Public Transport Priority for Multimodal Urban Traffic Control

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Abstract: *In order to improve the travel time of surface public transport vehicles (bus, tramway, etc.), several cities use Urban Traffic Control (UTC) systems enabling to give priority to public transport. This paper reviews these systems. Further on after a debate on their insufficiencies in the global regulation of the urban traffic on a whole network, the paper proposes intermodal regulation strategies, operating on intersection traffic lights to regulate the traffic, favouring the public transport. All these strategies are based on the Linear Quadratic (LQ) optimal control theory, but they are different in their ways of taking into account the public transport in the optimization problem. The simulation tests are carried out in a network of eight intersections and two public transport lines.*

Keywords: *Optimization, Traffic Control, Linear Quadratic, bus priority, multimodal control.*

1. Introduction

The mobility of the inhabitants in agglomerations is in continuous growth. Unfortunately, despite everything else, the harmful effects on the environment (pollution, noises, occupation of space, etc.), is due to the private cars which allure more the road users and the growth is due more to their travelling. Several measurements can be done in order to improve the quality of the public transport and favour its use in order to improve the comfort in the vehicles and in the stations, to improve the safety and security, etc. But a master reason for the users is the time they spent on their journey. However, the travel time of the surface public transport, like buses, trams and more generally, high occupancy vehicles depends on the congestions and the traffic lights.

Many measurements can be used in order to improve the travel time of the surface public transport. We can quote: exclusive rights of the way reserved for public transport, prohibition to a station on the roadway system, urban toll aiming at reducing the road traffic, static or dynamic guidance using the panels with variable messages to direct the private cars to roads less attended by trunk public transport and priority of the public transport vehicles at signal-controlled intersections.

Giving priority to the public transport vehicles in traffic lights makes improvements to their travel times. According to STIF (2001), facilitating the passage of buses on traffic lights could act on 40% on average on its total time of course.

17% of it is concerned with the stops at traffic lights, 15% are due to the decelerations/accelerations and the rest would be gained on the layover time which will be reduced by a better regularity. However, as it will be further described, the majority of the existent urban traffic strategies giving priority to buses do it in a local way in one or a small number of intersections. Although at peak hours, when the roads are very loaded, giving priority to buses on the basis of the local traffic conditions can send them quicker to the congested places and make worse the global traffic situation even for the buses themselves. That is why our objective in this work is to build a global strategy for a large scale network. Its aim is to act on the intersection traffic lights in order to give priority to the public transport vehicles and to regulate the traffic on the whole network. To achieve this objective we chose to develop a bi-modal optimal control strategy on the basis of the Linear Quadratic (LQ) optimization theory, which has the advantage to be appropriate for use in a closed loop. The LQ theory has already been applied for the regulation of urban intersections in TUC strategy (D i a k a k i et al. [6]). However, as it will be explained in Subsection 2.2.1, TUC gives an active priority to the public transport vehicles in the local area of the intersection.

In this paper, the following section will address the state of the art of the public transport priority systems. We underline the insufficiency of these systems for traffic regulation on the global level of a whole network. In the third section, we mention the various systems of global management for both modes (PC and PT) and give some elements, explaining the non-existence of the global regulation systems for both transport modes. In the fifth section we describe the used model of command. The sixth section deals with the definition of the optimal command problems, starting with the optimization criteria. The latter enables to regulate the traffic on the whole network and has an additional term, enabling to favour the arcs which support the public transport vehicles at instants of their presence on these arcs.

We show that the use of a strategy which combines different optimization criteria, one for the arcs where buses are present and another − for the arcs where there are only private cars, can solve this problem. The results of these various strategies applied in a simulation on a network with eight intersections, thirty two arcs and two lines of public transport, are given in the eighth section. The conclusion is given in the ninth section.

2. Different types of public transport priority at traffic light intersections

The bus priority at traffic lights can be operated in a passive or in an active way. The passive way is an off-line regulation in order to favour the buses. The active way needs real-time detection of buses and real-time modification of the traffic lights in order to take into account the buses information.

We can find several real-time urban traffic control systems in literature which have been developed to regulate the global traffic. They have been enlarged after that to include the public transport priority.

Real-time urban traffic control systems belong mainly to one of the two families (see Fig. 1). The first system family is adaptive time plan based. It uses a fixed traffic light cycle on a given period. These systems gradually adapt the signal colour duration to the variations in real time of the traffic conditions (for example: SCOOT (H u n t et al. [11]), SCATS (C h e n et al. [5]) and TUC (D i a k a k i et al. [6]). The second system family consists of adaptive commands, continually optimizing the traffic light plan on a sliding horizon (example: CRONOS (B o i l l o t et al. [4]), PRODYN (H e n r y and F a r g e s, [10]) and UTOPIA (M a u r o and T r a n t o [14])). In these systems the cycle duration is not constrained and varies from one cycle to the next. UTOPIA, PRODYN and SPPORT proceed by defining initially the different stages of the intersection and fixing the minimum and maximum green durations. On the other hand, CRONOS finds the green and red durations according to the traffic conditions only and the safety constraints. There are no repetitive stages as in further systems. This initial theoretic conception influences the way these systems take into account the buses priority at traffic lights. Generally, in the first category, the priority to the buses is given on the basis of pre-established rules. The strategies of the second category give priority to the buses further to optimization of some criteria.

