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# MODELS AND ALGORITHMS TO SUPPORT GROUP DECISION-MAKING

**ABSTRACT of PhD THESIS**

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The dissertation thesis was discussed and admitted to the defense of an extended session of the department of Information Processes and Decision Support Systems at IICT-BAS held on 14 June 2019.

The dissertation is structured as follow: introduction, three chapters, conclusion, declaration of originality of the results, list of papers in accordance to dissertation theme, citations and bibliography. The dissertation work contains 121 pages, 21 figures, 22 tables, and 136 references.

The defense of the dissertation will take place on .......................... 2019 from .......... hours in hall ........... The members of the scientific jury are:

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Title: **Models and Algorithms to Support Group Decision-Making**

## INTRODUCTION

Today, in many areas different group decisions are to be done. Although the notion of "decision-making" has a broad meaning, it can be illustrated as a choice of a preferable alternative from available set of alternatives. The growing competition due to globalization, make difficult to take successful decisions without the assistance of experts from different fields. Each of these experts must have proven capabilities to provide convincing information about their assessments of existing alternative solutions with respect to predetermined criteria. The essence of the decision-making-process through the choice of an alternative involving different quantitative and qualitative criteria, defines this process as a non-trivial and complex multi-criteria problem. Therefore, the goal of any decision-support system is to rely on well-founded models and algorithms that lead to an optimal choice of alternative from given set of alternatives.

The dissertation thesis is structured as follows: introduction, 3 chapters, conclusions, contributions, list of publications, declaration of originality and bibliography.

Chapter 1 provides an overview of the decision-making process and analysis of existing techniques, models and methods to support the group decision-making process are given. Prospective research directions, the purpose of the dissertation work and the tasks for its realization have been identified.

Chapter 2 describes proposed models for supporting group decision-making and the algorithms for their implementation, namely: modified simple additive weighting, modified weighted product model, and a modified model based on the SMART. For each of the proposed modified models to support group decision making, corresponding algorithms for their performance are also proposed. A generalized algorithm with three different strategies (for choosing an alternative; for choosing of several alternatives; or for ranking of all alternatives to their degree of preference) is also proposed. Four different group decision making models for selection of alternative(s) under uncertainty conditions considering principles of Wald, Laplace, Hurvitz and Savage are also proposed.

Chapter 3 describes the numerical experiments with the proposed modified models for group decision making and the algorithms for their implementation. The results of numerical experiments demonstrate the practical applicability of the proposed modifications of simple additive weighting model, the weighted product model, and the SMART-based model for choice of alternative(s) via group decision-making are presented. Numerous experiments using the proposed algorithm for group decision making with three different strategies are also described. The results of numerical testing, demonstrating the practical applicability of the group decision making models under uncertainty conditions using of Wald, Laplace, Hurvitz and Savage criteria, are presented.

The conclusion contains some directions for future research related to the area of group decision-making.

The numbering of the formulas, tables and figures in the current abstract is identical to that in the dissertation.

**Keywords**: decision making, group decision, modified weighted sum model, modified weighted prouct model, modified SMART model, generalized algorithm with integrated different strategies, group decision-making under uncertainty conditions.

# 1. Analysis of Techniques and Models to Support Group Decision Making

This chapter presents an analysis of techniques and models to support group decision-making. The most commonly used group-decision making techniques are described (as brainstorming; nominal group technique; Delphi technique; devil's advocates, and didactic interaction). The general statement of the decision-making problem using multiple attributes is presented. Different decision-making models for solving of the multiple attributes problems are described – simple additive weighting (SAW) model; simple multi-attribute rating technique (SMART); weighted product model (WPM); analytic hierarchy process (AHP); analytic network process (ANP); TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) and VIKOR (VIseKriterijumska Optimizacija I Kompromisno Resenje) models; autranking methods ELECTRE (elimination and choice translating reality) and PROMETHEE (preference ranking organization method for enrichment evaluation – multicriteria optimization and compromise solution), and interactive methods. Different decision-making criteria under uncertainty conditions, based on the principles of Wald, Laplace, Hurvitz and Savage, are presented.

As a result of the topic review, the main goal of the dissertation thesis, as well as the tasks for its realization, is determined.

The aim of the dissertation is to propose mathematical models to support the group decision making and algorithms for their realization, taking into account the differences in the expertise of the group' members. For the goal, the following tasks need to be fulfilled:

- to analyze existing models and techniques for group decision making;
- to propose models for group decision-making and algorithms their implementation, taking into account the expertise of each member of the group, using 1) weighted sum model, 2) weighted product model, and 3) SMART model;
- to propose a generalized algorithm for group decision-making with integrated different strategies – for selection of one best alternative, for choice of several good alternatives and rank of all alternatives;
- to propose models for group decision-making under uncertainty conditions using criteria Wald, Laplace, Hurvitz and Savage.

The methodology of research includes developing of mathematical models and algorithms, as well as evaluation toward their effectiveness and practical applicability by numerical testing using real data.

## 2. Models and algorithms for group decision considering differences in the group members expertise

A modification of decision matrix is proposed as shown in Table 2.1 [Borissova et al., 2016].

*Table 2.1. Modified of decision matrix for group decision makin*

| Group of experts | Weights for experts | Alternatives, $A_i$ | Decision variables, $x_i$ | Criteria/weights/evaluations | | | |
|---|---|---|---|---|---|---|---|
| | | | | $C_1$ | $C_2$ | ..... | $C_n$ |
| | | | | $w_1^1$ | $w_2^1$ | | $w_n^1$ |
| | | $A_1$ | $x_1$ | $e^1_{11}$ | $e^1_{12}$ | | $e^1_{1n}$ |
| $E^1$ | $\lambda^1$ | $A_2$ | $x_2$ | $e^1_{21}$ | $e^1_{22}$ | | $e^1_{2n}$ |
| | | $A_3$ | $x_3$ | $e^1_{31}$ | $e^1_{32}$ | | $e^1_{3n}$ |
| | | .... | | ... | ... | ... | ... |
| | | $A_m$ | $x_m$ | $e^1_{m1}$ | $e^1_{m1}$ | | $e^1_{mn}$ |
| | | | | $w_1^2$ | $w_2^2$ | | $w_n^2$ |
| | | $A_1$ | $x_1$ | $e^2_{11}$ | $e^2_{21}$ | | $e^2_{1n}$ |
| $E^2$ | $\lambda^2$ | $A_2$ | $x_2$ | $e^2_{21}$ | $e^2_{22}$ | | $e^2_{2n}$ |
| | | $A_3$ | $x_3$ | $e^2_{31}$ | $e^2_{32}$ | | $e^2_{3n}$ |
| | | .... | | ... | ... | ... | ... |
| | | $A_m$ | $x_m$ | $e^2_{m1}$ | $e^2_{m2}$ | | $e^2_{mn}$ |
| .... | | .... | | ... | ... | ... | ... |
| | | | | $w_1^k$ | $w_2^k$ | | $w_n^k$ |
| | | $A_1$ | $x_1$ | $e^k_{11}$ | $e^k_{12}$ | | $e^k_{1n}$ |
| $E^k$ | $\lambda^k$ | $A_2$ | $x_2$ | $e^k_{21}$ | $e^k_{22}$ | | $e^k_{2n}$ |
| | | $A_3$ | $x_3$ | $e^k_{31}$ | $e^k_{32}$ | | $e^k_{3n}$ |
| | | .... | | ... | ... | ... | ... |
| | | $A_m$ | $x_m$ | $e^k_{m1}$ | $e^k_{m2}$ | | $e^k_{mn}$ |

The following notations are used: $m$ – number of alternatives, $n$ – number of criteria), $k$ – number of group members, set of alternatives $A = \{A_1, A_2, ..., A_m\}$, set of criteria $C = \{C_1, C_2, ..., C_n\}$, set of experts $E = \{E^1, E^2, ..., E^k\}$, and set of weighted coefficients for expert expertice $\Lambda = \{\lambda^1, \lambda^2, ..., \lambda^k\}$. Evaluation of $i$-th alternative toward $j$-th criterion from $k$-th expert is expressed by $e^k_{ij}$, where $0 \le e^k_{ij} \le 1$. The coefficients $w^k_j$ express the relative importance between criteria from $k$-th expert point of view. The selction of most preferable altermative is realized by decision variables $\{x_i\}$ assigned to each alternative.

The distribution of responsibilities within the group is shown in Fig. 2.1 [Korsemov et al., 2018]:

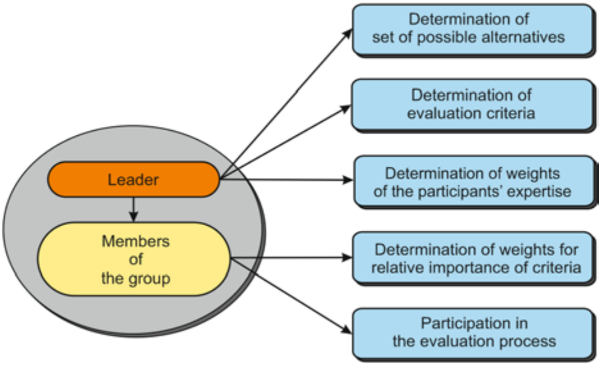



*Fig. 2.1. Distribution of the responsibilities in the group*

## 2.1. Modified weighted sum model for group decision making

Using the modified matrix structure of Table 2.1, the following modified model of the weighted sum for group decision making is proposed [Korsemov & Borissova, 2018]:

(2.1) $$\text{maximize}\left(\sum_{i=1}^{M} x_i \sum_{k=1}^{K} \lambda^k A_i^k\right)$$

subject to

(2.2) $$\forall i = 1, 2, \dots, M : (\forall k = 1, 2, \dots, K : A_i^k = \sum_{j=1}^{N} w_j^k e_{i,j}^k \;)$$

(2.3) $$\sum_{j=1}^{N} w_j^k = 1, \forall k = 1, 2, \dots, K$$

(2.4) $$\sum_{i=1}^{M} x_i = 1, \;\; x_i \in \{0, 1\}$$

(2.5) $$\sum_{k=1}^{K} \lambda^k = 1$$

The restriction (2.4) can be midified as follows:

(2.7) $$\sum_{i=1}^{M} x_i = z, \;\; x_i \in \{0,1\} \;\; \text{и} \;\; 1 < z < M$$

to select more than one pfererable alternative. The constant *z* is a number in the range (2, M-1).

An algorithm for implementation of the modified weighted sum model (2.1) - (2.6) for group decision making is proposed (Fig. 2.2).


*Fig. 2.2. Algorithm for implementation of modified weighted sum model for group decision making*

## 2.2. Modified weighted product model for group decision making

The proposed modified weighted product model for group decision making is as follows [Korsemov & Borissova, 2018]:

(2.8) $$\text{maximize} \sum_{i=1}^{M} x_i \sum_{k=1}^{K} \lambda^k R(A_i)^k$$

subject to

(2.9) $$\forall i = 1, 2, \ldots, M : (\forall k = 1, 2, \ldots, K : R(A_i)^k = \prod_{j=1}^{N} (e_{ij}^k)^{w_j^k}$$

(2.10) $$\sum_{j=1}^{N} w_j^k = 1, \forall k = 1, 2, \ldots, K$$

(2.11) $$\sum_{i=1}^{M} x_i = 1, \; x_i \in \{0, 1\}$$

(2.12) $$\sum_{k=1}^{K} \lambda^k = 1,$$

The restiction (2.11) can be changed to select several preferred alternatives.

