criterial optimization, the solution to a multi-criteria problem can be seen as a concept rather than a definition. With this type of optimization, there is usually no optimal solution that satisfies all the criteria, and most often at least two of the criteria are in conflict, i.e. improving one of the criteria leads to a deterioration of at least one other criterion among the other criteria in the problem. Therefore, it is necessary to make a compromise solution that sufficiently satisfies the preferences of the decision-maker. Harry Markowitz's classic theory of building an optimal portfolio addresses two criteria (two functions): the function of return $f_{\text{return}}(x)$, which must be maximized, and the function of risk $f_{\text{risk}}(x)$, which must be minimized.

Based on a detailed exposition in the dissertation research is proposed the following *modification* of the classic Markowitz model (1.1):

$$\min_x \ \frac{1}{2} x^T \zeta x \tag{2.22}$$

$$\max_x \ \mu^T x \tag{2.23}$$

subject to the constraints:

$$\left| \sum_{i=1}^{n} x_i^2 - div\_target \right| \le 0{,}05, \tag{2.24}$$

$$\sum_{i=0}^{n} \Delta x_{i,t} + \sum_{i=1}^{n} c_{i,t} \, |\Delta x_{i,t}| = 0, \quad t = 1,\ldots,T-1; \tag{2.25}$$

$$\sum_{i=1}^{n} (x_i) = 1 \tag{2.26}$$

$$l_i \le x_i \le u_i \,, \; i = 0,1,\ldots,n; \; \boldsymbol{t = 1,\ldots,T-1.} \tag{2.27}$$

The formulated bicriteria model (2.22) - (2.27) aims at the same time to optimize two mutually contradictory criteria, namely to minimize risk and at the same time maximize profit / return.

The advantages of the formulated optimization model (2.22) - (2.27) are the following: by using the constraint (2.24), the expected diversification of the portfolio is achieved; the constraint (2.25) ensures the robustness of the model, obliging the investor to fit within only the initial capital used; the constraints (2.27) set the lower and upper bonds of each of the assets involved in the portfolio and are relevant to the relative diversification of the portfolio; equality (2.26) ensures that exactly 100% of the invested capital will be used for the assets; the formulation of relevant optimization problems, based on the proposal, the bicriteria model (2.22) - (2.27) enables the solutions of these problems to determine Pareto-optimal solutions, i.e. the combination of portfolio assets.

The disadvantages of the bicriteria optimization model (2.22) - (2.27) are the following: the criterion (2.22) and the constraint (2.24) are nonlinear. This leads to the extreme sensitivity of the solutions to any change in the input parameters. Finding a global optimal solution in this case is the NP hard problem [10]. Also, getting a representative sample of solutions across the Pareto Front is a laborious task.

Therefore, for the above formulated multi-criteria optimization model (2.22) - (2.27) it is necessary to choose the appropriate approach to solve. To solve model (2.22) -

(2.27), a scalar approach was chosen, reformulating the model into a single criterial using the method of ε-constraints of Haimes et all. [11], analogous to the Markowitz model of mean variance (1.26) - (1.29). This method optimizes one of the criteria and transforms the other criteria into constraints. This results in proven weak Pareto-optimal solutions [19]. In the specific case, the criteria of the maximum of return (2.23) are reformulated as a constraint, requiring the return to be greater than or equal to a given percentage of R (see 2.29). The final appearance of the proposed modification of the model (1.1) and model (1.26) - (1.29) by adding the additional constraints (2.24) and (2.25) is:

$$\min_x \ \frac{1}{2} x^T \zeta x \tag{2.28}$$

subject to the constraints:

$$\mu^T x \geq R \tag{2.29}$$

$$\left|\sum_{i=1}^{n} x_i^2 - div\_target\right| \leq 0{,}05, \tag{2.30}$$

$$\sum_{i=0}^{n} \Delta x_{i,t} + \sum_{i=1}^{n} c_{i,t} \, |\Delta x_{i,t}| = 0, \quad t = 1,\ldots,T-1; \tag{2.31}$$

$$\sum_{i=1}^{n} (x_i) = 1 \tag{2.32}$$

$$l_i \leq x_i \leq u_i\,,\ i = 0,1,\ldots,n;\ \boldsymbol{t = 1,\ldots,T-1.} \tag{2.33}$$

The following methodology is proposed to generate many of the Pareto front solutions:

i. A series of ten optimization problems is solved at a different percentage of the minimum expected return R in the constraint (2.29). In doing so, R accepts the values: {6%, 6.5%, 7%, 7.5%, 8%, 8.5%, 9%, 9.5%, 10%, 10.5%}. As a result, ten Pareto front points are generated and the investor or decision-maker is enabled to choose the appropriate final compromise solution.

ii. The diversification target value is assumed to be *div_target* = 0,33333.

iii. The lower bound for each asset is assumed to be 0, and the upper bound is taken to be the value of all capital.

iv. The novelty in the proposed model (2.28) – (2.33) compared to the models in [7, 17], is taking into account the negative behavioral characteristics of the investor or decision-maker and ensuring robustness through the constraint (2.31) without going beyond the original capital. Moreover, the constraint (2.30) guarantees a sufficient degree of diversification of the resulting optimal portfolio. By varying the value of R in the constraint (2.29), the end result is an optimal portfolio with the expected rate of return.

2.4 A hybrid evolutionary algorithm for portfolio selection based on FFA and Pattern search

It should be noted that the formulated model (2.28) - (2.33) leads to the solution of a complex NP-hard problem of nonlinear programming. The traditional robust optimization techniques [2, 3, 5] may fail to obtain the optimal solution. In order to solve the portfolio model effectively, a hybrid algorithm for portfolio selection based on FFA and PS is proposed. This combines the ability to find a global optimal solution (in case the objectives function is multimodal) and the precise localization of the optimum through shrinking the mesh size to a predetermined solution mesh tolerance in Pattern search.

The proposed new hybrid algorithm for portfolio selection based on FFA-PS is presented below as follows:

Step 1. Determine the FFA parameters: α, β_0 and γ. Set iteration limit – *itlim*. Set diversification limit – *divlim*.

Set iteration counter k=0 and set diversification counter

divcount=0.

Step 2. Initialize fireflies' positions $\{P^k(1), \ldots, P^k(S)\}$, using the two- stage initialization strategy [12, 16].

While (there is improvement of at least one firefly brightness repeat):

Step 3. For each firefly $P^k(i)$ find the brightest firefly it can see.

Step 4. Calculate the new fireflies' positions and update the fireflies' swarm. Update iteration counter: $k = k+1$. Check the stopping criteria and if it is met - go to Step 6.

End While

Step 5. If mod (k/100) = 0, start the *Pattern search* procedure.

Step 6. Show the best obtained solution to the decision maker.

Step 7. Check the stopping criteria. If any of the stopping criteria is met - go to Step 8. Otherwise set a diversification search. Update the diversification counter:

divcont = *divcount* + 1.

Step 8. END.

Applying scalarization, the robust portfolio model (2.28) - (2.33) includes one criteria / single objective function. This model is a single objective model. It could be extended to a multi-objective model, taking into account the minimum transaction lots, tax and cardinality constraints, which exist in the real world.

2.5 Methodology for portfolio selection

There are several basic stages to a portfolio selection of different assets. This dissertation work proposes a generalized portfolio selection methodology that is implemented by performing the following procedures, as shown in Fig. 5.