2.1. Passive priority

The passive priority consists in generating the plans of the traffic lights so that they favour the roads supporting the public transport, without detecting these vehicles individually. Some measures can be applied to satisfy this objective, for example:

- To adjust the traffic lights coordination to the public transport speed instead of the private cars speed.
- To reduce the duration of the traffic light cycles, in order to reduce the waiting times of the public transport when they arrive at traffic lights. This measure cannot be applied in case of large density traffic since it reduces the intersection capacity.
- To split the green phase attributed to the road supporting the bus, when the traffic light cycle cannot be very short. However, this method also reduces the capacity of the intersection traffic flow.
- To design the traffic light diagrams, taking into account the number of passengers rather than the number of vehicles. However, it requires knowledge

about the load in terms of passengers number in each transport mode.

It is this latter method which is used to give priority to public transport vehicles in one of the most well known static urban traffic control system as TRANSYT system (V i n c e n t et al. [19]).

2.2. Active priority

The second method, called dynamic priority, consists in modification of the intersection signals to authorize the passage of the public transport vehicle which has been detected. This type of priority is performed on real-time urban traffic control systems. Real-time urban traffic control systems belong principally to two families (Fig. 1). The first one uses a fixed traffic light cycle on a given period. They gradually adapt the traffic light plan to the variations in real time of the traffic conditions (for example, SCOOT, SCATS and TUC). The second family of systems consists in adaptive commands, continually optimizing the traffic light plan on a sliding horizon (for example, CRONOS, PRODYN and UTOPIA). It influences their way in taking into account the priority at traffic lights. In the first category, the priority is given to the Public Transport Vehicle Crossing (PTVC) on the basis of the pre-established rules.
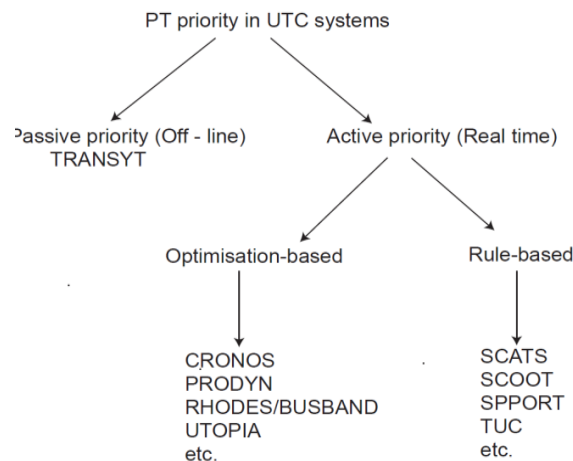

Fig. 1. Classification of the public transport priority strategies

The strategies of the second category give priority to the public transport vehicle further to optimization of some criteria.

2.2.1. Rule-based priority

These methods consist in a short term modification of the traffic light operation to favour the bus approaching the intersection. It is the most widely used method by the control strategies that gives priority to the buses. Together with the known control strategies of international level, such as SCOOT, SCATS, SPPORT, TRAFCOD, TUC, several less known systems which were developed by cities or by transport organization authorities, use also the rule-based priority.

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2.2.2. Determined priorities based on the global optimization at the intersection

It is based on the optimization theory to find the optimal durations of lights enabling the buses priority; and requires traffic models and a criterion to be optimized. The advantage of these strategies is that they are not constrained by a traffic light fixed cycle. Among the existing systems, CRONOS, PRODYN, RHODES/BUSBAND can be mentioned.

2.2.3. Limits of TPS regulation systems

The intersection control systems which enable giving priority to the public transport on the basis of rules cannot manage more than one bus per a traffic light cycle. As we have seen, these systems proceed, attributing additional duration of the green light at the approach of the bus and restore the order of the phases later on. This procedure cannot be repeated several times during a cycle since the green light durations are limited by maximum values imposed for safety reasons. Thus it limits the use of this type of strategy at the intersections which are not often used by the public transport vehicles. The systems which manage the priority through optimization algorithms can take several criteria into account before attributing the priority; for example they can attribute the priority to the public transport vehicle which deserves it most and not to the one which asks it first, etc. However, these systems are limited by the computational time. Some of them, since they are based on a microscopic modelling of the intersection and others – because they need large data information from the urban network. The computational time very often increases in a sequential way with the number of studied intersections.

From our viewpoint, a public transport priority strategy which is placed on the individual level of buses cannot have a global view of the traffic on a whole region. And as above noted, it can imply twisted effects, since it can feed the road network sections or congested intersections, resulting in deterioration of the traffic general conditions, including bus traffic conditions. Thus, it is necessary to develop regulation strategies which take into account the intermodal global situations of the traffic on a whole region (a whole route of the public transport for example).

A global multimodal strategy for public transport and private cars has been developed (S c e m a m a and T e n d j a o u i [18]). However, this strategy is an expert system trying to copy the operators' behaviour to give recommendation to the operators and does not give the optimal solution itself. Furthermore, it acts in a long period of time (15 minutes) and not in the cycle as the strategies that we propose.

B h o u r i et al. [3, 2] explored an approach using multi-agent modeling to process the traffic control strategy. The proposed strategy, ASUR, adapts the individual behaviours of buses given by buses agents to the collective behaviour of vehicles given by aggregate data and vice-versa. The method is promising but has to be tested on big networks to ensure a reasonable computing time.

The method bimodal control strategy proposed in this paper is a Linear Quadratic optimization method which is well adapted to real time control.

A network of the urban roads is composed of intersections linked by sections. In order to explain our objective in this work and the model used, we start by giving

variables which characterize the traffic light intersections, before developing the intermodal model of command.

3. Dynamic model

In this work we assume that the duration of the traffic light cycle, the phasing diagram and the gaps are fixed on the considered time horizon. The strategy acts on the duration of the green lights within the cycle in order to improve the traffic conditions. In order to obtain the dynamic equations for the mathematical model, we will consider the now well established Store and Forward model due to G a z i s and P o t t s [8]. The choice of this model is based on the simplifications it imposes on the equations that will allow us to write them as linear equations on the number of vehicles and the green time of the junctions.