An algorithm for implementation of the modified weighted product model (2.8) - (2.12) for group decision making is proposed (Fig. 2.3).


*Fig. 2.3. Algorithm for implementation of modified weighted product model for group decision making*

## 2.3. Modified SMART model for group decision making

The proposed modified SMART model for group decision making is as follows:

(2.13) $$\text{maximize} \left( \sum_{i=1}^{M} x_i \sum_{k=1}^{K} \lambda^k A_i^k \right)$$

subject to

(2.14) $$\forall i = 1, 2, \dots, \mathrm{M} : (\forall k = 1, 2, \dots, \mathrm{K} : A_i^k = \sum_{j=1}^{N} (w_j^k)^* e_{i,j}^k \;)$$

(2.15) $$(w_j^k)^* = w_j^k \left( \sum_{j=1}^{N} w_j^k \right)^{-1}$$

(2.16) $$\sum_{i=1}^{M} x_i \; = 1, \;\; x_i \in \{0,1\}$$

(2.17) $$\sum_{k=1}^{K} \lambda^k = 1$$

The restiction (2.16) can be changed to select several preferred alternatives.

An algorithm for implementation of the modified SMART model (2.13) - (2.17) for group decision making is proposed (Fig. 2.4):


*Fig. 2.4. Algorithm for implementation of modified SMART model for group decision making*

## 2.4. A generalized algorithm for group decision making with integrated different strategies

The proposed algorithm with three different strategies for making group solutions is shown in Fig. 2.6

The *preparation stage 1* of the algorithm starts with identification of the main goal of group decision making process. The next steps of this preparation stage includes: 1.1) identification the group of needed experts for group decision making; 1.2) determination of corresponding weights for each expert of group member in accordance to their knowledge and experience; and 1.3) identification of criteria and alternatives for evaluation.

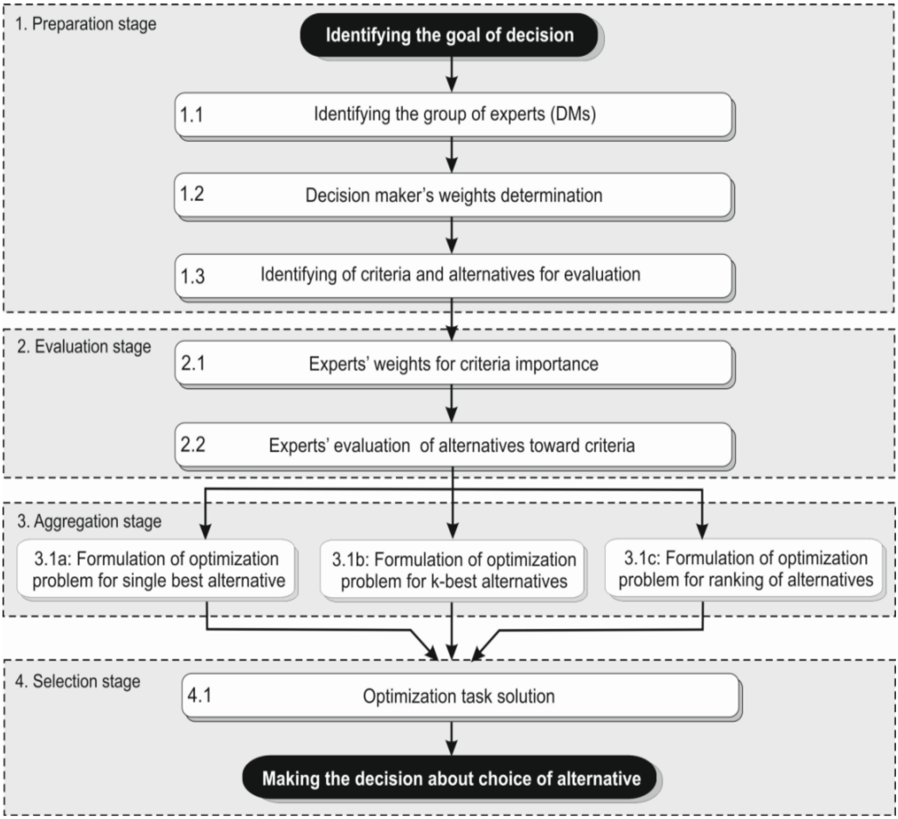



*Fig. 2.6. Group decision making algorithm with incorporated three different strategies*

The *evaluation stage 2* is composed of two steps: 2.1) assigning of weighted coefficients for the evaluation criteria by higher management; and 2.2) evaluation the set of alternatives against the criteria (set of predefined attributes) by members of group' experts. A key issue in MAGDM is aggregation of individual preference into one. Because of that, the *aggregation stage* 3 is the most important in define the aggregated group decision. In the current paper three types formulations of optimization problems are proposed depending on the particular goal for: 3.1a) selection of a single alternative; 3.1b) defining of *k*-best alternatives; 3.1c) ranking of all alternatives.

On the *selection stage 4*, the optimization tasks corresponding to simulation of three different decision making scenarios are solved. On this stage, the higher management could choose some of the described scenarios. The results of optimization tasks solutions on step 4.1 are presented to higher management to make final decision about alternative that is most suitable toward the defined main goal.

- **Mathematical model for selction of one best alternatives**

(2.18) $$\text{maximize} \left( \sum_{i=1}^{M} x_i \sum_{k=1}^{K} \lambda^k A_i^k \right)$$

subject to

(2.19) $$\forall i = 1, 2, \dots, M : (\forall k = 1, 2, \dots, K : A_i^k = \sum_{j=1}^{N} w_j^k e_{i,j}^k \ )$$

(2.20) $$\sum_{j=1}^{N} w_j^k = 1$$

(2.21) $$\sum_{i=1}^{M} x_i = 1, \ x_i \in \{0, 1\}$$

(2.22) $$\sum_{k=1}^{K} \lambda^k = 1$$

- **Mathematical model for selection of k-good alternatives**

The second scenario illustrates the identification of several (more than one) suitable alternatives by changing the restriction (2.21) as follows:

(2.23) $$\sum_{i=1}^{M} x_i = z \ , x_j \in \{0, 1\}, 1 < z < M$$

where *z* is a number in the range of (2, M-1).

- **Mathematical model for ranking of all alternatives**

The third scenario of the proposed algorithm for group decision making makes it possible to rank all alternatives through sequential solving of tasks of the type (2.18) - (2.22).

## 2.5. Models for group decision-making under uncertainty conditions

In general, the utility function can be expressed by the ratio between the expected costs and the expected revenue [Boardman et al., 2010]:

(2.24) $$CBE = \frac{Cost}{\text{Benefit}}$$

In this formulation, the alternatives evaluations are obtained by using of relation (2.24) under possible different states (perspectives). Therefore, a decision matrix in Table 2.1 is transformed as decision matrix under uncertainty conditions for group decision as shown in Table 2.2:

*Table 2.2. Group decision making matrix under uncertainty conditions*

| Group of experts | Weights for experts | Alternatives, ($A_i$) | Conditions/States | | | |
|---|---|---|---|---|---|---|
| | | | $S_1$ | $S_2$ | ..... | Sn |
| $E^1$ | $\lambda^1$ | $A_1$ | $CBE^{1}_{11}$ | $CBE^{1}_{12}$ | | $CBE^{1}_{1n}$ |
| | | $A_2$ | $CBE^{1}_{21}$ | $CBE^{1}_{22}$ | | $CBE^{1}_{2n}$ |
| | | .... | ... | ... | ... | ... |
| | | $A_m$ | $CBE^{1}_{m1}$ | $CBE^{1}_{m1}$ | | $CBE^{1}_{mn}$ |
| .... | | .... | ... | ... | ... | ... |
| $E^k$ | $\lambda^k$ | $A_1$ | $CBE^{k}_{11}$ | $CBE^{k}_{12}$ | | $CBE^{k}_{1n}$ |
| | | $A_2$ | $CBE^{k}_{21}$ | $CBE^{k}_{22}$ | | $CBE^{k}_{2n}$ |
| | | .... | ... | ... | ... | ... |
| | | $A_m$ | $CBE^{k}_{m1}$ | $CBE^{k}_{m2}$ | | $CBE^{k}_{mn}$ |

- **Model for group decision-making under uncertainty conditions and usage Wald criterion**

(2.25) $$\max\min \sum_{i=1}^{M} \sum_{k=1}^{K} \lambda^{k} CBE_{ij}^{k}$$

subject to

(2.26) $$\sum_{k=1}^{K} \lambda^{k} = 1$$

where $CBE_{ij}^{k}$ expresses evaluation of the i-th alternative toward j-state from k-expert.

- **Model for group decision-making under uncertainty conditions and usage Laplace criterion:**

(2.27) $$\max\left( \sum_{i=1}^{M} \sum_{k=1}^{K} \lambda^{k} \frac{CBE_{ij}^{k}}{M} \right)$$

subject to

(2.28) $$\sum_{k=1}^{K} \lambda^k = 1$$

- **Model for group decision-making under uncertainty conditions and usage Hurwicz criterion:**

(2.29) $$\max\left\{\alpha \max \sum_{\mathrm{i}=1}^{\mathrm{M}} \sum_{k=1}^{K} \lambda^{\mathrm{k}} \mathrm{CBE}_{\mathrm{ij}}^{k} + (1-\alpha) \min \sum_{\mathrm{i}=1}^{\mathrm{M}} \sum_{k=1}^{k} \lambda^{k} CBE_{ij}^{k}\right\}$$

subject to

(2.30) $$\sum_{k=1}^{K} \lambda^k = 1$$

where (1 – $\alpha$) represent the coefficient of pessimism.

- **Model for group decision-making under uncertainty conditions and usage Savage criterion:**

(2.31) $$\min \max \sum_{i=1}^{M} \sum_{k=1}^{K} \lambda^{\mathrm{k}} R_{ij}^{k}$$

subject to

(2.32) $$\forall i = 1, 2, \ldots, M : (\forall k = 1, 2, \ldots, K : R_{ij}^{k} = \left| CBE_{ij}^{k} - \max CBE_{ij}^{k} \right|$$

(2.33) $$\sum_{k=1}^{K} \lambda^k = 1$$

where $R_{ij}^{k}$ represent the regret as result of opportunity loss if $A_i$ is chosen and state $S_j$ happens in accordance to $k$-th expert point of view.

### 2.5.2. Algorithm for implementation of group decision-making under uncertainty conditions

The main stages of the algorithm for implementation of models for group decision-making under uncertainty conditions are shown in Fig. 2.5.


*Fig. 2.5. Algorithm for implementation of group decision-making under uncertainty conditions*

## 3. NUMERICAL TESTING OF PROPOSED MODELS AND ALGORITHMS FOR GROUP DECISION-MAKING

This chapter describes the results of the numerical testing of the proposed models and algorithms for group decision making, namely:

- numerical testing based on real examples to demonstrate the practical applicability of the modified simple additive weighting and the weighted product models for selection of alternative(s) under group decision making,
- numerical testing of the proposed modification of the SMART model for selection of alternative(s) under group decision making,
- numerical testing of the proposed algorithm and models for group decision making with three different strategies,
- numerical testing of the proposed group decision making models under uncertainty conditions.