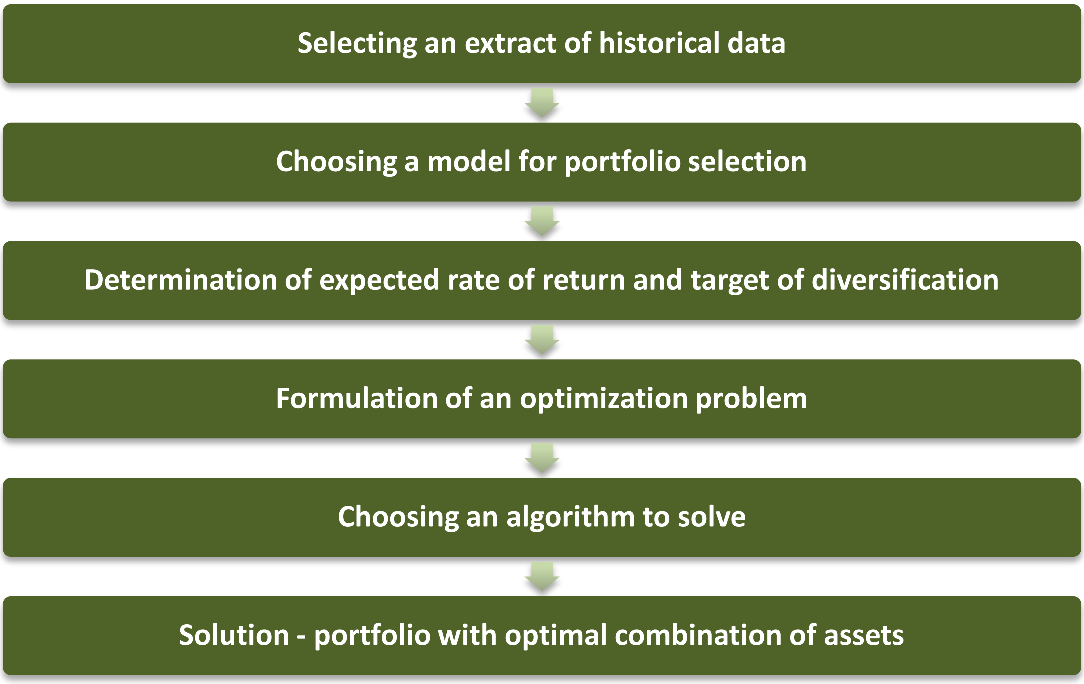

Fig. 5 A generalized methodology for portfolio selection

Chapter 3. Results of numerical experiments

This chapter describes the numerical experiments performed on the problem based on the (2.28) - (2.33) model for portfolio optimization with specific constraints. Based on historical data, two experimental models for three and six assets have been formulated to research the effectiveness of the proposed hybrid algorithm for portfolio selection. Through two optimization approaches: *Interior Point* in Matlab (*fmincon* solver) and the proposed new hybrid algorithm for portfolio selection results were obtained for the two models and compared.

3.1 A three-asset model and 10 years historical data

In the first experiment, a modified Markowitz's mean variance model (2.28) - (2.33) is applied, to construct an optimal portfolio of French stocks, US Treasury bonds and deposits with a 1% *constant positive* interest rate. Historical returns on these three assets over 10 years are used to calculate the geometric mean, the correlation matrix, and the covariance matrix, which served to formulate of the portfolio optimization problem as a quadratic programming problem, so that the summarized expected rate of return on the three assets in the portfolio is not less than in advance given target value.

The formulation of the portfolio optimization problem is:

$$\text{MIN } F = [0.010616427928 x_{BONDS}{}^2 + 2.(-0.000337020935) x_{BONDS}.x_{STOCKS} + \\ + 2.(-0.000000539066) x_{BONDS}.x_{DEPOSITS} + 0.002284457879 x_{STOCKS}{}^2 + \\ + 2.0.000000462459 x_{STOCKS}.x_{DEPOSITS} + 0.000000002090 x_{DEPOSITS}{}^2], \quad (3.6)$$

subject to:

$$\mu^T \boldsymbol{x} = -0.22\, \boldsymbol{x}_{BONDS} + 0.18 \boldsymbol{x}_{STOCKS} + 0.08456\, \boldsymbol{x}_{DEPOSITS} \geq \boldsymbol{R}$$

$$|\sum_{i=1}^{3} x_i^2 - 0{,}33333| \leq 0{,}05,$$

$$\sum_{i=0}^{3} \Delta x_{i,t} + \sum_{i=1}^{3} c_{i,t} \,|\, \Delta x_{i,t} \,| = 0, \quad t = 1,\ldots,T-1;$$

$$\boldsymbol{x}_{BONDS} + \boldsymbol{x}_{STOCKS} + \boldsymbol{x}_{DEPOSITS} = \mathbf{1}$$

$$\boldsymbol{x}_{BONDS}\, , \boldsymbol{x}_{STOCKS}\, , \boldsymbol{x}_{DEPOSITS} \geq \mathbf{0}$$

The above problem is solved 10 times for different expected rates of return in percentages: R = 6%, R = 6,5%, ..., R = 10,5% using incremental step of 0,5%. First the optimization has been carried out with help of the developed hybrid algorithm for portfolio selection, and after than by *fmincon* solver of Matlab “Optimization Toolbox” [18], by means of the *Interior point* algorithm, whose advantage is, that allows for all kinds of constraints.

3.1.1 Results from optimizing a 3-asset portfolio through the proposed bicriteria model and using the hybrid algorithm and with the Matlab fmincon solver

Table 9 presents the results of the numerical experiments performed with the hybrid algorithm. For each parameter value R from 6% to 10.5%, ten evaluations with the Hybrid algorithm were performed. The red solution indicates the best solution, in advance given target value of expected return for optimization of portfolio with three assets based on historical data for a period of 120 months or 10 years.

Except Table 9. Results via hybrid algorithm for three assets and a 6 % returm

№	Value of Objective function	Optimal portfolio		
		Bonds	Stocks	Deposits
6% expected rate of return				
1	5.794066940371048e-04	0.195533308441747	0.305914684499346	0.498582406818212
2	5.431084013674748e-04	0.159477056833486	0.370032551199147	0.470295740570779
3	5.871732296605341e-04	0.148121541017727	0.416191315799532	0.436124992962780

4	6.599734985032965e-04	0.152215198424261	0.448699495334937	0.397648450327024
5	6.051161402039595e-04	0.194374079230470	0.328854690650089	0.476588428394402
6	5.822353927709721e-04	0.149571215636529	0.411098896797715	0.439676629466057
7	6.626166141358330e-04	0.150578769257309	0.452484170030794	0.394181626971399
8	6.417347482106582e-04	0.153503697558590	0.437228280942269	0.410088866107027
9	7.333492161955965e-04	0.160118041513668	0.473496518659992	0.365081714458605
10	5.352082093821857e-04	0.162491508733274	0.358816119183875	0.478759042850245

In Fig. 6 shows trends using three assets and different expectation rates of return.