The network is represented by a directed graph composed of nodes and arcs. The nodes $j \in J$ represent intersections and the arcs $a \in A$ – the unidirectional travel links.

On every arc the model consists of two equations, one of them modelling the progress of the total number of vehicles on the arc, expressed as a Private Vehicle Unit (PVU) (for example, the bus equals 2.3 PVU). The second equation models the number of public transport vehicles on the arc.

3.1. The general traffic dynamic equations

As said in the introduction, this strategy adopts the same bases as TUC inter-section regulation strategy and both are based on the Store and Forward model. The traffic on each arc a is modelled using the vehicle-conservation equation (D i a k a k i et al. [6]).

(1) $$x_a(k + 1) = x_a(k) + T[q_a(k) - u_a(k)],$$

where x_a is the number of cars on the link expressed in PVU, q_a and u_a are the inflow and the outflow of the link a during $[kT, (k + 1)T]$ where k is the discrete time step and T is the sampling time. Fig. 2 clarifies the relations between the variables. Herein we have neglected the traffic generated and consumed in each link, it would be easy to include them without substantially changing the current development.
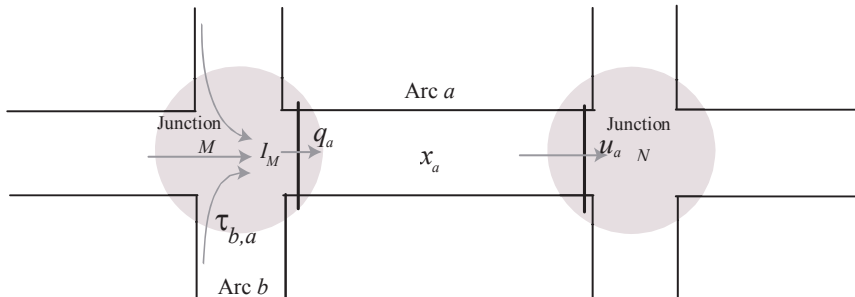

Fig. 2. Variable definition

In order to clarify the equations for q and u we will consider the saturation flow of each link S_a, that represents the maximum traffic flow that can exit the link, expressed in PVU/s. The *Store and Forward* model assumes that the vehicles reaching the arc's end are stored there and exit with rate S_a during the green light. Hence, we can write

$$(2) \qquad u_a(k) = \frac{S_a.G_a(k)}{C},$$

where C is the cycle time and $G_a(k)$ is the efficient green time of link a, i.e., the green light duration attributed to arc a during the traffic light cycle C of the intersection situated at the arc exit, and it will be the control variable in our approach. If the green light periods are attributed to arc a during different phases, $G_a(k)$ is equal to the sum of all these green light durations

$$(3) \qquad G_a(k) = \sum_{i \in P_N^a} G_{N,i}(k),$$

where $G_{N,i}(k)$ is the green light duration for phase i on the junction N, the summation is made over all the phases such that arc a has the right of way (green light), this set is called P_N^a. It also assumes that the outflow is distributed among the different following links according to the coefficients τ_{ab}, called turning rates that represent the proportion of the outflow from a entering arc b.

If the link a originates at junction M, the inflow traffic rate entering arc a can be written as the sum of the outflow traffic rates coming from the arcs entering junction M (other than a). If the arc b precedes arc a, the corresponding flow is $\tau_{ba}u_b$, so that the total flow entering arc a is

$$(4) \qquad q_a(k) = \sum_{b \in I_M} \tau_{b,a} u_b(k),$$

where I_M is the set of arcs entering junction M, and we have defined $\tau_{aa} = 0$.

Replacing all the previous definitions in Equation (1), we obtain the following model:

$$(5) \qquad x_a(k+1) = x_{a(k)} + \frac{T}{C}\left[\sum_{b \in I_M} \tau_{b,a} S_b G_{M,i_b}(k) - S_a \sum_{j \in P_N^a} G_{N,j}(k)\right]$$

or in a matrix form

$$(6) \qquad X(k+1) = X(k) + B\,G(k),$$

where B is a matrix of dimension $N \times M$, N is the number of links and M is the total number of phases on the network.

This modelling is possible under the following assumptions:

- the sampling time interval T is at least equal to the duration of the light cycle C, we will use $T = C$;
- the gaps between the intersections are not taken into account;
- variations in the queue are neglected, which means that the model considers that all of the input flows on the arc have the green phase at the same time.

3.2. The public transport traffic dynamic equations

Since we will be considering two kinds of traffic, the general one and the public transport one, we will distinguish the state variables as x^v for the number of vehicles and x^b for the number of public transport vehicles (buses). Knowing the sequence of arcs which are used by each public transport line, the progress of the public transport vehicles is modelled by a delay equation:

(7)
$$x_a^{b_i}(k) = x_{a'}^{b_i}(k - \zeta_a^i),$$

where $x_a^{b_i}$ is the number of vehicles of the public transport line number i on arc a, a' is the arc preceding a for the line i and ζ_a^i is a parameter which expresses the mean travel time of the vehicles on line b_i to travel from arc a' to arc a. These values should be real ones, however, in order to be able to write the precedent equation, we take ζ_a^i as integer, meaning that the travel time is a multiple of the sampling interval T. Thus we consider that ζ_a^i is equal to 1 if the bus line has no station on arc a, otherwise ζ_a^i is equal to 2 (for example). Substituting these values in equation (7), the model of the public transport becomes the following:

(8)
$$x_a^{bi}(k+1) = \begin{cases} x_{a'}^{b_i}(k-1) & \text{if the bus line } b_i \text{ has a commercial stop on the arc } a', \\ x_{a'}^{b_i}(k) & \text{otherwise.} \end{cases}$$

This simplification complies with the dynamical modelling of the PC, since it consists in assuming that both the PC and public transport are "stored" during the red light period and then are "distributed" during the green light period, thus they spend a light cycle on the arc. However, the choice of the cycle duration should be done carefully.