## 3.1. Testing the modified simple additive weighting model for group decision making

### 3.1.1. Selection of enterprise resource planning system (ERP)

The proposed modified simple additive weighting model for selection of alternative via group decision making is verified by using of the input data adapted from an ERP selection problem [Efe, 2016], where 4 criteria have been considered: 1) price (C1); 2) supplier specifications (C2); 3) software technical specifications (C3); and 4) ease of use (C4). The alternatives are evaluated by group of 5 experts composed of: financial consultant (E-1), business analysts (E-2 and E-3) and database administrators (E-4 and E-5). Normalized alternatives' estimations against the criteria, together with the coefficients for criteria importance and coefficients for the expertise of group' members are shown in Table 3.1.

*Table 3.1. Input data in case of ERP selection*

| Group of experts | Weights for experts | Alternatives | Decision variables, $x_i$ | Criteria / Weights / Evaluations | | | |
|---|---|---|---|---|---|---|---|
| | | | | C1 | C2 | C3 | C4 |
| E-1 | 0.10 | | | 0.3 | 0.2 | 0.25 | 0.25 |
| | | A-1 | $x_1$ | 0.806 | 0.85 | 0.777 | 0.823 |
| | | A-2 | $x_2$ | 0.762 | 0.894 | 0.765 | 0.841 |
| | | A-3 | $x_3$ | 0.818 | 0.813 | 0.785 | 0.877 |
| E-2 | 0.27 | | | 0.21 | 0.26 | 0.26 | 0.27 |
| | | A-1 | $x_1$ | 0.792 | 0.865 | 0.767 | 0.843 |
| | | A-2 | $x_2$ | 0.785 | 0.904 | 0.775 | 0.905 |
| | | A-3 | $x_3$ | 0.763 | 0.893 | 0.792 | 0.89 |
| E-3 | 0.21 | | | 0.16 | 0.28 | 0.28 | 0.28 |
| | | A-1 | $x_1$ | 0.822 | 0.876 | 0.815 | 0.915 |
| | | A-2 | $x_2$ | 0.793 | 0.884 | 0.838 | 0.885 |
| | | A-3 | $x_3$ | 0.808 | 0.831 | 0.808 | 0.897 |
| E-4 | 0.25 | | | 0.18 | 0.22 | 0.28 | 0.33 |
| | | A-1 | $x_1$ | 0.788 | 0.798 | 0.807 | 0.873 |
| | | A-2 | $x_2$ | 0.764 | 0.815 | 0.868 | 0.916 |
| | | A-3 | $x_3$ | 0.749 | 0.855 | 0.835 | 0.905 |
| E-5 | 0.17 | | | 0.2 | 0.25 | 0.25 | 0.3 |
| | | A-1 | $x_1$ | 0.797 | 0.877 | 0.786 | 0.865 |
| | | A-2 | $x_2$ | 0.811 | 0.897 | 0.798 | 0.897 |
| | | A-3 | $x_3$ | 0.823 | 0.854 | 0.812 | 0.925 |

Using the proposed modified model (2.1) – (2.5) and data from Table 3.1, the following optimization task is formulated:

(3.1) $$\text{maximize} \left( \sum_{i=1}^{3} x_i \sum_{k=1}^{5} \lambda^k A_i^k \right)$$

subject to

(3.2) $$\forall i = 1, 2, \dots, 3 : (\forall k = 1, 2, \dots, 5 : A_i^k = \sum_{j=1}^{4} w_j^k e_{i,j}^k \ )$$

(3.3) $$\sum_{j=1}^{4} w_j^k = 1, \forall k = 1, 2, \dots, 5$$

(3.4) $$x_1 + x_2 + x_3 = 1, \ \ x_i \in \{0,1\}$$

(3.5) $$\lambda^1 = 0.10; \lambda^2 = 0.27; \lambda^3 = 0.21; \lambda^4 = 0.25; \lambda^5 = 0.17$$

The results of different coefficients using about the importance of expert assessments for choosing 1 and 2 alternatives are shown in Table 3.2 [Korsemov & Borissova, 2018]:

*Table 3.2 Results for selection of ERP*

| Case | Weights for experts | | | | | Chosen 1 alternative | Chosen 2 alternatives |
|---|---|---|---|---|---|---|---|
| | E-1 | E-2 | E-3 | E-4 | E-5 | | |
| (1) | 0.10 | 0.27 | 0.21 | 0.25 | 0.17 | A-2 | A-2 & A-3 |
| (2) | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | A-2 | A-2 & A-3 |
| (3) | 0.35 | 0.13 | 0.12 | 0.20 | 0.20 | A-3 | A-2 & A-3 |

### 3.1.2. Selection of a personal computer vendor for public procurement purposes

The proposed modified simple additive weighing model is used to select a PC vendor for the purpose of a public contract with three bids (A-1, A-2, A-3). Each supplier is evaluated by 12 quality and numeral criteria: 1) technical performance; 2) bid price; 3) price breaks and quantity discounts; 4) payment terms – possibility of deferred payment; 5) warranty; 6) out-of-warranty service; 7) number of available repair shops; 8) availability of experienced staff; 9) certifications; 10) previous experience; 11) lead time; 12) customer recommendations. A group of one financial consultant (E-1), two IT specialists (E-2 and E-3) 4) and one manager (E-5) is used. The normalized evaluations of alternatives against criteria, along with weights for relative importance between criteria, as well as the coefficients for group members' expertise, in case of PCs supplier selection are shown in Table 3.3 [Korsemov et al., 2018].

*Table 3.3. Input data in case of PCs supplier selection*

| Group of experts | Weights for experts | Alterna-tives | Criteria / Weights / Evaluations | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 |
| E-1 | 0.20 | | 0.08 | 0.10 | 0.09 | 0.09 | 0.08 | 0.07 | 0.08 | 0.08 | 0.09 | 0.08 | 0.07 | 0.10 |
| | | A-1 | 0.76 | 1.00 | 0.72 | 0.82 | 0.76 | 0.56 | 0.69 | 1.00 | 0.68 | 0.66 | 0.66 | 0.56 |
| | | A-2 | 0.83 | 0.88 | 0.78 | 0.93 | 0.77 | 0.58 | 0.65 | 0.93 | 0.79 | 0.76 | 0.68 | 0.55 |
| | | A-3 | 0.81 | 0.90 | 0.76 | 1.00 | 0.72 | 0.66 | 0.72 | 0.92 | 0.74 | 0.75 | 0.70 | 0.63 |
| E-2 | 0.20 | | 0.11 | 0.07 | 0.07 | 0.06 | 0.09 | 0.09 | 0.09 | 0.09 | 0.08 | 0.08 | 0.08 | 0.09 |
| | | A-1 | 0.81 | 0.85 | 0.73 | 0.67 | 0.67 | 0.89 | 0.73 | 0.67 | 0.74 | 0.85 | 0.80 | 0.93 |
| | | A-2 | 0.84 | 0.78 | 0.76 | 0.65 | 0.77 | 0.91 | 0.75 | 0.69 | 0.78 | 0.82 | 0.81 | 0.85 |
| | | A-3 | 0.82 | 0.74 | 0.73 | 0.72 | 0.72 | 0.82 | 0.78 | 0.65 | 0.79 | 0.78 | 0.73 | 0.91 |
| E-3 | 0.20 | | 0.15 | 0.06 | 0.06 | 0.06 | 0.11 | 0.10 | 0.09 | 0.09 | 0.06 | 0.06 | 0.07 | 0.09 |
| | | A-1 | 0.86 | 0.81 | 0.78 | 0.77 | 0.95 | 0.82 | 0.84 | 0.82 | 0.81 | 0.76 | 0.96 | 0.79 |
| | | A-2 | 0.72 | 0.79 | 0.76 | 0.81 | 0.83 | 0.76 | 0.70 | 0.81 | 0.78 | 0.79 | 0.89 | 0.82 |
| | | A-3 | 0.81 | 0.78 | 0.79 | 0.69 | 1.00 | 0.80 | 0.79 | 0.88 | 0.83 | 0.72 | 1.00 | 0.76 |
| E-4 | 0.20 | | 0.13 | 0.06 | 0.06 | 0.06 | 0.12 | 0.11 | 0.09 | 0.08 | 0.06 | 0.07 | 0.08 | 0.08 |
| | | A-1 | 1.00 | 0.85 | 0.61 | 0.62 | 0.95 | 0.93 | 0.73 | 0.73 | 0.94 | 0.85 | 0.85 | 0.93 |
| | | A-2 | 0.88 | 0.74 | 0.66 | 0.66 | 0.96 | 1.00 | 0.75 | 0.79 | 0.88 | 1.00 | 0.92 | 0.88 |
| | | A-3 | 0.92 | 0.76 | 0.73 | 0.75 | 0.83 | 0.91 | 0.78 | 0.76 | 1.00 | 0.88 | 0.78 | 0.90 |
| E-5 | 0.20 | | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.06 | 0.06 | 0.07 | 0.06 | 0.09 | 0.10 | 0.06 |
| | | A-1 | 0.92 | 0.79 | 0.88 | 0.73 | 0.83 | 0.78 | 1.00 | 0.73 | 0.72 | 0.76 | 0.63 | 0.84 |
| | | A-2 | 0.78 | 0.88 | 1.00 | 0.72 | 0.84 | 0.71 | 0.88 | 0.74 | 0.75 | 0.78 | 0.68 | 1.00 |
| | | A-3 | 0.84 | 0.76 | 0.82 | 0.68 | 0.88 | 0.73 | 0.91 | 0.79 | 0.81 | 0.82 | 0.69 | 0.86 |

The obtained results for three different cases of coefficients expertise of group members are shown in Table 3.4:

*Table 3.4. Results for selction of PCs vendor under public procurement*

| Case | Weights for experts | | | | | Chosen 1 alternative | Chosen 2 alternatives |
|---|---|---|---|---|---|---|---|
| | E-1 | E-2 | E-3 | E-4 | E-5 | | |
| (1) | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | A1 | A1 & A3 |
| (2) | 0.25 | 0.10 | 0.15 | 0.25 | 0.25 | A3 | A2 & A3 |
| (3) | 0.27 | 0.12 | 0.13 | 0.18 | 0.30 | A2 | A2 & A3 |

### 3.1.3. Selection of a contractor in the field of software engineering

The numerical example of selecting a software developer adapted by [Krapohrl, 2014] is described in [Korsemov et al., 2018]. The problem consists of 3 alternatives (A-1, A-2, A-3), evaluated by 19 criteria from a group of 6 experts (E-1, E-2, E-3, E-4, E-5 , E-6) as shown in Table 3.5.