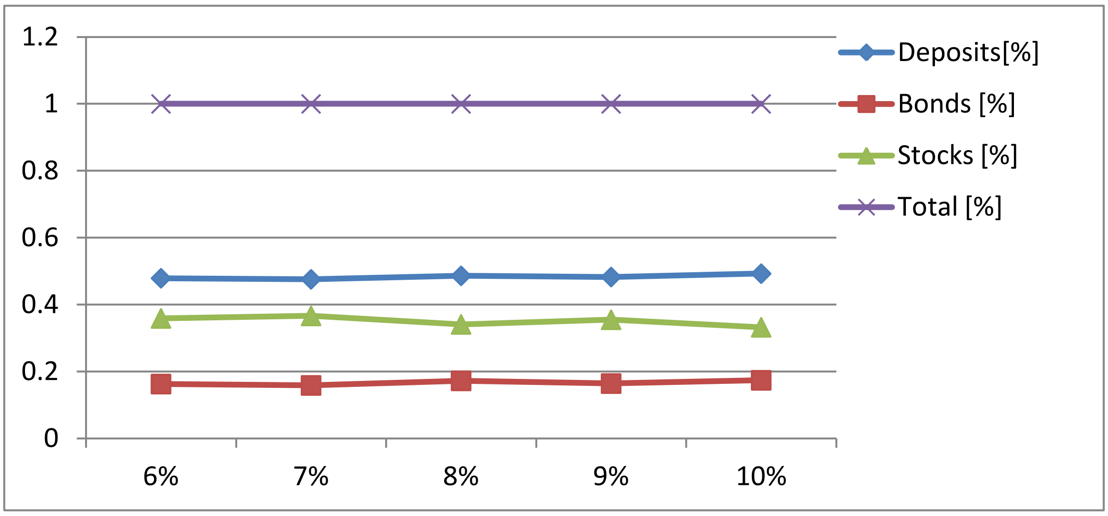

Fig. 6 Trend for experimental model with three assets

3.1.2 Results on optimizing a three-asset portfolio through the proposed bicriteria model and using the Matlab's fmincon solver

The results of the optimization for all ten optimization problem with the second approach for the different expected rate of returns on the *fmincon* solver by Matlab's "Optimization Toolbox" are summarized in Table 10 as follows:

*Table 10. Results via Matlab's **fmincon** solver*

Value of Objective function *f*	**Exp. rate of return *R* [%]**	**Iteration**	**Total objective function evaluations**	**Optimal portfolio**		
				Bonds [%]	Stocks [%]	Deposits [%]
5.387437690212220E-4	6	22	98	0.168966	0.347022	0.484012
5.388831287378297E-4	6.5	18	89	0.168262	0.348713	0.483025
5.430593589223452E-4	7	22	94	0.161816	0.365296	0.472888
5.640361640922369E-4	7.5	22	94	0.154267	0.392602	0.453131
6.311263811064216E-4	8	17	76	0.151272	0.435088	0.413640
6.403115399398162E-4	8.5	18	98	0.144184	0.445191	0.410626
6.710914897734869E-4	9	21	118	0.144424	0.465421	0.387285
6.821441041356491E-4	9.5	16	90	0.138080	0.464479	0.393496
6.883647044897766E-4	10	16	84	0.130139	0.467585	0.396123
6.959992168393732E-4	10.5	20	108	0.127702	0.469306	0.396252

In Fig. 7 and Fig. 8 shows the optimal configurations of a portfolio with three assets at different expected rates of return

	Bonds		*Stocks*		*Deposits*

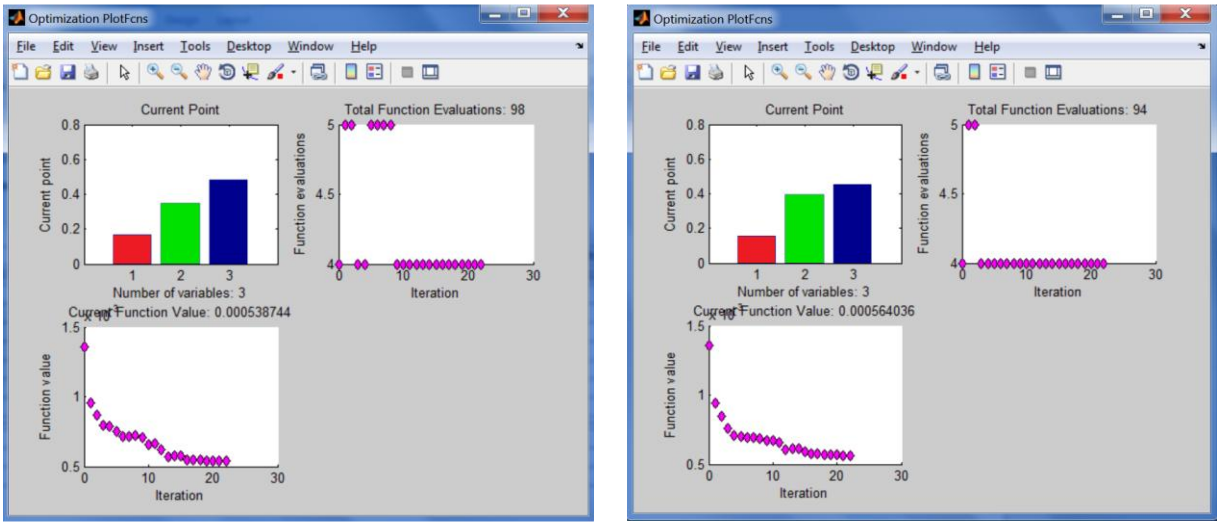

Fig. 7 Optimal portfolios for expected rates of return 6% and 7,5%

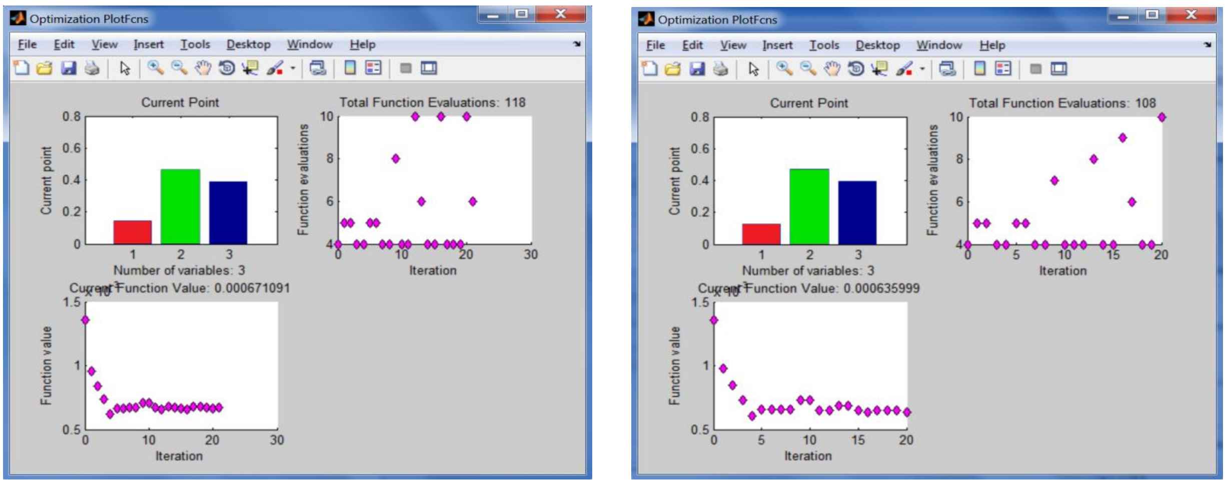

Fig. 8 Optimal portfolios for expected rates of return 9 % and 10,5 %

The points obtained for the Pareto front are shown in Fig. 9. Each calculated Pareto-optimal portfolio is represented as a triangle located on the Pareto-surface in this case with coordinates 1) the risk measures (standard deviation) and 2) the expected return.