The last equation, written in a vector form, gives

(9)
$$X^b(k+1) = A_0^b X^b(k) + A_1^b X^b(k-1),$$

where matrix A_0^b is the adjacency matrix corresponding to the bus line for the arcs without stops, A_1^b is the adjacency matrix corresponding to the bus line for the arcs with a stop, and $X^b(k)$ is the vector of numbers of buses at each traversed arc. It can be further simplified, adding if necessary supplementary state variables, such as

(10)
$$X^B(k+1) = A^b X^B(k),$$

where X^B is the vector obtained after stacking $X^b(k)$ and $X^b(k-1)$, and matrix A^b is the block matrix given by

(11)
$$A^b = \begin{pmatrix} A_0^b & A_1^b \\ I & 0 \end{pmatrix}.$$

3.3. The public transport PC model

Here we do not talk of a coupled model, because, as it is easy to see, both dynamics are not coupled, in fact they will be coupled, but for the objective function in the optimal control problem to be presented in the next section.

The state variable of the whole system consists of a vector of dimension $N + 2N_b$, where N is the number of arcs in the system, N_b is the number of arcs crossed by the public transport lines. The dynamics of the system thus is represented by the following equation

(12)
$$X(k+1) = AX(k) + BG(k),$$

where A is a matrix of dimension $(N + 2N_b)\times(N + 2N_b)$. Matrix B is composed of two stacked blocks − the upper one is defined by the topology of the road network, i.e., when the coefficient B_{a_j} is different from 0 means that phase j is found entering or leaving arc a and its value is defined according to (5). The lower block corresponds to the influence of the green lights on the bus, which, as it is neglected, has to be 0. We have then

$$(13) \qquad A = \begin{pmatrix} I & 0 \\ 0 & A^b \end{pmatrix}, \quad B = \begin{pmatrix} B \\ 0 \end{pmatrix}.$$

With these matrices, it is clear that it will not be possible to command the public transport because of the null block of matrix B. However, it does not set any problem because in the definition of the model we suppose that the travel times of the public transport are fixed. What we want is to act in such way that the buses can comply with their schedules.

4. Optimal control problem

Here we pose the optimal control problem. The control part of the problem means that we will be able to choose the green light times in order to modify the flows. The optimality will be measured in terms of the number of private cars that share the roads with the buses. We will explain here these definitions. When doing so, we keep in mind that we want to obtain a simply computable global green time. As we have linear dynamics, choosing a quadratic objective function and imposing no restrictions will make the optimal control problem over an infinite horizon belonging to the LQ class. The importance of this relies on the fact that the optimal solution can be written as a linear (constant in time) feedback law and the matrix that defines this law is the solution of a matrix equation (Ricatti equation) stated in terms of the given data.

4.1. Optimization criteria

From the viewpoint of the traffic regulation, our objective is to improve the traffic conditions of PT on the network, relative to the PC flow, without deteriorating the global traffic conditions. The objective function needs to be quadratic in terms of the state and control variables to rest in the LQ case, the general form of these functions is:

$$(14) \qquad J(x,u) = \int_0^\infty \alpha_x \|x\|_{Q_x}^2 + \alpha_u \|u\|_{Q_u}^2,$$

where Q_x and Q_u are positive definite matrices that allow to weigh differently the components of x and u; α_x and α_u are non-negative coefficients. These conditions guarantee that the function J will be convex (strongly if $\alpha_{x,u} > 0$),

which in turn guarantees the existence (and uniqueness) of the solution over the closed convex set defined by the linear dynamic equations.

In our (discrete time) case we propose the following objective function:

$$(15) \qquad \min_{G} J(G) = \sum_{k=0}^{\infty} \left[\alpha(X(k)'X^{b}(k)) + \beta\|X(k)\|^{2} + \gamma\|G(k)\|^{2} \right],$$

where α, β and γ are non-negative weighting parameters and X are given by the dynamic equations (5) and (8).

Even if the introduction of the objective function was made for computational simplicity, we can give an interpretation to each term. The first term of the criteria, $(X(k), X^{b}(k))$ puts forward the traffic conditions on the arcs crossed by the PT at the time these PT vehicles are present on it. The second member aims at reducing the number of vehicles on every arc in the network and thus to equalize the congestion on every arc. The role of this second term is mainly to not degrade too much the traffic in the other arcs. The last term is used in order to avoid large variations of the control (green light times).

The optimization criteria (15) have three different terms weighted by parameters α, β and γ. The choice of the values of these parameters enables the modification of the objective of the regulation. For example, for $\alpha = 0$, $\beta = \gamma = 1$, the strategy is equivalent to TUC, which does not take into account the presence of the PT. On the other hand, a significant parameter α ($\alpha >> \beta$) will strongly penalize the arcs which do not support the PT.

4.2. Control law

The problem of optimal control consists in minimizing the criteria given by (15) respecting the dynamics of the system given by (12). In order to avoid working with the input and exit flows we define a nominal green time G^{N} that solves $BG^{N} = 0$. In such case the corresponding nominal state is constant and we can work with the following dynamic equation

$$(16) \qquad X(k+1) = AX(k) + B\Delta G(k),$$

where $\Delta G(k) = G(k) - G^{N}$, and now X represents the deviation from the nominal state. Writing the objective (or performance) function as:

$$(17) \qquad J = \sum_{k} X(k)^{\mathrm{T}} QX(k) + G(k)^{\mathrm{T}} RG(k),$$

following the LQ optimization method, the applied command law is given by the following equation

$$(18) \qquad G(k) = G^{N} - FX(k),$$

where F is the Feedback matrix defined as

$$F = (R + B^{\mathrm{T}}PB)^{-1}B^{\mathrm{T}}PA,$$

and the matrix P solves the Riccati matrix equation

$$P = Q + A^{\mathrm{T}}PA - A^{\mathrm{T}}PBF.$$

The dependence of the objective function on the coefficients α, β, and γ is carried by matrices Q and R.