*Table 3.5. Input data for the software engineering example*

| Experts | Alter-natives | Criteria / Weights / Assessments | | | | | | | | | | | | | | | | | | | Coeff. for expertise | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 | C13 | C14 | C15 | C16 | C17 | C18 | C19 | Case 1 | Case 2 |
| E-1 | | 0.010 | 0.075 | 0.075 | 0.045 | 0.075 | 0.075 | 0.045 | 0.075 | 0.045 | 0.075 | 0.065 | 0.070 | 0.070 | 0.060 | 0.020 | 0.020 | 0.040 | 0.040 | 0.020 | 0.167 | 0.11 |
| | A-1 | 2.000 | 9.000 | 9.000 | 10.000 | 3.000 | 4.000 | 8.000 | 6.000 | 5.000 | 2.000 | 6.000 | 2.000 | 1.000 | 3.000 | 5.000 | 7.000 | 10.000 | 4.000 | 4.000 | | |
| | A-2 | 10.000 | 9.000 | 10.000 | 10.000 | 5.000 | 10.000 | 9.000 | 8.000 | 6.000 | 4.000 | 8.000 | 10.000 | 8.000 | 9.000 | 8.000 | 7.000 | 3.000 | 2.000 | 8.000 | | |
| | A-3 | 8.000 | 3.000 | 6.000 | 8.000 | 3.000 | 5.000 | 7.000 | 5.000 | 6.000 | 10.000 | 7.000 | 6.000 | 5.000 | 4.000 | 4.000 | 7.000 | 5.000 | 1.000 | 3.000 | | |
| E-2 | | 0.035 | 0.075 | 0.095 | 0.020 | 0.040 | 0.020 | 0.060 | 0.075 | 0.095 | 0.020 | 0.075 | 0.040 | 0.020 | 0.060 | 0.075 | 0.040 | 0.060 | 0.075 | 0.020 | 0.167 | 0.12 |
| | A-1 | 4.000 | 8.000 | 2.000 | 6.000 | 1.000 | 5.000 | 9.000 | 4.000 | 6.000 | 10.000 | 5.000 | 8.000 | 1.000 | 6.000 | 4.000 | 8.000 | 9.000 | 4.000 | 2.000 | | |
| | A-2 | 5.000 | 8.000 | 4.000 | 2.000 | 10.000 | 5.000 | 6.000 | 7.000 | 10.000 | 8.000 | 9.000 | 1.000 | 2.000 | 5.000 | 10.000 | 9.000 | 8.000 | 6.000 | 8.000 | | |
| | A-3 | 4.000 | 1.000 | 5.000 | 8.000 | 7.000 | 9.000 | 4.000 | 10.000 | 5.000 | 10.000 | 4.000 | 6.000 | 10.000 | 4.000 | 8.000 | 10.000 | 9.000 | 10.000 | 8.000 | | |
| E-3 | | 0.050 | 0.050 | 0.050 | 0.030 | 0.072 | 0.050 | 0.030 | 0.072 | 0.050 | 0.050 | 0.072 | 0.072 | 0.071 | 0.050 | 0.030 | 0.050 | 0.071 | 0.050 | 0.030 | 0.167 | 0.05 |
| | A-1 | 6.000 | 7.000 | 8.000 | 4.000 | 9.000 | 3.000 | 4.000 | 6.000 | 8.000 | 8.000 | 6.000 | 2.000 | 2.000 | 2.000 | 2.000 | 1.000 | 10.000 | 1.000 | 1.000 | | |
| | A-2 | 8.000 | 7.000 | 7.000 | 8.000 | 7.000 | 7.000 | 10.000 | 8.000 | 6.000 | 8.000 | 8.000 | 7.000 | 8.000 | 8.000 | 6.000 | 9.000 | 4.000 | 1.000 | 6.000 | | |
| | A-3 | 6.000 | 5.000 | 7.000 | 8.000 | 2.000 | 2.000 | 6.000 | 6.000 | 6.000 | 6.000 | 2.000 | 5.000 | 5.000 | 2.000 | 2.000 | 1.000 | 2.000 | 1.000 | 2.000 | | |
| E-4 | | 0.050 | 0.050 | 0.020 | 0.030 | 0.070 | 0.035 | 0.050 | 0.050 | 0.070 | 0.035 | 0.070 | 0.035 | 0.035 | 0.070 | 0.050 | 0.070 | 0.070 | 0.070 | 0.070 | 0.167 | 0.24 |
| | A-1 | 1.000 | 7.000 | 7.000 | 4.000 | 1.000 | 1.000 | 2.000 | 2.000 | 2.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 7.000 | 1.000 | 1.000 | | |
| | A-2 | 9.000 | 1.000 | 10.000 | 7.000 | 3.000 | 8.000 | 7.000 | 7.000 | 2.000 | 7.000 | 8.000 | 6.000 | 7.000 | 7.000 | 8.000 | 8.000 | 5.000 | 1.000 | 7.000 | | |
| | A-3 | 8.000 | 1.000 | 8.000 | 6.000 | 8.000 | 7.000 | 7.000 | 7.000 | 5.000 | 9.000 | 7.000 | 6.000 | 7.000 | 6.000 | 8.000 | 7.000 | 8.000 | 1.000 | 6.000 | | |
| E-5 | | 0.027 | 0.048 | 0.048 | 0.027 | 0.067 | 0.027 | 0.048 | 0.048 | 0.067 | 0.028 | 0.067 | 0.048 | 0.048 | 0.067 | 0.067 | 0.067 | 0.067 | 0.067 | 0.067 | 0.166 | 0.38 |
| | A-1 | 1.000 | 6.000 | 6.000 | 4.000 | 2.000 | 2.000 | 2.000 | 2.000 | 2.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 5.000 | 1.000 | 2.000 | | |
| | A-2 | 9.000 | 2.000 | 9.000 | 8.000 | 2.000 | 7.000 | 5.000 | 5.000 | 1.000 | 6.000 | 9.000 | 6.000 | 6.000 | 8.000 | 8.000 | 7.000 | 4.000 | 1.000 | 6.000 | | |
| | A-3 | 9.000 | 2.000 | 8.000 | 7.000 | 9.000 | 7.000 | 8.000 | 8.000 | 7.000 | 9.000 | 8.000 | 6.000 | 6.000 | 6.000 | 7.000 | 8.000 | 8.000 | 2.000 | 5.000 | | |
| E-6 | | 0.067 | 0.020 | 0.067 | 0.040 | 0.055 | 0.055 | 0.040 | 0.067 | 0.055 | 0.040 | 0.068 | 0.068 | 0.068 | 0.055 | 0.067 | 0.068 | 0.040 | 0.040 | 0.020 | 0.166 | 0.10 |
| | A-1 | 1.000 | 7.000 | 7.000 | 3.000 | 1.000 | 1.000 | 8.000 | 2.000 | 8.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 10.000 | 1.000 | 1.000 | | |
| | A-2 | 9.000 | 10.000 | 10.000 | 8.000 | 7.000 | 7.000 | 10.000 | 10.000 | 5.000 | 9.000 | 7.000 | 8.000 | 9.000 | 10.000 | 10.000 | 9.000 | 5.000 | 1.000 | 8.000 | | |
| | A-3 | 2.000 | 10.000 | 10.000 | 2.000 | 9.000 | 3.000 | 8.000 | 3.000 | 9.000 | 9.000 | 2.000 | 5.000 | 5.000 | 7.000 | 8.000 | 7.000 | 7.000 | 1.000 | 6.000 | | |

The obtained results in Table 3.6 for the Case-1 are identical to the results described in [Krapohrl, 2014, Mustakerov & Borissova, 2014], namely identifying the A-2 as the best choice. This comparison proves the correctness of the proposed optimization model for group decision making (2.1) – (2.5). For Case-2, the decision identifies the most preferred alternative A-3. In selection of 2 preferred alternatives, the alternatives A-2 and A-3 are determined for both cases of weighted coefficients for the experts (Table 3.6). It is unknown which of these two alternatives is better than other, but both alternatives A-2 and A-3 are certainly better than the alternative A-1.

*Table 3.6 Results for selction of software company*

| **Case** | **Weights for experts** | | | | | **Chosen 1 alternative** | **Chosen 2 alternatives** |
|---|---|---|---|---|---|---|---|
| | E-1 | E-2 | E-3 | E-4 | E-5 | | |
| (1) | 0.10 | 0.27 | 0.21 | 0.25 | 0.17 | A-2 | A-2 & A-3 |
| (2) | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | A-2 | A-2 & A-3 |
| (3) | 0.35 | 0.13 | 0.12 | 0.20 | 0.20 | A-3 | A-2 & A-3 |

## 3.2. Testing the modified weighted product model for group decision making

### 3.2.1. Selection of enterprise resource planning system (ERP)

Using the data from Table 3.1 and the proposed modified weighted product model (2.8) – (2.12) for selection of alternative via group decision making, the following optimization task is formulated:

(3.6) $$\text{maximize} \sum_{i=1}^{3} x_i \sum_{k=1}^{5} \lambda^k R(A_i)^k$$

subject to

(3.7) $$\forall i = 1, 2, \dots, 3 : (\forall k = 1, 2, \dots, 5 : R(A_i)^k = \prod_{j=1}^{4} (e_{ij}^k)^{w_j^k}$$

(3.8) $$\sum_{j=1}^{4} w_j^k = 1, \forall k = 1, 2, \dots, 5$$

(3.9) $$x_1 + x_2 + x_3 = 1,\ x_i \in \{0,1\}$$

(3.10) $$\lambda^1 = 0.10;\ \lambda^2 = 0.27;\ \lambda^3 = 0.21;\ \lambda^4 = 0.25;\ \lambda^5 = 0.17$$

The solution of the task determines the values of the binary integer variables $x_2 = 1$ and $x_1 = x_3 = 0$. The obtained results for 3 different coefficients about the importance of the experts' assessments for selction of 1 and respectively 2 alternatives are shown in Table 3.7.

*Table 3.7 Results for selection of ERP*

| Case | Weights for experts | | | | | Chosen 1 alternative | Chosen 2 alternatives |
|---|---|---|---|---|---|---|---|
| | E-1 | E-2 | E-3 | E-4 | E-5 | | |
| (1) | 0.10 | 0.27 | 0.21 | 0.25 | 0.17 | A-2 | A-2 & A-3 |
| (2) | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | A-2 | A-2 & A-3 |
| (3) | 0.35 | 0.13 | 0.12 | 0.20 | 0.20 | A-3 | A-2 & A-3 |

### 3.2.2. Selection of a personal computer vendor for public procurement purposes

The results for personal computer vendor selction for the purpose of public procurement using the data from Table 3.3 and the proposed modified weighted product model (2.8) – (2.12) are shown in Table 3.8:

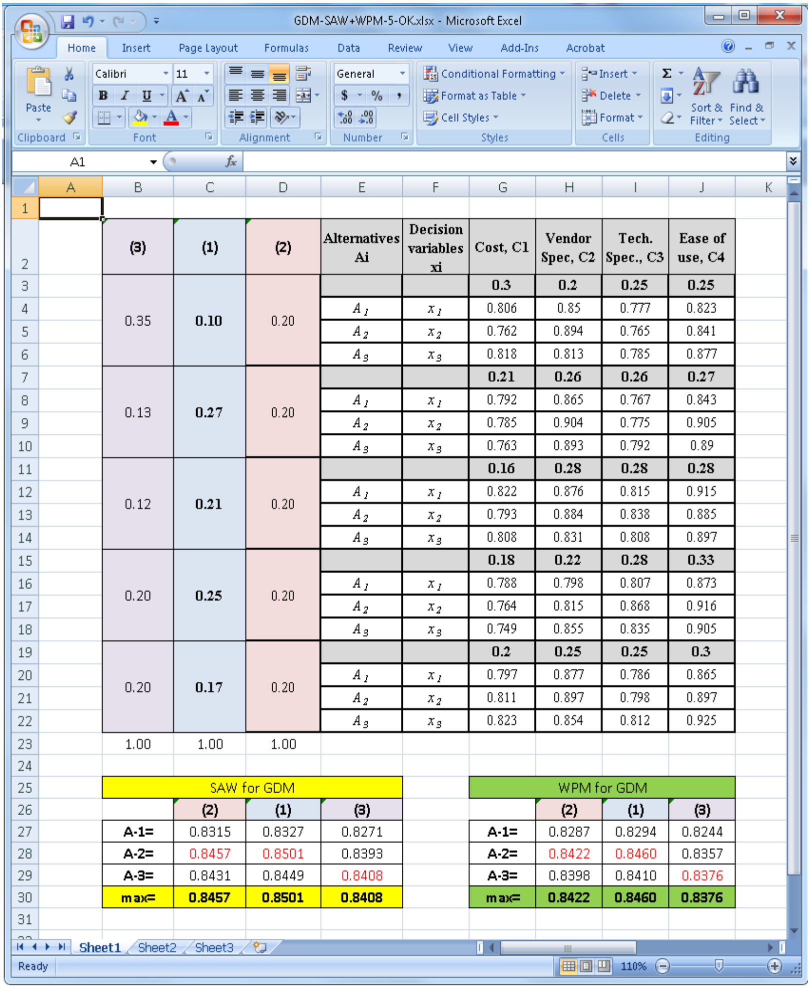
*Table 3.8. Results for selction of PCs vendor under public procurement*

| Case | Weights for experts | | | | | Chosen 1 alternative | Chosen 2 alternatives |
|---|---|---|---|---|---|---|---|
| | E-1 | E-2 | E-3 | E-4 | E-5 | | |
| (1) | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | A3 | A2 & A3 |
| (2) | 0.25 | 0.10 | 0.15 | 0.25 | 0.25 | A3 | A2 & A3 |
| (3) | 0.27 | 0.12 | 0.13 | 0.18 | 0.30 | A3 | A2 & A3 |

## 3.3. Numerical testing of modified weighted sum model and weighted product model for group decision making in MS Excel enviroment

The numerical examples of the modified modified simple additive weighting and weighted product models for group decision making are presented in an MS Excel spreadsheet, Fig. 3.8 [Korsemov et al., 2018].

The obtained results in the middle of MS Excel (Figure 3.8) are identical to the results obtained in solving the optimization tasks using the Lingo solver. The small differences in the values obtained for the target functions at Lingo v.12 and the values shown in Fig. 3.5, are due to the fact that the corresponding cells of the MS Excel spreadsheet are formatted to show the numbers with a precision of 4 decimal places.



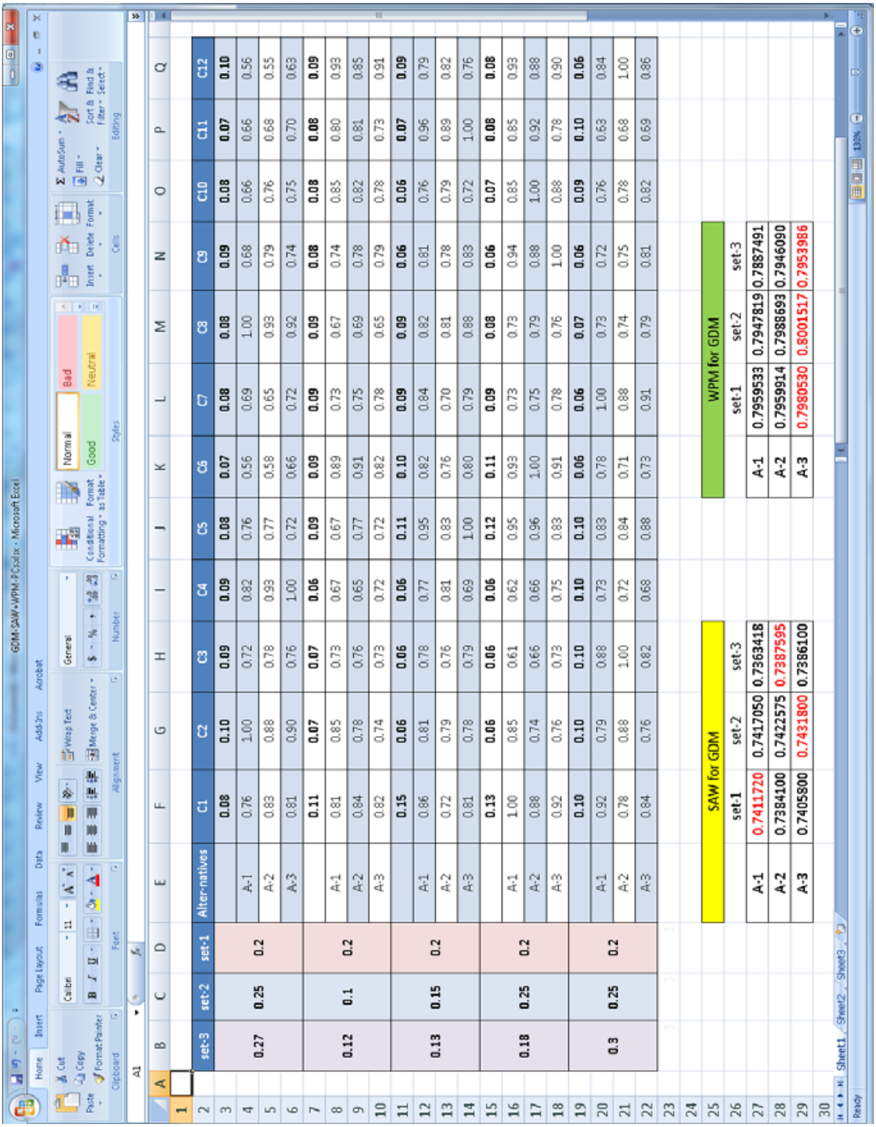
| (3) | (1) | (2) | Alternatives Ai | Decision variables xi | Cost, C1 | Vendor Spec, C2 | Tech. Spec., C3 | Ease of use, C4 |
|---|---|---|---|---|---|---|---|---|
| 0.35 | 0.10 | 0.20 | | | 0.3 | 0.2 | 0.25 | 0.25 |
| | | | $A_1$ | $x_1$ | 0.806 | 0.85 | 0.777 | 0.823 |
| | | | $A_2$ | $x_2$ | 0.762 | 0.894 | 0.765 | 0.841 |
| | | | $A_3$ | $x_3$ | 0.818 | 0.813 | 0.785 | 0.877 |
| 0.13 | 0.27 | 0.20 | | | 0.21 | 0.26 | 0.26 | 0.27 |
| | | | $A_1$ | $x_1$ | 0.792 | 0.865 | 0.767 | 0.843 |
| | | | $A_2$ | $x_2$ | 0.785 | 0.904 | 0.775 | 0.905 |
| | | | $A_3$ | $x_3$ | 0.763 | 0.893 | 0.792 | 0.89 |
| 0.12 | 0.21 | 0.20 | | | 0.16 | 0.28 | 0.28 | 0.28 |
| | | | $A_1$ | $x_1$ | 0.822 | 0.876 | 0.815 | 0.915 |
| | | | $A_2$ | $x_2$ | 0.793 | 0.884 | 0.838 | 0.885 |
| | | | $A_3$ | $x_3$ | 0.808 | 0.831 | 0.808 | 0.897 |
| 0.20 | 0.25 | 0.20 | | | 0.18 | 0.22 | 0.28 | 0.33 |
| | | | $A_1$ | $x_1$ | 0.788 | 0.798 | 0.807 | 0.873 |
| | | | $A_2$ | $x_2$ | 0.764 | 0.815 | 0.868 | 0.916 |
| | | | $A_3$ | $x_3$ | 0.749 | 0.855 | 0.835 | 0.905 |
| 0.20 | 0.17 | 0.20 | | | 0.2 | 0.25 | 0.25 | 0.3 |
| | | | $A_1$ | $x_1$ | 0.797 | 0.877 | 0.786 | 0.865 |
| | | | $A_2$ | $x_2$ | 0.811 | 0.897 | 0.798 | 0.897 |
| | | | $A_3$ | $x_3$ | 0.823 | 0.854 | 0.812 | 0.925 |
| 1.00 | 1.00 | 1.00 | | | | | | |

| SAW for GDM | | | |
|---|---|---|---|
| | (2) | (1) | (3) |
| A-1= | 0.8315 | 0.8327 | 0.8271 |
| A-2= | 0.8457 | 0.8501 | 0.8393 |
| A-3= | 0.8431 | 0.8449 | 0.8408 |
| max= | 0.8457 | 0.8501 | 0.8408 |

| WPM for GDM | | | |
|---|---|---|---|
| | (2) | (1) | (3) |
| A-1= | 0.8287 | 0.8294 | 0.8244 |
| A-2= | 0.8422 | 0.8460 | 0.8357 |
| A-3= | 0.8398 | 0.8410 | 0.8376 |
| max= | 0.8422 | 0.8460 | 0.8376 |

*Fig. 3.8. Selection of the best alternative using modified models for group decision making*

The example of selecting a personal computer vendor under public procurement purposes is also implemented in the MS Excel environment and the obtained results are identical to the results from Lingo solver using (Fig. 3.9).