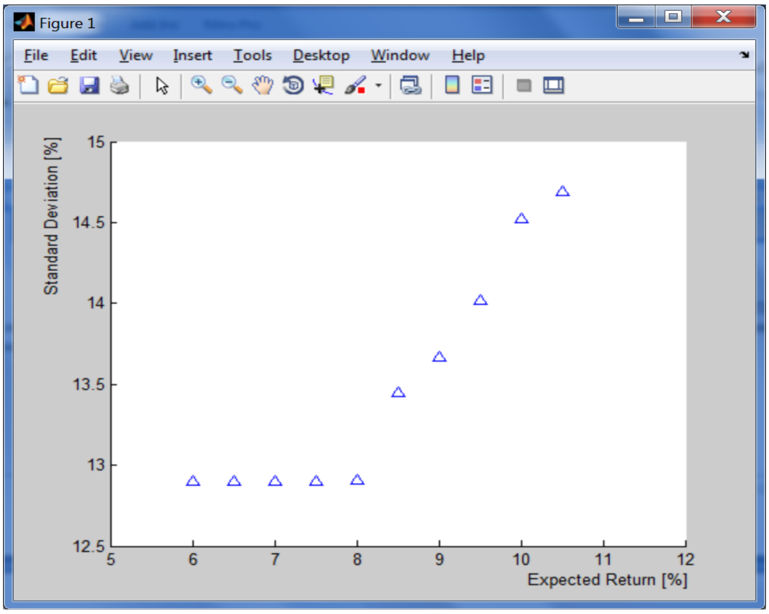

Fig. 9. Pareto front approximation via fmincon solver

3.1.3 Comparison of obtained results for the three-asset model with the both approaches

This section compares the results for the first model with the hybrid algorithm and the Matlab's *fmincon* solver. Table 12 presents the results of the calculations for the "120 months, 3 assets" model, with the red highlighting is the best solution by expected rate of return of 6% to 10.5%.

Except Table 12. Comparison of the results for the first model with the both approaches

The best solution	Value of Objective function	Optimal portfolio		
		Bonds	Stocks	Deposits
6% expected rate of return				
Hyb. A	5.352082093821857e-04	0.162491508733274	0.358816119183875	0.478759042850245
MATLAB	5.387437690212220e-04	0.168966	0.347022	0.484012
6,5% expected rate of return				
Hyb. A	5.261467870846590e-04	0.162489036358221	0.352846326472330	0.483859796759362
MATLAB	5.388831287378297e-04	0.168262	0.348713	0.483025

It can be seen that in all ten evaluations with different expected rates of return, the Hybrid algorithm has obtained more accurate solutions than the *fmincon* solver. In this case, the differences in the value of objective function reach 1.6e-04 (see Table 12, return 10.5%). This difference seems insignificant, but it is related to significant differences in the percentage of assets in the portfolio. Also, the more is the volume of investment in a portfolio, and the greater will be the profit/gain due to use of more precise solutions.

In Fig. 10 shows a comparison of the results obtained with the both approaches with a return of 10.5%.

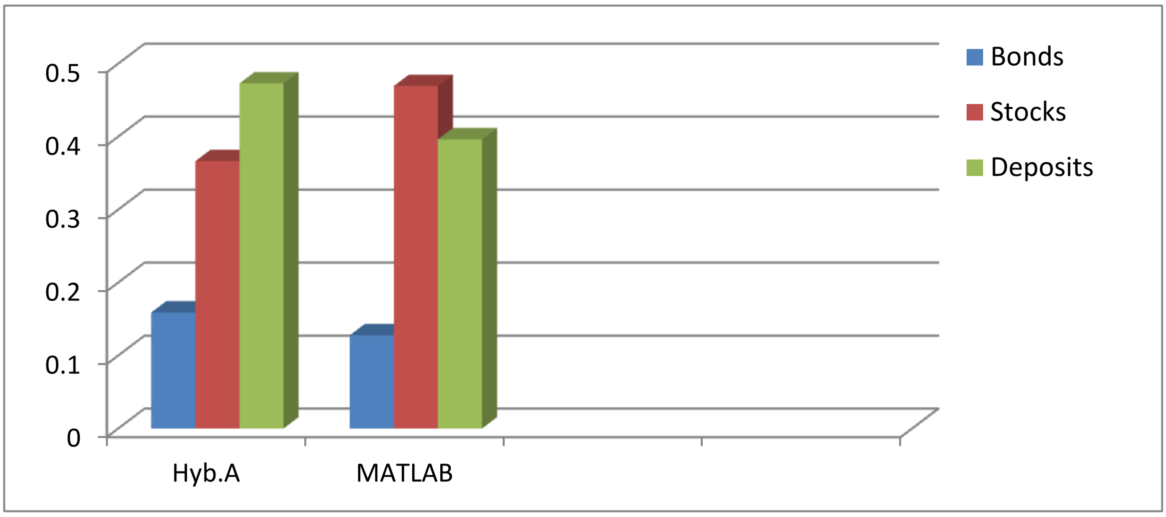

Fig. 10: Optimal portfolio with the both approaches and exp. rate of return 10,5%.

In Fig. 11 shows the trend of the value of the objective function for the three assets obtained by the hybrid algorithm and the Matlab's ***fmincon*** solver over the whole interval of expected rates of return.

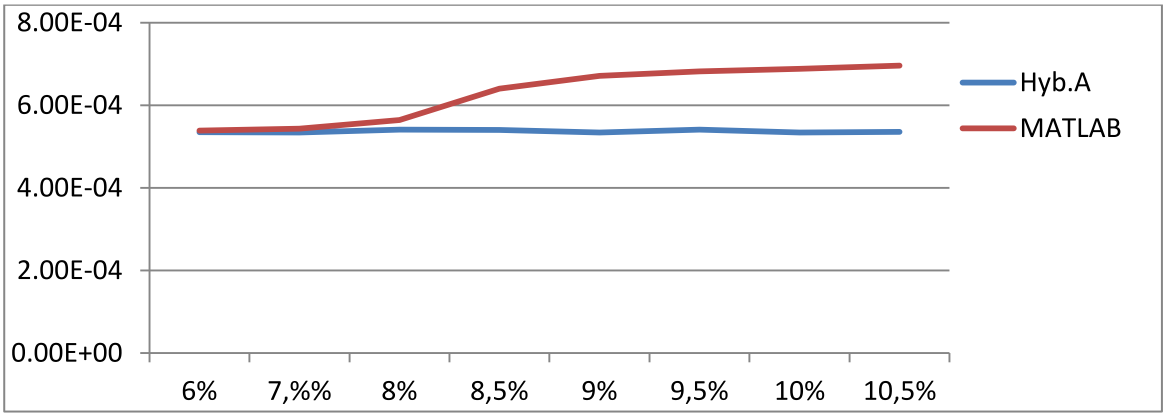

Fig. 11: Trend of the value of objective function for the model with three assets

3.2 A six-asset model and historical data for 10 years and 11 months

In the second experiment, a modified Markowitz's mean variance model (2.28) - (2.33) is applied, to construct an optimal portfolio of American stocks, Japanese stocks, French and Swiss stocks, American Treasury bonds and 1% *constant negative* interest rate. The historical 131-month returns on these six assets are used to calculate the geometric means, the correlation matrix, and the covariance matrix, which served to formulate of the portfolio optimization problem as a quadratic programming problem, so that the summarized expected rate of return on the six assets in the portfolio is not less than in advance given target value.