Considering (18) for k and $k-1$, by a simple substraction, we have

$$(19) \qquad G(k) = G(k-1) - F\big(X(k) - X(k-1)\big).$$

The use of this equation rather than of equation (18) avoids the estimation of the nominal values of the control.

It should be noted that the choice of an infinite time horizon in (15) implies that the feedback matrix F is time independent. This choice is justified by the will for a real time command of the intersection lights and thus by the simplification of the calculations for each command. However, it has the drawback to consider the time average of the criteria, reducing the significance of our main objective which is to reduce the number of vehicles on the arcs at the instants when the PT vehicles are on these arcs. This led to the idea to test various strategies, whether a single Riccati matrix, or at most a finite matrix combination is used, each of them being calculated for a different system state. We explain this idea in the following sections.

Strategy with PT priority (PPT)

As it was above said, the choice of the parameters α, β and γ enables to model various control objectives. The first strategy tested consists in slightly favouring the PT with the choices $\alpha = \beta = 1$ in (15).

Strategy with strong PT priority (PFPT)

In this second strategy, big significance is given to the first term of the criteria consisting in favouring the arcs that support the buses at the instants when they are on it. In this case we choose $\alpha >> \beta$ (B h o u r i and L o t i t o [1]).

Combined strategy

This strategy is based on the ability of detecting the presence of buses on the arcs which can be accomplished using appropriate sensors. We use two different criteria (different Riccati equations) according to the presence or the absence of the PT vehicle on the arc.

This is a good compromise between a single Riccati matrix (the same for all k) and an infinite or very large sequence of Riccati matrices (one for every k). The idea is to give the control law in a practical and implementable way. The intersection controllers have two Riccati matrices calculated in the following way: the first one does not take the PT into account (TUC for example); on the contrary, the second one strongly takes them into account (α very large). The matrix which corresponds to the situation of the PT is used on each of the intersections on the network.

More precisely, let us consider F_1 - the feedback matrix obtained with TUC (or another independent criteria of the PT position). Given F_2, the feedback matrix is obtained with a criterion which takes into account the position of PT ($\alpha >> \beta$). The optimal command is given by:

(20) $$G_k = G_{\text{nom}} + (\lambda P_k F_1 + (I - \lambda P_k) F_2)(X_k - X_k - 1),$$

where P_k is a diagonal matrix, every element of which is equal to 1, if at the moment k there is a PT vehicle waiting on the corresponding arc, and λ is a coefficient to be determined in order to further improve congestion (see the Numerical experiments section for more details).

This strategy appears to be cleverer since it enables to reduce the congestion on the arcs where there are PT vehicles at the instants when they are on it, without increasing it on the arcs where there is none. The simulation results seem to support this affirmation and will be detailed in the forthcoming section about numerical results.

4.3. The constraints

The LQ methodology used to obtain the solution of the optimal control problem does not allow taking into account the constraints, and for the Riccati equation will no longer be valid when the optimal control problem is constrained. However, for operative needs, at every intersection j, the durations of green lights should comply with a certain number of constraints:
- the cycle duration (C),
- the phase diagram: all phases P_j must have their green light within the cycle,
- the clearance times between the phases R_j,

which implies:

(21)
$$\sum_{i \in P_j} G_{j,i} + R_j = C.$$

Also, the duration of every green light is limited by a maximum and minimum time. Indeed, a too long red light duration can be interpreted by the users as a malfunction of the intersection lights and imply their non-compliance :

(22)
$$G_{j,i,\min} \leq G_{j,i} \leq G_{j,i,\max.}$$

In order to obtain green times according to the previous constraints, the same method that appeared in (D i a k a k i et al. [6]) is applied, i.e., the obtained control values are projected onto the set of feasible values defined by the constraints. It means to obtain the closest (for some distance) values to the optimal, but not the feasible ones. The projection step means to solve the following quadratic optimization problem that includes the constraints (21) and (22),

(23)
$$\min_G \quad \sum_{i \in P_j} (G_{j,i} - \overline{G}_{j,i})^2,$$
$$\text{s.t.} \quad (21) \text{ and } (22).$$

This problem belongs to the class of Quadratic Knapsacks problems and the numerical solution was done according to the algorithm presented in L o t i t o [13] (see P a t r i k s s o n [17] for a survey of available algorithms).

After all the simplifications made in order to apply standard tools, solving directly the full, optimal control, the problem could appear to be easier. The full problem would be

$$\min_F J(X,G) = \sum_0^T \alpha_x \|X(k)\|_{Q_x}^2 + \alpha_u \|G(k)\|_{Q_u}^2,$$
$$\text{s.t.} \quad X(k+1) = AX(k) + BG(k),$$
$$G(k) = G^N - F(k).X(k),$$

$$\sum_{i \in P_j} G_{j,i}(k) + R_j = C,$$

$$G_{j,i,\min} \le G_{j,i}(k) \le G_{j,i,\max},$$

where T is the considered time horizon and the free variables are non-constant feedback matrices $F(k)$. When solving it for real examples where the time horizon is large, the number of variables is also large and so the infinite horizon control problem could be more adequate, because (assuming a nominal state), a constant feedback matrix is obtained, giving along the facility of practical implementation of a constant feedback.