| set-3 | set-2 | set-1 | Alter-natives | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 | C11 | C12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.27 | 0.25 | 0.2 | | **0.08** | **0.10** | **0.09** | **0.09** | **0.08** | **0.07** | **0.08** | **0.08** | **0.09** | **0.08** | **0.07** | **0.10** |
| | | | A-1 | 0.76 | 1.00 | 0.72 | 0.82 | 0.76 | 0.56 | 0.69 | 1.00 | 0.68 | 0.66 | 0.66 | 0.56 |
| | | | A-2 | 0.83 | 0.88 | 0.78 | 0.93 | 0.77 | 0.58 | 0.65 | 0.93 | 0.79 | 0.76 | 0.68 | 0.55 |
| | | | A-3 | 0.81 | 0.90 | 0.76 | 1.00 | 0.72 | 0.66 | 0.72 | 0.92 | 0.74 | 0.75 | 0.70 | 0.63 |
| 0.12 | 0.1 | 0.2 | | **0.11** | **0.07** | **0.07** | **0.06** | **0.09** | **0.09** | **0.09** | **0.09** | **0.08** | **0.08** | **0.08** | **0.09** |
| | | | A-1 | 0.81 | 0.85 | 0.73 | 0.67 | 0.67 | 0.89 | 0.73 | 0.67 | 0.74 | 0.85 | 0.80 | 0.93 |
| | | | A-2 | 0.84 | 0.78 | 0.76 | 0.65 | 0.77 | 0.91 | 0.75 | 0.69 | 0.78 | 0.82 | 0.81 | 0.85 |
| | | | A-3 | 0.82 | 0.74 | 0.73 | 0.72 | 0.72 | 0.82 | 0.78 | 0.65 | 0.79 | 0.78 | 0.73 | 0.91 |
| 0.13 | 0.15 | 0.2 | | **0.15** | **0.06** | **0.06** | **0.06** | **0.11** | **0.10** | **0.09** | **0.09** | **0.06** | **0.06** | **0.07** | **0.09** |
| | | | A-1 | 0.86 | 0.81 | 0.78 | 0.77 | 0.95 | 0.82 | 0.84 | 0.82 | 0.81 | 0.76 | 0.96 | 0.79 |
| | | | A-2 | 0.72 | 0.79 | 0.76 | 0.81 | 0.83 | 0.76 | 0.70 | 0.81 | 0.78 | 0.79 | 0.89 | 0.82 |
| | | | A-3 | 0.81 | 0.78 | 0.79 | 0.69 | 1.00 | 0.80 | 0.79 | 0.88 | 0.83 | 0.72 | 1.00 | 0.76 |
| 0.18 | 0.25 | 0.2 | | **0.13** | **0.06** | **0.06** | **0.06** | **0.12** | **0.11** | **0.09** | **0.08** | **0.06** | **0.07** | **0.08** | **0.08** |
| | | | A-1 | 1.00 | 0.85 | 0.61 | 0.62 | 0.95 | 0.93 | 0.73 | 0.73 | 0.94 | 0.85 | 0.85 | 0.93 |
| | | | A-2 | 0.88 | 0.74 | 0.66 | 0.66 | 0.96 | 1.00 | 0.75 | 0.79 | 0.88 | 1.00 | 0.92 | 0.88 |
| | | | A-3 | 0.92 | 0.76 | 0.73 | 0.75 | 0.83 | 0.91 | 0.78 | 0.76 | 1.00 | 0.88 | 0.78 | 0.90 |
| 0.3 | 0.25 | 0.2 | | **0.10** | **0.10** | **0.10** | **0.10** | **0.10** | **0.06** | **0.06** | **0.07** | **0.06** | **0.09** | **0.10** | **0.06** |
| | | | A-1 | 0.92 | 0.79 | 0.88 | 0.73 | 0.83 | 0.78 | 1.00 | 0.73 | 0.72 | 0.76 | 0.63 | 0.84 |
| | | | A-2 | 0.78 | 0.88 | 1.00 | 0.72 | 0.84 | 0.71 | 0.88 | 0.74 | 0.75 | 0.78 | 0.68 | 1.00 |
| | | | A-3 | 0.84 | 0.76 | 0.82 | 0.68 | 0.88 | 0.73 | 0.91 | 0.79 | 0.81 | 0.82 | 0.69 | 0.86 |

| SAW for GDM | set-1 | set-2 | set-3 |
|---|---|---|---|
| A-1 | 0.7411720 | 0.7417050 | 0.7363418 |
| A-2 | 0.7384100 | 0.7422575 | 0.7387595 |
| A-3 | 0.7405800 | 0.7431800 | 0.7386100 |

| WPM for GDM | set-1 | set-2 | set-3 |
|---|---|---|---|
| A-1 | 0.7959533 | 0.7947819 | 0.7887491 |
| A-2 | 0.7959914 | 0.7988693 | 0.7946090 |
| A-3 | 0.7980530 | 0.8001517 | 0.7953986 |

*Fig. 3.9. Selection of supplier under public procurement via group decision making*

The advantage of MS Excel spreadsheet using is that once the corresponding formulas have been entered, it can easily be modified by various adjustments to the problem. As a shortcoming, it can be noted that MS Excel models can not identify several good alternatives at the same time, unlike those solved by means of Lingo system.

### 3.1.3.4. Numerical testing of the proposed modified SMART model for group decision making

As input data for the numerical testing of the proposed model, adapted data from the ERP selection problem [Efe, 2016] are used. The evaluation of the software and its provider is done by the following criteria: 1) price (C1); 2) supplier specifications (C2); 3) software technical specifications (C3); and 4) ease of use (C4). The selection is made by a group consisting of a database administrator (E-1), a financial consultant (E-2) and a business analyst (E-3).

The normalized evaluations of alternatives against criteria, along with weights for relative importance between criteria, as well as the coefficients for group members' expertise, in case of ERP selection are shown in Table 3.9.

*Table 3.9 Input data in case of ERP selection*

| Group of experts | Weights for experts | Alternatives | Criteria / Weights / Evaluations | | | |
|---|---|---|---|---|---|---|
| | | | C1 | | | |
| E-1 | 0.33 | | 0.10 | 0.25 | 0.40 | 0.25 |
| | | A-1 | 0.92757 | 0.89738 | 0.14286 | 0.92757 |
| | | A-2 | 0.55556 | 0.33333 | 0.42857 | 0.42857 |
| | | A-3 | 0.55556 | 0.81087 | 0.33333 | 0.42857 |
| | | A-4 | 0.77778 | 0.14286 | 0.89738 | 0.89738 |
| | | A-5 | 0.68785 | 0.46577 | 0.63224 | 0.58876 |
| E-2 | 0.33 | | 0.50 | 0.25 | 0.10 | 0.15 |
| | | A-1 | 0.55556 | 0.68712 | 0.33333 | 0.55556 |
| | | A-2 | 0.92757 | 0.33333 | 0.42857 | 0.92757 |
| | | A-3 | 0.77778 | 0.89738 | 0.42857 | 0.77778 |
| | | A-4 | 0.55556 | 0.92757 | 0.33333 | 0.42857 |
| | | A-5 | 0.46577 | 0.68785 | 0.63224 | 0.63224 |
| E-3 | 0.34 | | 0.15 | 0.15 | 0.30 | 0.40 |
| | | A-1 | 0.33333 | 0.89738 | 0.14286 | 0.33333 |
| | | A-2 | 0.81087 | 0.92757 | 0.24560 | 0.92757 |
| | | A-3 | 0.24560 | 0.24560 | 0.42857 | 0.33333 |
| | | A-4 | 0.33333 | 0.33333 | 0.33333 | 0.14286 |
| | | A-5 | 0.63224 | 0.63224 | 0.68785 | 0.46577 |

Using the modified SMART model for group decision making (2.13) – (2.17), the following optimization task is formulated:

(3.11) $$\text{maximize} \sum_{i=1}^{5} x_i \sum_{k=1}^{3} \lambda^k A_i^k$$

subject to

(3.12) $$\forall i = 1, 2, \dots, 5 : (\forall k = 1, 2, \dots, 3 : A_i^k = \sum_{j=1}^{4} (w_j^k)^* e_{i,j}^k )$$

(3.13) $$(w_j^k)^* = w_j^k \left( \sum_{j=1}^{N} w_j^k \right)^{-1}$$

(3.14a) $$x_1 + x_2 + x_3 + x_4 + x_5 = 1,\ x_i \in \{0,1\}$$

(3.14б) $$x_1 + x_2 + x_3 + x_4 + x_5 = 3,\ x_i \in \{0,1\}$$

(3.15) $$\sum_{k=1}^{3} \lambda^k = 1$$

The solution of the task using the Lingo v.12 software and three different sets for coefficients expressing the group members' expertise are shown in Table 3.10.

*Table 3.10 Results for ERP selection*

| **Case** | **Weights for experts** | | | **Chosen 1 alternative** | **Chosen 3 alternatives** |
|---|---|---|---|---|---|
| | E-1 | E-2 | E-3 | | |
| (1) | 0.33 | 0.33 | 0.34 | A-2 | A-2, A-3, A-5 |
| (2) | 0.35 | 0.20 | 0.45 | A-2 | A-1, A-2, A-5 |
| (3) | 0.46 | 0.22 | 0.32 | A-5 | A-2, A-4, A-5 |

Table 3.10 shows that the use of coefficients expressing the expertise of group members influences the final decision in determining the most preferred alternative.

## 3.5. Numerical testing of the proposed algorithm for group decision making with 3 different strategies

The numerical example is taken from practice of company producing CNC metalworking machines. The company faces the problem of supplier selection for some auxiliary details. There are five potential suppliers corresponding to five alternatives (A-1, A-2, A-3, A-4, and A-5). The decision should be taken based on the evaluations of a group of five managers: design manager (E1), production manager (E2), marketing manager (E3), purchasing manager (E4), and R&D manager (E5). The evaluation attributes are adapted from (Chou & Chang, 2008): 1) unit price of elements; 2) discount rate; 3) input control rejection rate; 4) customer rejection rate; 5) lead time; 6) flexibility in unexpected orders; 7) capability in management; 8) compatibility in production strategy; 9) innovation capabilities; and 10) unexpected problem-solving.

The normalized evaluations of alternatives against criteria, along with weights for relative importance between criteria, as well as the coefficients for group members' expertise are shown in Table 3.11.

*Table 3.11. Input data for supplier selction*

| Group of experts | Weights for experts | Alternatives | Criteria / Weights / Evaluations | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | **C1** | | | | **C1** | | | | **C1** | |
| E-1 | 0.20 | | 0.2 | 1 | 1 | 0.6 | 1 | 1 | 0.6 | 1 | 0.6 | 0.4 |
| | | A-1 | 0.2 | 0.9 | 0.9 | 1 | 0.3 | 0.4 | 0.8 | 0.6 | 0.5 | 0.4 |
| | | A-2 | 0.8 | 0.9 | 1 | 0.9 | 0.5 | 0.8 | 0.9 | 0.8 | 0.6 | 0.4 |
| | | A-3 | 0.8 | 0.3 | 0.6 | 0.8 | 0.3 | 0.5 | 0.7 | 0.5 | 0.6 | 1 |
| | | A-4 | 0.2 | 0.6 | 0.2 | 0.4 | 0.3 | 0.5 | 0.7 | 1 | 0.4 | 0.4 |
| | | A-5 | 0.4 | 0.8 | 1 | 0.8 | 0.9 | 0.8 | 0.7 | 0.3 | 0.2 | 0.8 |
| E-2 | 0.20 | | 0.4 | 0.8 | 1 | 0.2 | 0.4 | 0.2 | 0.6 | 0.8 | 1 | 0.2 |
| | | A-1 | 0.4 | 0.8 | 0.2 | 0.6 | 0.1 | 0.5 | 0.9 | 0.4 | 0.6 | 1 |
| | | A-2 | 0.5 | 0.8 | 0.4 | 0.2 | 1 | 0.5 | 0.6 | 0.7 | 1 | 0.8 |
| | | A-3 | 0.4 | 0.1 | 0.5 | 0.8 | 0.7 | 0.9 | 0.4 | 1 | 0.5 | 1 |
| | | A-4 | 1 | 0.5 | 0.8 | 0.1 | 0.6 | 0.4 | 0.8 | 0.9 | 0.4 | 0.2 |
| | | A-5 | 0.8 | 0.9 | 0.1 | 0.2 | 0.5 | 1 | 0.9 | 0.8 | 0.6 | 0.8 |
| E-3 | 0.20 | | 0.8 | 0.8 | 0.8 | 0.6 | 1 | 0.8 | 0.6 | 1 | 0.8 | 0.8 |
| | | A-1 | 0.6 | 0.7 | 0.8 | 0.4 | 0.9 | 0.3 | 0.4 | 0.6 | 0.8 | 0.8 |
| | | A-2 | 0.8 | 0.7 | 0.7 | 0.8 | 0.7 | 0.7 | 1 | 0.8 | 0.6 | 0.8 |
| | | A-3 | 0.6 | 0.5 | 0.7 | 0.8 | 0.2 | 0.2 | 0.6 | 0.6 | 0.6 | 0.6 |
| | | A-4 | 0.8 | 0.6 | 0.2 | 0.2 | 0.2 | 0.2 | 0.1 | 1 | 0.1 | 0.7 |
| | | A-5 | 0.8 | 0.8 | 0.7 | 0.8 | 0.8 | 0.6 | 0.9 | 0.4 | 0.1 | 0.6 |
| E-4 | 0.20 | | 0.8 | 0.8 | 0.4 | 0.6 | 1 | 0.6 | 0.8 | 0.8 | 1 | 0.6 |
| | | A-1 | 0.1 | 0.7 | 0.7 | 0.4 | 0.1 | 0.5 | 0.2 | 0.3 | 0.2 | 0.1 |
| | | A-2 | 0.9 | 0.1 | 1 | 0.7 | 0.3 | 0.8 | 0.7 | 0.7 | 0.2 | 0.7 |
| | | A-3 | 0.8 | 0.1 | 0.8 | 0.6 | 0.8 | 0.7 | 0.7 | 0.7 | 0.5 | 0.9 |
| | | A-4 | 0.5 | 0.1 | 0.1 | 0.3 | 0.3 | 0.5 | 0.4 | 0.7 | 0.4 | 0.1 |
| | | A-5 | 0.7 | 0.8 | 0.6 | 0.7 | 0.7 | 0.8 | 0.8 | 0.5 | 0.1 | 0.7 |
| E-5 | 0.20 | | 0.6 | 0.8 | 0.8 | 0.6 | 1 | 0.6 | 0.8 | 0.8 | 1 | 0.6 |
| | | A-1 | 0.1 | 0.6 | 0.6 | 0.4 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.1 |
| | | A-2 | 0.9 | 0.2 | 0.9 | 0.8 | 0.2 | 0.7 | 0.5 | 0.5 | 0.1 | 0.6 |
| | | A-3 | 0.9 | 0.2 | 0.8 | 0.7 | 0.9 | 0.7 | 0.8 | 0.8 | 0.7 | 0.9 |
| | | A-4 | 0.4 | 0.8 | 0.2 | 0.7 | 0.1 | 0.6 | 0.1 | 0.5 | 0.9 | 0.2 |
| | | A-5 | 0.6 | 0.9 | 0.6 | 0.6 | 0.8 | 0.8 | 0.7 | 0.4 | 0.1 | 0.6 |