The experiment was complemented by a similar approach proposed by Cesarone et al. [8, 9]. The authors include in the portfolio optimization model the real constraints that no more than K assets should be contained in the portfolio (cardinal constraints). They also include a quantitative constraint so that each asset in the portfolio is limited at a certain interval. The second constraint is too strict and can be omitted to obtain an optimal solution of higher quality. The cardinal constraint is not introduced in the present dissertation and the optimization process is divided into two stages. In the first stage, the optimization is done with all available assets and for a certain return. Then the best K assets (in this case, K = 3) are included in the second optimization stage, where the optimization problem is solved several times with different returns. Finally, a specific return and optimal solution is chosen.

The formulation of the portfolio optimization problem is:

$$\begin{aligned} \text{MIN } F = [&0.001745x_1^2 + 2.(\ 0.000490\ )x_1.x_2 + 2.(\ 0.000521)x_1.x_3 + 2.(\ 0.000898\ )x_1.x_4 + \\ &+ 2.(0.0000003)\ x_1.x_5 + 2.(0.000367)x_1.x_6 + 0.002190x_2^2 + \\ &+ 2.(0.001883)x_2.x_3 + 2.(0.001306)x_2.x_4 - 2.(0.00000008)x_2.x_5 + \\ &+ 2.(0.001641)x_2.x_6 + 0.003336x_3^2 + 2.(0.002022)x_3.x_4 - 2.(0.0000002)x_3.x_5 + \\ &+ 2.(0.001703)x_3.x_6 + 0.009293x_4^2 - 2.(0.00000009)x_4.x_5 + \\ &+ 2.(\ 0.001174)x_4.x_6 + 0.00000000008x_5^2 - 2.(0.00000008)x_5.x_6 + \\ &+ 0.001815x_6^2], \end{aligned} \tag{3.7}$$

subject to:

$$\mu^T x = 0.01259x_1 + 0.010284x_2 + 0.24x_3 - 0.35x_4 - 0.084426x_5 + 0.48x_6 \geq R$$

$$|\sum_{i=1}^{6} x_i^2 - 0,33333| \leq 0,05,$$

$$\sum_{i=0}^{6} \Delta x_{i,t} + \sum_{i=1}^{6} c_{i,t} \,|\, \Delta x_{i,t} | = 0, \quad t = 1,\dots,T-1;$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1$$

$$x_1\ , x_2\ , x_3\ , x_4\ , x_5\ , x_6 \quad \geq 0$$

After this stage, only the 3 assets with the highest percentage are included in the new portfolio and construct a new optimization problem, which is solved in the second stage of optimization.

The formulation of the portfolio optimization problem for the second stage is:

$$\min f = [0.0017453940x_1^2 + 2.(0.0000002754)x_1.x_2 + 2.(0.0003672134)x_1.x_3 + \\ + 0.0000000008x_2^2 - 2.(0.0000000767)x_2.x_3 + 0.0018149142x_3^2] \qquad (3.8)$$

subject to:

$$\mu^T x = 0.011259x_1 - 0.00084426x_2 + 0.0048x_3 \geq R$$

$$\sum_{i=1}^{3} x_i^2 - 0{,}33333| \leq 0{,}05,$$

$$\sum_{i=0}^{3} \Delta x_{i,t} + \sum_{i=1}^{3} c_{i,t} \,|\Delta x_{i,t}| = 0, \quad t = 1,\ldots,T-1;$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1 , x_2 , x_3 \geq 0$$

The problem was solved 10 times using a different rate of return for R = 6%, R = 6.5%, ..., R = 10.5% in increments of 0.5%. The Matlab's *fmincon* solver was used by Interior point algorithm.

3.2.1 Results on optimizing a six-asset portfolio through the proposed bicriteria model and using the Matlab's *fmincon* solver

Optimization for solving the six-asset model problem is accomplished in two steps. In the first stage, the optimization is performed with all six assets for the expected rate of return. The obtained results for the first stage through the Matlab's *fmincon* solver are presented in Table 18.

*Except Table 18. Results for the 1st stage via Matlab's **fmincon** solver*

Optimal value of objective function	R [%]	iter	f eval	Swiss stocks	French stocks	Japanese stocks	American bonds	Deposits 1% neg. int.rate	American stocks
2.2088264574732 003e-4	6	86	802	0.171817	0.028714	0.039459	0.001748	0.557528	0.200734
2.2459003508634 788e-4	6.5	43	435	0.170496	0.022932	0.039854	0.001661	0.554468	0.210590

For expected rate of returns R = 6%, R = 6.5%, R = 7%, R = 9% and R = 9.5%, the *fmincon* solver finds the optimal solution with the highest values of the variables: x_1 , x_5 , x_6 (Fig. 12).

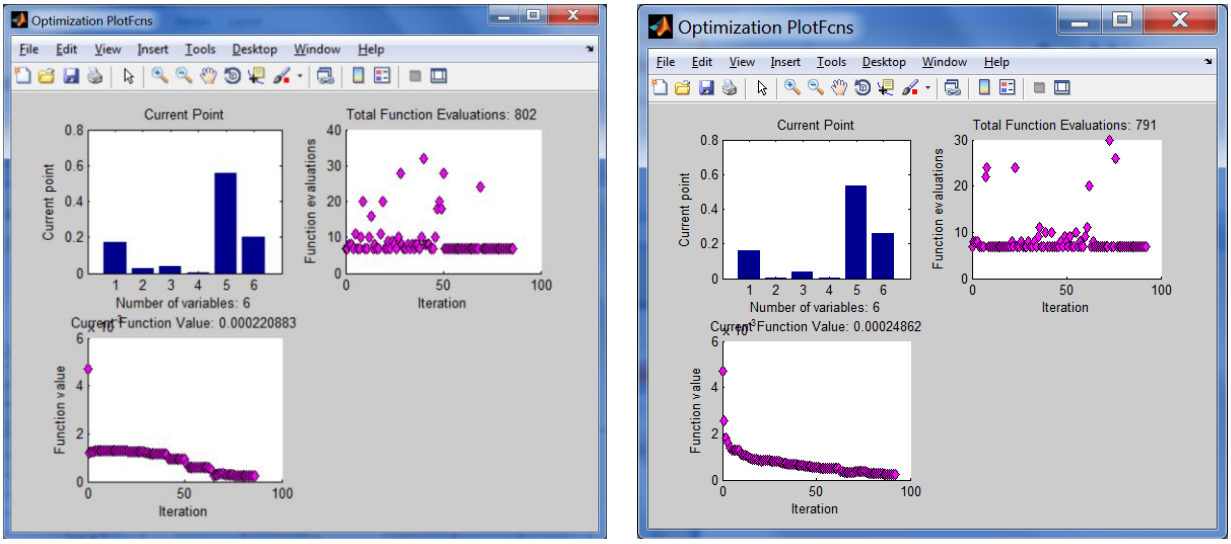

Fig. 12 Optimal portfolios for expected rates of return 6% u 9%

For returns R = 7.5%, R = 8%, R = 8.5%, R = 10% and R = 10.5%, the *fmincon* solver finds the optimal solution with the highest values of the variables x_1 , x_3 , x_6 (Fig. 13). The final message of the solver in these cases is: "Possible local minimum. The constraints are satisfied. The *fmincon* solver stopped because the current step size is less than the default step size tolerance value and the constraints are satisfied within the constraint tolerance default value".