5. Numerical experiments

In this section we expose numerical tests that we have made with a small academic example network. The numerical tests have been made using a micro-simulator designed adhoc and based on car following models (see G a b a r d [7] and H e l b i n g et al. [9]). The idea is to use different models for control derivation and for control testing.

5.1. Microsimulator

The availability of mathematical models describing the dynamics of vehicles is fundamental in order to apply the control theory. The model presented before, stated in terms of continuous vehicle flows, is considered as a macroscopic model in contraposition to the microscopic models that consider the position of each vehicle.

In order to make computational tests of the designed strategies we have considered as an important step to use a model of different nature from the one used to design the strategy. Microscopic simulators are based mostly in Cellular Automata (N a g e l [15], L o t i t o et al. [12]) or on the Car-following model (P a p a g e o r g i o u [16]). The last one was chosen to develop our simulator. Hence, we consider a discrete event system, such that at each time step there are vehicles entering at fixed rates and interacting among each other following certain rules. These rules model the movement on straight lines and the lane change.

The positions of vehicles evolve according to the equations

(24) $$x_n - x_{n+1} = L + S\dot{x}_{n+1},$$

where n is the precedent vehicle, L is the vehicle length and S is a separation coefficient. In this formula, the vehicle $n + 1$ stays separated from the precedent by a fixed distance (L) plus a distance proportional to its speed.

After differentiating Equation (24) it results in

(25) $$\ddot{x}_{n+1} = \frac{1}{S}(\dot{x}_n(t) - \dot{x}_{n+1}(t)),$$

showing that the acceleration (or deceleration) is proportional to the relative speed between successive vehicles. Defining the factor $1/S$ and introducing a delay coefficient the following formula for the speeds it is obtained

$$(26) \qquad \ddot{x}_{n+1}(t + \tau) = \frac{1}{S}(\dot{x}_n(t) - \dot{x}_{n+1}(t)).$$

Even if these considerations are common in practice, a set of parameters that are randomly distributed among the different vehicles is used here. These parameters include: maximum speed, length of the vehicle, behaviour, anxiety, etc. Now, in the proposed model, the vehicle position is given by a set of rules that include: precedent vehicle position, relative speed, maximum speed, maximum acceleration (or deceleration). A simple version of the used rule is given in the following excerpt of a pseudo code:

if (pos_Veh_prec − pos_Veh)> vel_Veh*(S+deltaT) then {should accelerate}
vel_Veh := vel_Veh + (acel_Veh*deltaT); {max acceleration in order to not over pass the precedent vehicle}
vel_Veh := Min(vel_Veh, (pos_Veh_prec − pos_Veh − (vel_Veh*S))/deltaT);
else {should decelerate}
vel_Veh := Max(vel_Veh + (decel_Vel*deltaT),(pos_Veh_Adel − pos_Veh − (vel_Veh*S))/deltaT)
end if
vel_Veh := Min(max_Vel_Veh,vel_Veh); {in order to not exceed max speed}
pos_Veh := pos_Veh + (vel_Veh*deltaT);

So, at each time step new vehicles enter the system according to predefined rates at entering arcs and interact with the existing ones following the described rules. The simulation is shown with the aid of a graphical interface which also serves to enter the entering rates. We show a screen-shot of the simulator in Fig. 3a.
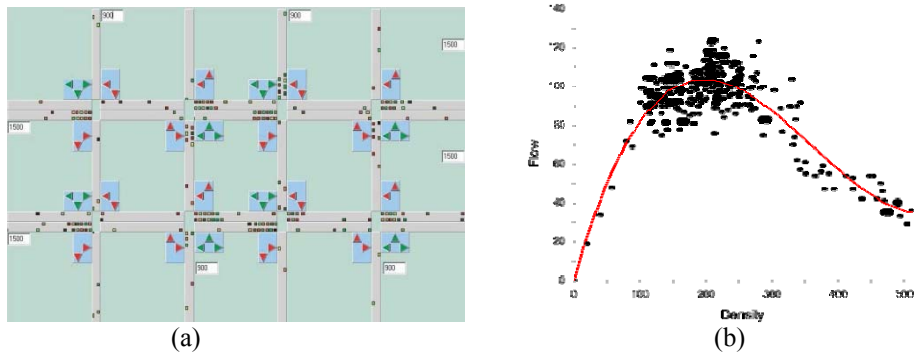

(a)   (b)

Fig. 3. Screen-shot of the micro simulator (a) and the resulting traffic fundamental diagram (b)

More complex equations could be used to model the dynamic equation (24). However, the calibration made for the city of Tandil that is under a study, asked by the Transit Authorities has shown that the simulation is correct. Indeed, the simulation results approach very well the Fundamental Diagram of Traffic provided by the city (see Fig. 3b)

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5.2. Example network

The chosen example network has 8 intersections, 28 links and two bus lines (yellow and blue) as shown in Fig. 4.

Each intersection has the general form given in Fig. 5 and three phases. In the nominal state, each one is given 50%, 10% and 30% of the green time respectively, as it is shown in Fig. 5. In this figure the turning rates for each movement are also shown.

The saturation flow is 0.5 veh/s everywhere and the entering flows are given in the Table 1.