### 3.5.1. Selection the best single alternative

The first scenario of the proposed algorithm in Chapter 2 (Figure 2.6) requires formulating and solving an optimization task whose solution determines a single alternative:

(3.16) $$\text{maximize} \left( \sum_{i=1}^{5} x_i \sum_{k=1}^{5} \lambda^k A_i^k \right)$$

subject to

(3.17) $$\forall i = 1, 2, \ldots, 5 : (\forall k = 1, 2, \ldots, 5 : A_i^k = \sum_{j=1}^{10} w_j^k e_{i,j}^k \ )$$

(3.18) $$\sum_{j=1}^{N} w_j^k = 1$$

(3.19) $$x_1 + x_2 + x_3 + x_4 + x_5 = 1,\ x_i \in \{0,1\}$$

(3.20) $$\sum_{k=1}^{5} \lambda^k = 1$$

The optimization task (3.16) – (3.20) is solved for three different sets of coefficients expressing the group members' expertise as shown in Table 3.12..

*Table 3.12. Selected the best single alternative via group decision making*

| Case | Weights for experts | | | | | The best single alternative |
|---|---|---|---|---|---|---|
| | E-1 | E-2 | E-3 | E-4 | E-5 | |
| (1) | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | A-2 |
| (2) | 0.10 | 0.05 | 0.25 | 0.30 | 0.30 | A-3 |
| (3) | 0.05 | 0.10 | 0.40 | 0.05 | 0.40 | A-4 |

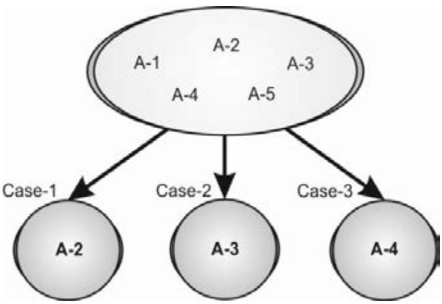
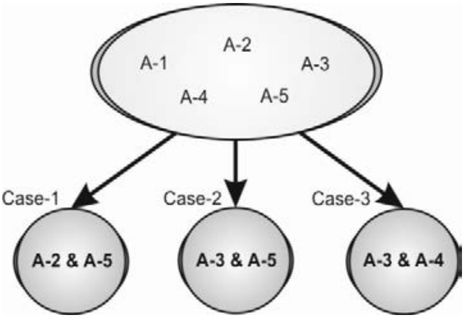



*Fig. 3.10. The best single alternative selection*

The obtained results in first scenario where one best alternative is to be selected under 3 different cases for weights of group members expertise are illustrated in Fig. 3.10.

### 3.5.2. Choice of several good alternatives




*Fif. 3.11. Selection of 2 good alternatives*

The obtained results in second scenario where 2 good alternatives are to be selected under 3 different cases for weights of group members expertise are illustrated in Fig. 3.11

### 3.5.3. Ranking of all alternatives

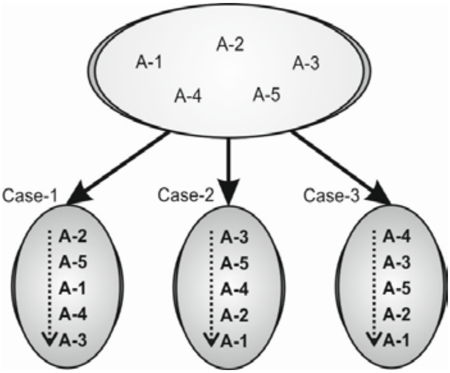



*Fig. 3.12. Ranking of all alternatives*

The obtained results in third scenario where all of the alternatives are to be ranked under 3 different cases for weights of group members expertise are illustrated in Fig. 3.12

## 3.6. Numerical testing of the models for group decision-making under uncertainty conditions

The numerical testing of group decision making models under uncertainty conditions concern problem related with purchasing of ERP. Because of the dynamics of the economic environment, the company's prospects are represented through 3 possible situations: increasing, reducing or maintaining the current state of revenue. For this purpose, the following utility function is formulated:

$$CBE = \frac{Cost}{Revenue} = \frac{C_{ac} + C_{cust} + C_{inst} + C_{test} + C_{staff} + C_{proc} + C_{file} + C_{un} + C_{np}}{R_{lcr} + R_{kis} + R_{lc} + R_{ap}} \qquad (3.21)$$

The *Cost* express the total cost obtained by summing costs for acquisition ($C_{ac}$), customization ($C_{cust}$), installation ($C_{inst}$), testing for technically and functionally after installation ($C_{test}$), staff training ($C_{staff}$), change some working procedures ($C_{proc}$), conversion of files ($C_{file}$), uninstalling the old system ($C_{un}$), new policy for loyal clients ($C_{np}$). The *Revenue* is the expected incomes related with labor costs reducing ($R_{lcr}$), reducing the need for keeping stock ($R_{kis}$), improving reliability by new policy for loyal clients ($R_{lc}$), using of more automated processes ($R_{ap}$).

The use of (3.21) makes evaluation possible as a result of the implementation of certain situations, as shown in Table 3.15.

*Table 3.15. Modified group decision making matrix under uncertainty conditions*

| Group of experts | Weights for experts | Alternatives | Conditions | | |
|---|---|---|---|---|---|
| | | | Increase in requests (S-1) | Decrease in requests (S-2) | Without changes (S-3) |
| E-1 | 0.25 | A-1 | 0.52 | 0.83 | 0.75 |
| | | A-2 | 0.53 | 0.86 | 0.72 |
| | | A-3 | 0.54 | 0.84 | 0.71 |
| E-2 | 0.35 | A-1 | 0.56 | 0.87 | 0.78 |
| | | A-2 | 0.55 | 0.80 | 0.74 |
| | | A-3 | 0.58 | 0.81 | 0.72 |
| E-3 | 0.40 | A-1 | 0.62 | 0.85 | 0.72 |
| | | A-2 | 0.60 | 0.88 | 0.76 |
| | | A-3 | 0.61 | 0.82 | 0.70 |

### 3.6.1. Numerical testing of the model for group decision making using the Wald's criterion

Using the Wald criterion, the following optimization task is formulated to determine the most preferred alternative:

(3.22)
$$\max\min \sum_{i=1}^{3}\sum_{k=1}^{3} \lambda^{k} CBE_{ij}^{k}$$

subject to

(3.23)
$$\lambda^{1} = 0.25;\ \lambda^{2} = 0.35;\ \lambda^{3} = 0.40$$

The task (3.22) – (3.23) solution defines as the most preferred alternative A-3.

### 3.6.2. Numerical testing of the model for group decision making using the Laplace criterion

The use of the Laplace criterion leads to the formulation of the following optimization task to determine the most preferred alternative:

(3.24) $$\max\left(\sum_{i=1}^{3}\sum_{k=1}^{3}\frac{\lambda^k CBE_{ij}^k}{M}\right)$$

subject to

(3.25) $\lambda^1 = 0.25;\ \lambda^2 = 0.35;\ \lambda^3 = 0.40$

The task (3.24) – (3.25) solution determine alternative A-2 as the most preferred alternative.

### 3.6.3. Numerical testing of the model for group decision making using the Hurwitz criterion

Using the Hurwitz criterion, the following optimization task is formulated:

(3.26) $$\max\left\{\alpha\max\sum_{i=1}^{3}\sum_{k=1}^{3}\lambda^k \mathrm{CBE}_{ij}^k + (1-\alpha)\min\sum_{i=1}^{3}\sum_{k=1}^{3}\lambda^k CBE_{ij}^k\right\}$$

subject to

(3.27) $\lambda^1 = 0.25;\ \lambda^2 = 0.35;\ \lambda^3 = 0.40$

(3.28) $\alpha = 0.4$

The task (3.27) – (3.29) solution determines as the most preferred alternative A-1, for $\alpha = 0.4$ and for $\alpha = 0.15$ the most preferable is the alternative A-3.

### 3.6.4. Numerical testing of the model for group decision making using the Savage criterion

To use the Savage criterion, a regret matrix is required and calculated from the point of view of each expert by following reletion:

(3.29) $$R_{ij}^k = \left|CBE_{ij}^k - \max CBE_{ij}^k\right|$$

The regret matrix determined by (3.29) is shown in Table 3.16.