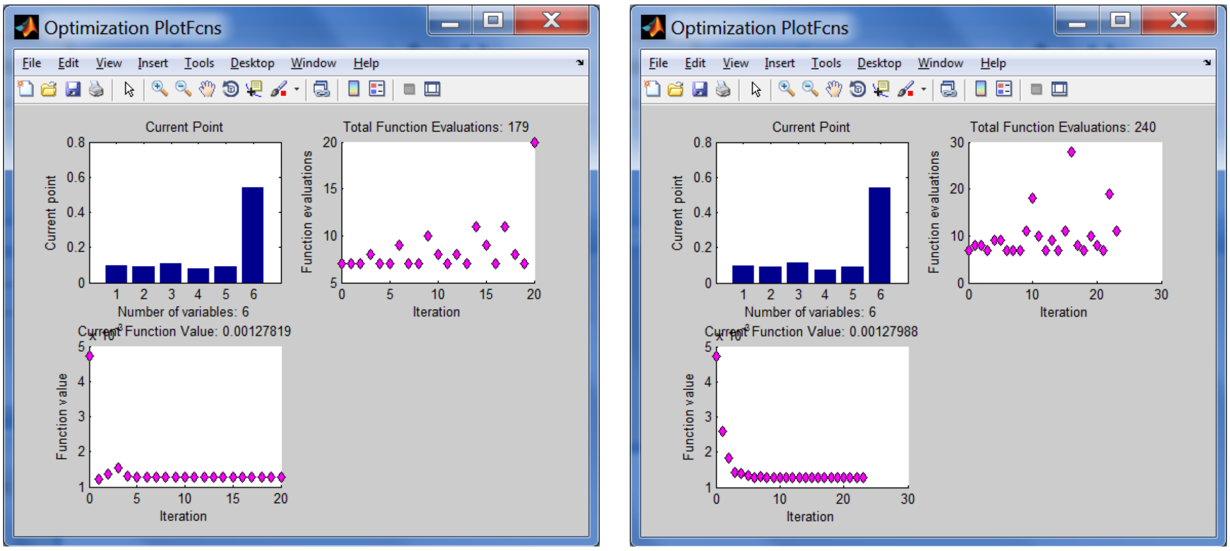

Fig. 13 Optimal portfolios for expected rates of return 8,5% u 10,5%

In view of the obtained values for the objective function, it can be assumed that the obtained solutions by *fmincon* for returns R = 7,5%, R = 8%, R = 8,5%, R = 10% and R = 10,5% are only local optimal. For this reason, in the second stage of optimization, a portfolio is constructed, including the Swiss stocks, Deposits 1% negative interest rate and American stocks, i.e. variables x_1, x_5, x_6.

The Hybrid algorithm also shows that the variables x_1, x_5, x_6 have the highest values. Considering only the variables with the highest values in the ten best solutions found with the Hybrid algorithm (Table 19), variable x_1 participates 5 times, variable x_2 participates 4 times, variable x_3 participates 3 times, variable x_5 participates 10 times and the variable x_6 participated 8 times. As a result, it is confirmed that the portfolio in the second stage must contain the indices: Swiss stocks, Deposits 1% negative interest rate and American stocks (SP 500 USA), i.e. variables x_1, x_5, x_6.

3.2.2 Results on optimizing a six-asset portfolio through the proposed bicriteria model (2.28) - (2.33) and using the Hybrid algorithm

This section presents the results of the tests performed on the second model with the Hybrid Algorithm (see Table 19), with the red indicating the best solution, with the expected rate of return in the interval (6-10.5) % for optimizing the portfolio with six assets. For each different return value, six evaluations were performed using the Hybrid algorithm.

Except Table 19. Results for second model via Hybrid algorithm, the red is the best solution, with a return of 6% to 10.5% and 6 evaluations.

№	Value of objective function	Optimal portfolio					
		Swiss stocks	French stocks	Japanese stocks	American bonds	Deposits 1% neg. int.rate	American stocks
6% expected rate of return							
1	16.68187620659e-04	0.136707436136592	0.012823048196617	0.353371471013417	0.093445663495940	0.005362563416308	0.397333630693842
2	5.829186983791690e-04	0.073832799008721	0.255312612469978	0.029745219454827	0.000841643904796	0.385196199912611	0.257976723120115
3	10.26571748e-04	0.514936832794863	0.231238291511671	0.012675074571854	0.016454872848148	0.042955348399033	0.182625295735171
4	5.829169994432790e-04	0.073850168560546	0.255339717475985	0.029722985266706	0.000841841039746	0.385193591459225	0.257963328059998
5	15.04299253011e-04	0.081961234329811	0.530955148712454	0.070369337813015	0.028139301566989	0.048741430339631	0.240054959214900
6	5.828098768309748e-04	0.073861974276137	0.255383534204454	0.029564277267465	0.000914538076972	0.385136096866381	0.257970696123819

3.2.3 Comparison of obtained results for the six-asset model with the both approaches

This section compares the results for the second model with the hybrid algorithm and the Matlab's *fmincon* solver. Table 20 presents the results of the expected rate of returns in the interval (6-10.5) %.

Except Table 20. Results for the1st stage via Matlab's fmincon solver and Hybrid algorithm

	Value of objective function	Optimal portfolio					
		Swiss stocks	French stocks	Japanese stocks	American bonds	Deposits 1% neg. int.rate	American stocks
6 % expected rate of return							
Hyb.A	5.82809877e-4	0.073862	0.255384	0.029564	0.000915	0.385136	0.257971
MATLAB	2.20882646e-4	0.171817	0.028714	0.039459	0.001748	0.557528	0.200734
6,5 % expected rate of return							
Hyb.A	5.9857358e-04	0.080005	0.071475	0.110320	0.000285	0.368099	0.367313
MATLAB	2.24590035e-4	0.170496	0.022932	0.039854	0.001661	0.554468	0.210590

When comparing the results, it can be seen that better values for the minimum of the objective function are obtained by means of the Hybrid algorithm. It is more accurate in calculations than the Matlab's *fmincon* solver in 6 out of 10 evaluations with different expected rates of return. The difference in favor of the Hybrid Algorithm is increasing as the desired rate of return increases (see Table 20, return 9.5%, 10%, 10.5%).

Stage 2: In the second stage of optimization, an optimal portfolio is calculated using the modified Markowitz's mean variance model, comprising three types of assets: Swiss stocks, Deposits 1% constant negative interest rate and American stocks (SP 500 USA). Each of the ten tests with different rates of return, and the six corresponding evaluations were performed by the Hybrid algorithm. As in the other tests described above, the Matlab's *fmincon* solver found similar solutions, but with less precision.

Figures 14 and 15 present the results for optimal portfolios with expected rates of returns of 6%, 7.5%, 9% and 10.5% obtained through the Matlab's *fmincon* solver.