Table 1

Arc	1	3	11	16	20	26	30
Flow (veh./s)	0.25	0.15	0.15	0.25	0.25	0.15	0.25

The flow d originated and consumed at each link is determined in such a way that $B_* \ \overline{G} + d = 0$, thus guaranteeing that the proposed "nominal" state is indeed nominal. The yellow-bus line enters node 12 and traverse intersections 1, 5, 9, 14, 27 and 30, making a stop before intersections 1, 3 and 5 (it is shown by S in Fig. 4. The frequency of the buses is 1 bus at each 3 time steps. The blue-bus line enters node 12 and traverse intersections 1, 5, 9, 14, 27 and 30, making a stop before intersections 1, 3 and 5 (it is shown by S in Fig. 4. The frequency of the buses is 1 bus at each 3 time steps. Most of the following examples were obtained considering only the yellow-bus line. The blue-bus line is introduced to further test the NetPrior strategy.
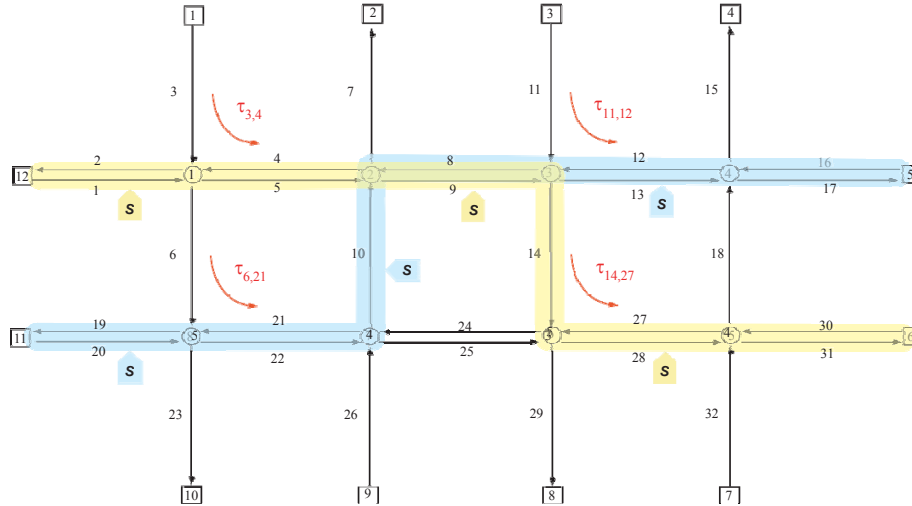

Fig. 4. The example network

5.3. The proposed strategies

Many parameters should be set in order to compute the feedback matrices for each strategy. The first description was given in Subsection 4.2 where the distinction was made between the PC model and the PC-PT model. Here the computed strategies are described more explicitly. The proposed strategies and its parameters are:

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TUC The strategy proposed in D i a k a k i et al. [6], which is based on the PC model, where the matrix Q in 17 is the identity matrix, i.e., all the arcs are given the same weight.

Pr-L1 The same strategy as before, except for the weights of the arcs traversed by line 1, which are increased by 400%.

Comb Combination of TUC and Pr-L1, when the presence of a bus is detected in a junction, the green times used are those given by Pr-L1, otherwise the green times are given by TUC.

PrArc5 In this case, only the weight of arc 5 is increased.

NetPrior The strategy computed using the PC-PT model described in Subsection 4.2 only considering line 1.

NetPrior2 Same as before but considering both line 1 and line 2.

In order to see the impact of the arc weights in the computed feedback matrices; for the phase 1 of the intersection 2, which allows the movements $8 \rightarrow 4$, $8 \rightarrow 2$ and $5 \rightarrow 9$, the corresponding rows of the feedback matrices (in absolute values) are shown in Fig. 6.
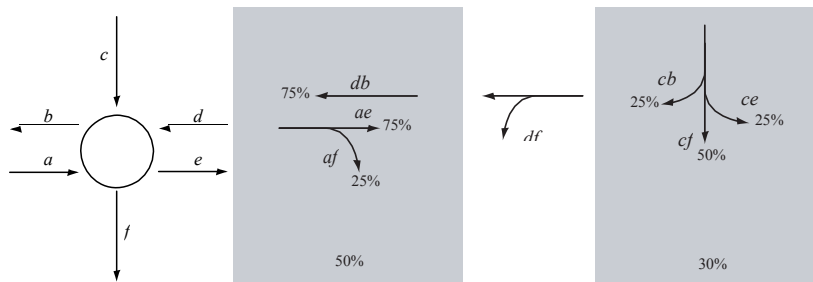

Fig. 5. Diagram of a general intersection and the different phases with the proportion of the green time and turning rates for a given junction

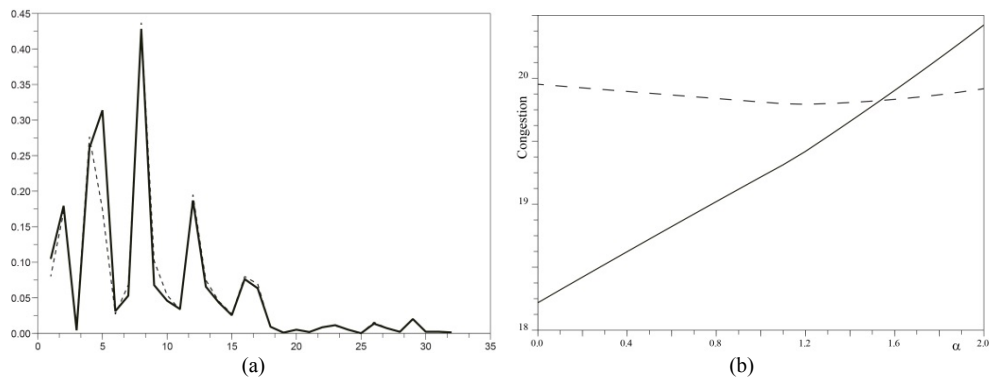(a)   (b)

Fig. 6. Value of the coefficient in the rows of the feedback matrices for phase 1 on intersection 2 (a); variation on the congestion for different values of λ (b)

The interest of this plot is that in some sense, it shows the impact of dotted line lights of the flows on the network arcs. As it can be observed, this importance is bigger for the second feedback matrix. It is so because it was computed, giving more importance to this arc.