*Table 3.16. Regret matrix*

| Group of experts | Weights for experts | Alternatives ($A_i$) | Regrets | | |
|---|---|---|---|---|---|
| **E-1** | 0.25 | A-1 | 0.02 | 0.03 | 0 |
| | | A-2 | 0.01 | 0 | 0.03 |
| | | A-3 | 0 | 0.02 | 0.04 |
| **E-2** | 0.35 | A-1 | 0.02 | 0 | 0 |
| | | A-2 | 0.03 | 0.07 | 0.04 |
| | | A-3 | 0 | 0.06 | 0.06 |
| **E-3** | 0.40 | A-1 | 0 | 0.03 | 0.04 |
| | | A-2 | 0.02 | 0 | 0 |
| | | A-3 | 0.01 | 0.06 | 0.06 |

The formulated optimization task, taking into account the the Savage criterion is:

(3.30) $$\min \max \sum_{i=1}^{3} \sum_{k=1}^{3} \lambda^{k} R_{ij}^{k}$$

subject to

(3.31) $$\forall i = 1, 2, \ldots, 3 : (\forall k = 1, 2, \ldots, 3 : R_{ij}^{k} = \left| CBE_{ij}^{k} - \max CBE_{ij}^{k} \right|$$

(3.32) $$\lambda^{1} = 0.25; \quad \lambda^{2} = 0.35; \quad \lambda^{3} = 0.40$$

The solution of the task determines as the most preferred alternative A-2.

### 3.6.5. Comparative analysis of the proposed models for group decision making under uncertainty conditions

The obtained results by using of different weighted coefficients for experts' expertise and using of different strategies expressed by corresponding criteria of Wald, Laplace, Hurvitz and Savage are shown in Table 3.17.

As a result of the comparison, it can be concluded that the use of the different criteria of Wald, Laplace, Hurwitz and Savage, combined with the viewpoints of the group's experts, leads to different preferences in determining the group alternative. Therefore, it is important to prioritize the most appropriate decision-making strategy in the context of uncertainty.

The described models in the dissertation could be used with other formulations of utility functions.

*Table 3.17. Comparison between different optimization criteria and chosen alternative*

| Criterion | Weights for experts' expertise | | | Chosen alternative |
|---|---|---|---|---|
| **Case-1** | $\lambda^1$ | $\lambda^2$ | $\lambda^3$ | |
| Wald | 0.25 | 0.35 | 0.40 | A-3 |
| Laplace | 0.25 | 0.35 | 0.40 | A-2 |
| Hurwicz ($\alpha$=0.40) | 0.25 | 0.35 | 0.40 | A-1 |
| Hurwicz ($\alpha$=0.15) | 0.25 | 0.35 | 0.40 | A-3 |
| Savage | 0.25 | 0.35 | 0.40 | A-2 |
| **Case-2** | $\lambda^1$ | $\lambda^2$ | $\lambda^3$ | |
| Wald | 0.34 | 0.46 | 0.20 | A-3 |
| Laplace | 0.34 | 0.46 | 0.20 | A-1 |
| Hurwicz ($\alpha$=0.4) | 0.34 | 0.46 | 0.20 | A-1 |
| Hurwicz ($\alpha$=0.15) | 0.34 | 0.46 | 0.20 | A-3 |
| Savage | 0.34 | 0.46 | 0.20 | A-2 |

The proposed group decision-making models under uncertainty conditions are implemented in MS Excel as shown in Fig. 3.13.

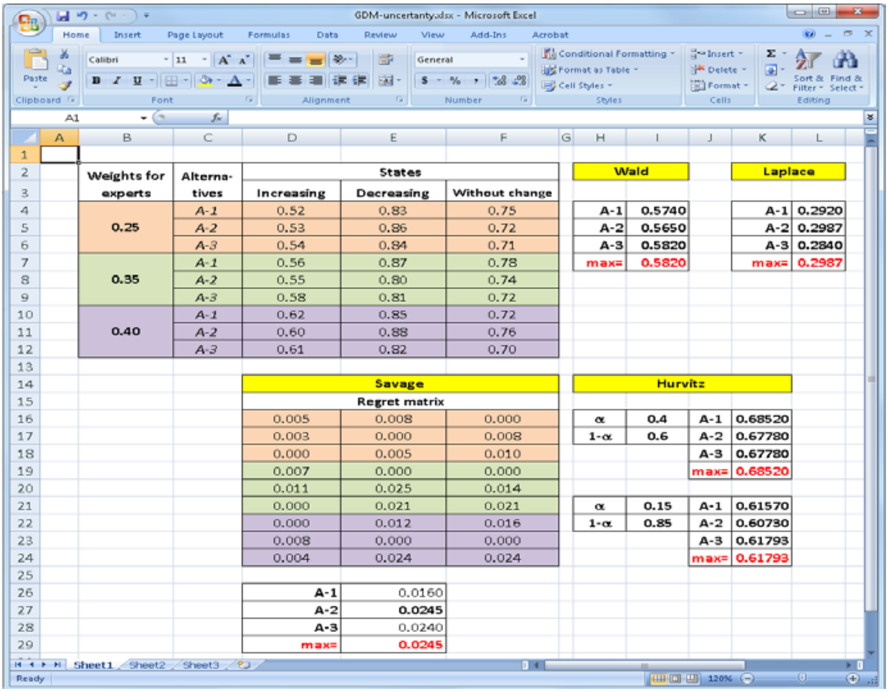



*Fig. 3.13. Group decision-making under uncertainty conditions in MS Excel*

# CONCLUSION

The dissertation deals with the problems that arise in decision-making and in particular in group decision-making. Many application areas require decision-making of a different nature. The need for decision-making leads to the development of various well-grounded methods and algorithms. The essence of these methods and algorithms aim to support the process of group decision-making by taking into account different quantitative and qualitative criteria. All of this determines the actuality of the research related to the development of models and algorithms to support group decision-making. An important stage in group decision making is the formation of the group of experts. In group decision-making, it is important to consider the influence of the competencies and responsibilities of the group's experts on the quality of the final decision. The expertise of each member of the group can be determined on the basis of experience and knowledge in the field, which are generally different for the different experts.

The current thesis presents modifications of models that take into account the qualifications and experience of the individual members of the group participating in the group selection of the most suitable alternative. Also, some algorithms for their practical application are proposed. Some typical cases of uncertainty are investigated when the goals to be achieved are known, but information on alternatives and future events is incomplete. Modifications of models based on the principles of Wald, Laplace, Hurvitz and Savage are proposed.

The practical applicability of the proposed models and algorithms for group decision-making has been proven through numerical experiments based on real problems. Some of the proposed models and algorithms are implemented in appropriate software tools to support group decision-making.

As a future development of the dissertation research it is planned to study the capabilities of other models in order to modify them for group decision-making as well as to create new models and algorithms to support decision-making that take into account various essential parameters and situations for the goals of group decision-making.

The obtained results in relation of the dissertation thesis are published in 5 scientific papers and and 2 papers presented on international conferences.

## SUMMARY OF THE DISSERTATION CONTRIBUTIONS

The main results of the dissertation paper could be summarized as follows:

1. Modifications of weighted sum model, weighted product model and SMART model for selection of alternative(s) under group decision-making are formulated. The proposed modifications take into account the differences in the experience and knowledge of the group members by using of weighted coefficients to express the level of expertise. Modifications of the models allow to select one the best alternative or to select several good alternatives. The proposed models allow formulation of combinatorial optimization tasks, whose solutions determine the optimal preferred alternative(s). The corresponding algorithms for practical application of these models are proposed.
2. A generalized algorithm for group decision making with three different strategies is porposed: 1) for choice one best alternative, 2) for selection of several good alternatives, 3) for ranking of all alternatives). For each strategy, appropriate optimization models for group decision making are formulated, taking into account the expertise of each member of the group.
3. Modified models for group decision making under uncertainty conditions using the criteria of Wald, Laplace, Hurvitz and Savage are proposed. The proposed modifications take into account the differences in the experience and knowledge of the group experts by introducing corresponding weighted coefficients for each expert. Appropriate optimization tasks have been formulated to determine the optimal alternative for each of these criteria.
4. The proposed modifications of the models for group decision-making are implemented in MS Excel spreadsheets enviroment. The results of conducted tests in MS Excel and in Lingo enviroment are identical, that prove their practical applicability.

## Publications in Conjunction of the Dissertation


1. Borissova, D., **D. Korsemov**, I. Mustakerov. Multi-attribute group decision making considering difference in experts knowledge: An Excel application. In Proc. *12th Int. Management Conference – Management Perspectives in the Digital Era, 1-2 Nov. 2018, Bucharest, Romania*, ISSN 2286-1440, pp. 387-395.
2. **Korsemov, D.**, D. Borissova, I. Mustakerov. Group decision making for selection of supplier under public procurement. In: Kalajdziski S., Ackovska N. (eds) ICT Innovations 2018. Engineering and Life Sciences. ICT 2018. Communications in Computer and Information Science, Vol. 940, Springer, Cham, ISBN: 978-3-030-00824-6, DOI: https://doi.org/10.1007/978-3-030-00825-3_5. (SJR = 0.17)
3. **Korsemov, D.**, D. Borissova, I. Mustakerov. Combinatorial optimization model for group decision-making. *Cybernetics and Information Technologies*, ISSN: 1311-9702, Vol. 18(2), 2018, pp. 65-73. (SJR = 0.204)
4. **Korsemov, D.**, D. Borissova. Modifications of simple additive weighting and weighted product models for group decision making. *Advanced Modeling and Optimization*, ISSN: 1841-4311, Vol. 20(1), 2018, pp. 101-112.
5. Borissova, D., I. Mustakerov, **D. Korsemov**, V. Dimitrova. Selection of ERP via cost-benefit analysis under uncertainty conditions. *Advanced Modeling and Optimization*, ISSN: 1841-4311, Vol. 19(2), 2017, pp. 177-186.
6. Borissova D., I. Mustakerov, **D. Korsemov.** Business intelligence system via group decision making. *Cybernetics and Information Technologies*, ISSN: 1311-9702, vol. 16(3), 2016, pp. 219-229. (SJR = 0.198)
7. Borissova, D., I. Mustakerov, **D. Korsemov**, V. Dimitrova. Evaluation and selection of ERP software by SMART and combinatorial optimization. *Advanced Modeling and Optimization*, ISSN: 1841-4311, Vol. 18(1), 2016, pp. 145-152.

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- **Korsemov, D., D. Borissova. Modifications of simple additive weighting and weighted product models for group decision making. *Advanced Modeling and Optimization*, ISSN: 1841-4311, Vol. 20(1), 2018, pp. 101-112.**
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- **Borissova, D., I. Mustakerov, D. Korsemov. Business Intelligence System via Group Decision Making. *Cybernetics and Information Technologies*, Vol. 16(3), 2016, pp. 219-229.**
  3. Nicklas Wall. The business intelligence mediator. Master's Thesis, M.Sc. Industrial Management and Engineering, Department of Industrial Economics Blekinge Institute of Technology Karlskrona, Sweden 2017, http://www.diva-portal.org/smash/get/diva2:1137770/FULLTEXT01.pdf

- **Borissova, D., I. Mustakerov, D. Korsemov, V. Dimitrova. Evaluation and selection of ERP software by SMART and combinatorial optimization. Int. Journal Advanced Modeling and Optimization, ISSN: 1841-4311, Vol. 18(1), 2016, pp. 145-152 .**
  4. Efe, B. ERP Software Selection Based on Intuitionistic Fuzzy VIKOR Method. In book: Multi-Criteria Decision-Making Models for Website Evaluation, 2019, Pages: 17, DOI: 10.4018/978-1-5225-8238-0.ch006, https://www.igi-global.com/chapter/erp-software-selection-based-on-intuitionistic-fuzzy-vikor-method/227559

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