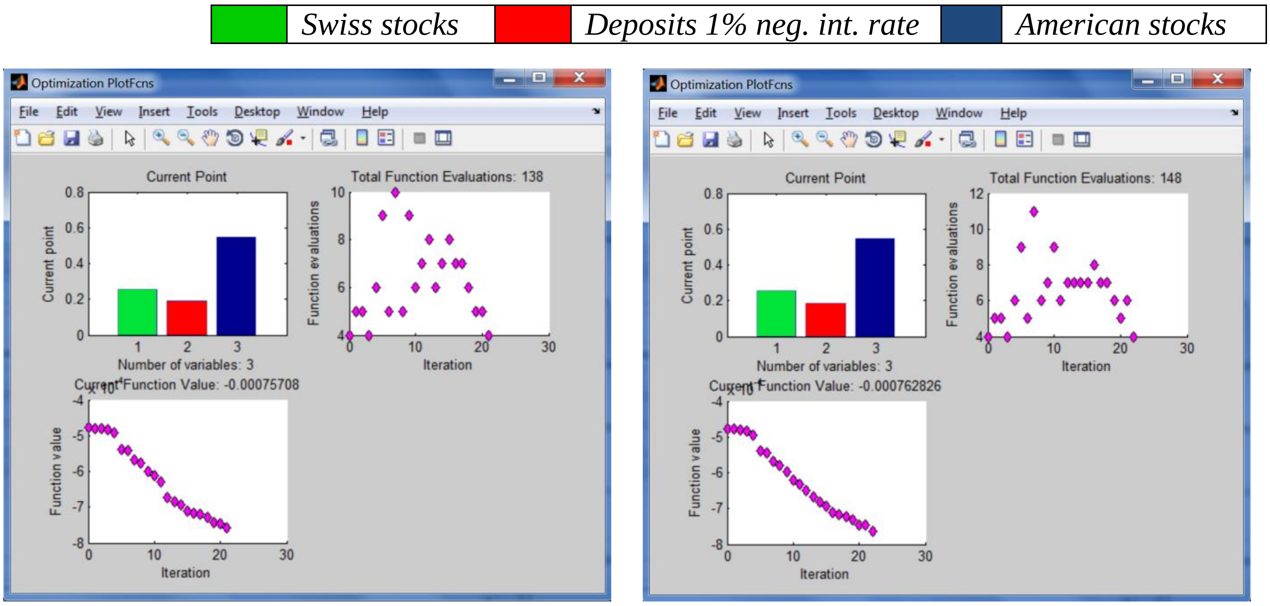

Fig. 14 Optimal portfolios for expected rates of return 6 % u 7,5 %

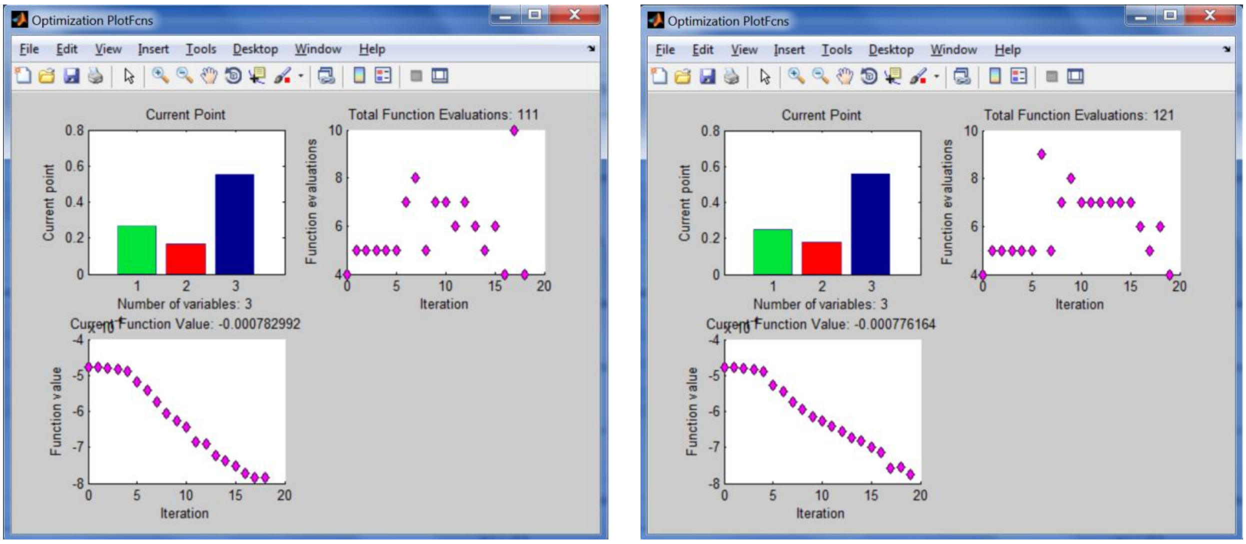

Fig. 15 Optimal portfolios for expected rates of return 9 % u 10,5 %

In all 10 cases considered for different expected rates of return, the Hybrid algorithm received more accurate solutions compared to the Matlab's *fmincon* solver. The difference in the values of the objective function reaches 4,289f-04 at R = 6,5% (see Table 21 and Table 22).

Except Table 21. Results for the 2nd stage via Matlab's ***fmincon*** solver

Value of objective function *f*	***R* [%]**	**Iteration**	**Total evaluations of objective function**	**Optimal portfolio**		
				Swiss stocks	**Deposits 1% neg. int. rate**	**American stocks**
7.570803525333860E-4	6	21	138	0.255126	0.187342	0.546048
7.418840314847011E-4	6.5	21	143	0.261102	0.189784	0.540048

Except Table 22. Results for the 2nd stage via Hybrid algorithm
(The best of 6 solutions obtained for any R value)

Value of objective function *f*	***R* [%]**	**Iteration**	**Total evaluations of objective function**	**Optimal portfolio**		
				Swiss stocks	**Deposits 1% neg. int. rate**	**American stocks**
3.454245734754128e-04	6	100	4000	0.17643308	0.4546998	0.3663634
3.129365174569988e-04	6.5	100	4000	0.16513090	0.4835514	0.3503956

3.3 Implementation of the proposed bicriteria model and hybrid algorithm by portfolio management

When choosing an investment policy, the goals of the investor and the volume of investments are defined, the types of assets are evaluated and the most favorable ones are selected, taking into account the factors of return and risk. Forming a securities portfolio involves a two-step optimization process. In the first stage, the specific assets in which the investment will be invested are determined. This is clearly illustrated in the dissertation with the 6-assets experimental model. After the first optimization phase, the three most profitable assets were identified to be included in the final portfolio and the remaining three assets were rejected. In doing so, the diversification of the portfolio is taken into account in order to maximize the expected return. In this way, by applying the formulated bicriteria model and the Hybrid algorithm developed, the decision maker is significantly assisted in the selection of the final assets in the portfolio (Fig. 16).

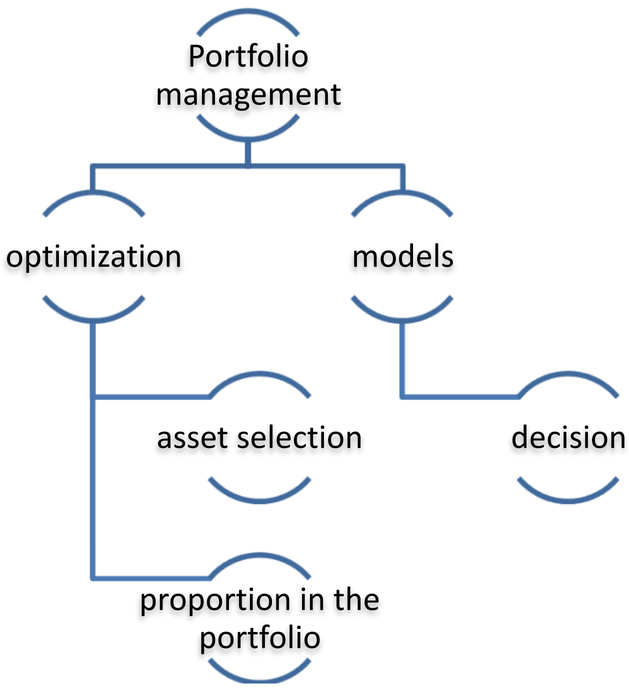

Fig. 16. Portfolio management

In the second optimization stage, the proportions of invested capital for the selected assets are calculated. Again, the decision-maker is substantially assisted in their decisions, as they are provided with optimal solutions for 10 different investment policies - with an expected rate of return in the interval [6%, 10.5%] in 0.5% increments. When making the investment, the optimal quantities of each asset type are purchased accordingly.

An initial portfolio formed after a given period of time may not be optimal for the investor.

After the portfolio is formed, a certain time interval is usually waited, after which the effectiveness of the portfolio is evaluated. At this stage, the yield from the portfolio is calculated, and the result obtained is compared with the selected benchmark. If the effectiveness of the portfolio is positive, it will keep the portfolio so formed for a new period of time. A new effectiveness evaluation is then carried out. If the portfolio effectiveness is negatively assessed, the portfolio is re-evaluated, which is associated with changes in the investment objectives and a possible re-selection of assets. The formulated bicriteria model and the proposed hybrid algorithm can be used cyclically in the portfolio management process. Their ability to work with a portfolio of six assets has been proven with a time series of 131 months (almost 11 years). This leads to the expectation that the model and algorithm can be applied to solve real portfolio optimization problems.

3.4 Summary

Optimization by population of fireflies of the two experimental models was realized with the hybrid algorithm for portfolio selection developed in the thesis, based on Yang's FFA method from 2007 and the Hook&Jeeves's Pattern Search method from 1961.

Throuth used two optimization approaches *Interior point* by nonlinear optimization with constraints in Matlab and the created hybrid algorithm, were tested different optimization problem, namely 30 portfolio optimization tasks were solved. The effectiveness of the approaches was evaluated by the following criteria: number of iterations, number of objective function evaluations and time for which the evaluations were performed. The size of the evaluation tasks varied. In the first experiment, ten calculations for the expected rate of return in interval (6-10.5) % in increments of 0.5, and in the second one, six evaluations, identical to the above expected returns. As a result, it was found that the Hybrid algorithm for portfolio selection solved the tasks of the two experiments more accurately than the result obtained through the Matlab's *fmincon* solver and reached the final solution in a relatively short time. It provides important information to decision-maker in the selection of assets to participate in the portfolio, taking into account predefined criteria. In this way, the financial analyst, the manager or other decision-maker is assisted in deciding which assets to exclude/include in the portfolio. At the next optimization stage, the exact proportions of the assets included in the portfolio are determined. Obtaining high accuracy results is related to adjusting the parameters in the algorithm.

Conclusion

The dissertation research is devoted to problems, related to the development of new, highly efficient mathematical models and methods for portfolio optimization and management, which is not only of scientific but also practical interest for the exchanges, banks, insurance and investment companies as well as the state for the functioning of the financial system.

The study uses mathematical models for decision-makers in a stable economic situation, while extending the model of classical finance theory that the decision-maker seeks only to maximize its utility. In real life, decision makers face a number of constraints: cardinal, transaction costs, turnover constraints, commercial, etc., which are described in detail. The behavioral factors of the decision-maker are also influenced, as the actions of the investors/managers/decision-maker sometimes deviate from the rational.

Both optimization approaches – *Interior point* in the Matlab and the created hybrid evolutionary algorithm for portfolio selection make possible the work with time series of historical real data indexes over 10 years of all available international public information.

Numerical calculations were performed for two experimental models with different numbers of assets participating in the portfolio, and with different series of output real data is able to test the feasibility of the new hybrid algorithm for portfolio optimization and be able to compare the two approaches to solving the problem for portfolio optimization with constraints.

From all conducted experiments, it can be concluded that both approaches are applicable to the nonlinear optimization of a portfolio with constraints.

A detailed examination of the results of the two experimental models shows that the Hybrid evolutionary algorithm easily overcomes the disadvantages of the time series method for significant computational difficulty, another advantage is its efficiency/accuracy with which it finds optimal solutions. It is more accurate in calculations than Matlab's *fmincon* solver.

In conclusion, the created hybrid algorithm for portfolio selection with constraints is applicable and effective for solving a wide range of tasks in real financial processes. In this way, the working hypothesis of the present work was confirmed that evolutionary algorithms can be successfully applied in the optimization of portfolio with constraints.

Solving larger-scale real-world problems could be a direction for future research that will enrich the research area.

The obtained and described results in the dissertation research are presented and published in a total of seven publications, one of which is a book chapter, four of which have been reported at international conferences.

Contribution summary

As a result of the research carried out in this paper, have been made the following scientific and applied contributions:

- A bicriteria optimization model for portfolio selection has been formulated, which is a modification of the Markowitz's mean variance model
- A summary methodology for portfolio selection is proposed.
- A Hybrid evolution algorithm is created based on FFA and PS. The advantage of the proposed algorithm is the accuracy in the calculation of the optimal solution and the relatively short time to solve the optimization problems. The polynomial computational complexity of the Hybrid evolutionary algorithm allows it to be successfully used to solve large-scale optimization problems.
- On the basis of the proposed bicriteria optimization model, corresponding tasks are formulated, which are solved by the created Hybrid algorithm and by the standard Matlab's *fmincon* solver. The obtained results confirm the working capacity of the Hybrid algorithm.
- A set of software modules of Matlab has been developed to implement the Hybrid algorithm to solve the problem of portfolio optimization while minimizing risk, with different difersification and expected rates of return.

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5. Guliashki, V., Stoyanova, K. Portfolio Risk Optimization Based on MVO Model. In Proc. of the LIII International Scientific Conference on Information, Communication and Energy Systems and Technologies ICEST'2018, ISSN: 2603-3259 (Print), 2603-3267 (Online), pp. 67-70.
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7. Stoyanova K., Guliashki V. (2020). Two-Stage Portfolio Risk Optimization Based on MVO Model. *International Journal for Reasoning-based Intelligent Systems*, Special Issue: "Intelligent Sensor Data Processing, Mobile Telecommunications and Air Traffic Control", (Guest Eds: K. Dimitrov, I. Draganov), Vol. 12, No. 1, pp. 70-79, ISSN online: 1755-0564, ISSN print: 1755-0556, SJR (0,125), SJR (Q3), DOI: 10.1504/IJRIS.2020.105011

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