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The strategy Comb is a combination of TUC with Pr-L1, as said in Subsection 4.2 there is a parameter to be determined in order to reduce the congestion. The reason is that an improvement for buses may penalize too much the arcs not traversed by the bus line. Hence, there is a trade-off among reducing the congestion on bus traversed arcs and augmenting the congestion on the remaining arcs. This should be analyzed for each particular case. For this example different values of the coefficient λ that appear in formula 20 were analyzed. In Fig. 6b, the values of the congestion over bus-traversed (dotted line) and bus-non-traversed arcs (down line) for different values of λ are shown. The correct choice of λ depends on the practitioner decision about the priority according to the public transport.

Table 2. Simulation results for Case 1

Strategy	Total	Line 1	Line 2	Arc 5	L1 Bus	L2 Bus	Mean time
None	628.29	142.45	129.76	38.80	43.71	35.12	1202.94
TUC	682.91	130.49	146.10	29.98	40.80	39.38	1182.86
Pr-L1	665.58	125.99	143.58	27.22	39.42	38.66	1180.59
Comb	670.10	130.11	145.91	30.07	40.35	38.78	1195.15
Pr-Arc5	684.03	129.76	146.64	27.57	40.95	39.31	1169.41
NetPrior	677.05	133.19	145.53	31.87	42.20	39.18	1181.76
NetPrior2	628.38	120.72	128.86	24.87	38.70	35.38	1161.21

Table 3. Simulation results for Case 2

Strategy	Total	Line 1	Line 2	Arc 5	L1 Bus	L2 Bus	Mean time
None	1007.07	247.80	386.55	87.84	84.04	91.72	2776.38
TUC	718.50	161.94	168.56	46.70	55.73	41.00	1261.50
Pr-L1	717.66	162.34	165.64	48.37	58.64	41.99	1233.74
Comb	725.29	163.55	169.20	47.45	56.36	41.89	1242.19
Pr-Arc5	716.60	158.78	166.52	47.49	57.41	41.61	1242.82
NetPrior	716.91	163.55	165.24	47.16	56.06	41.47	1233.39
NetPrior2	719.78	161.36	169.55	47.50	56.99	41.83	1254.99

5.4. Numerical results

In order to see the control in action two perturbed cases were considered:

• In the first case, the entering rates on arcs 1 and 20 are increased 30% during 10 minutes each hour.

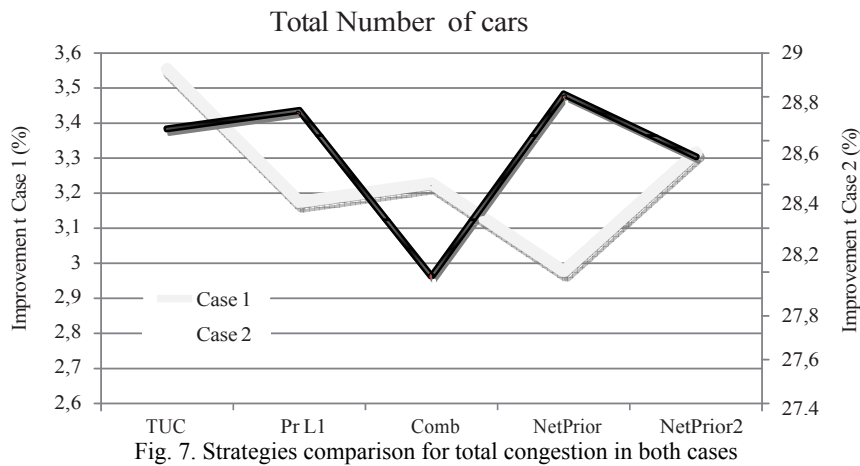• In the second case the perturbation stayed for 30 minutes each hour.

Fig. 7. Strategies comparison for total congestion in both cases

The congestion experimented by the buses for each strategy on a given arc a can be measured as $\sum_k x_a(k)$. The results obtained with the simulations are presented in Table 1. It corresponds to an average of 100 runs of the simulator. The total congestion for each strategy is shown in the column "Total", in columns "Line 1" and "Line 2" the measured simulated congestion corresponding to the arcs belonging to those lines is given. In column "Arc 5" the congestion is computed only on that arc. In columns "L1 Bus" and "L2 Bus" the congestion is only computed when the buses are present. Finally, in the last column the mean travel time of the buses is shown.
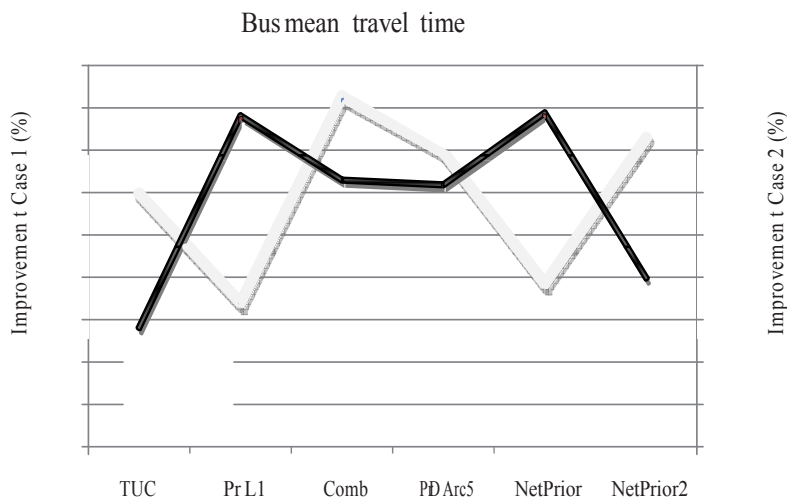

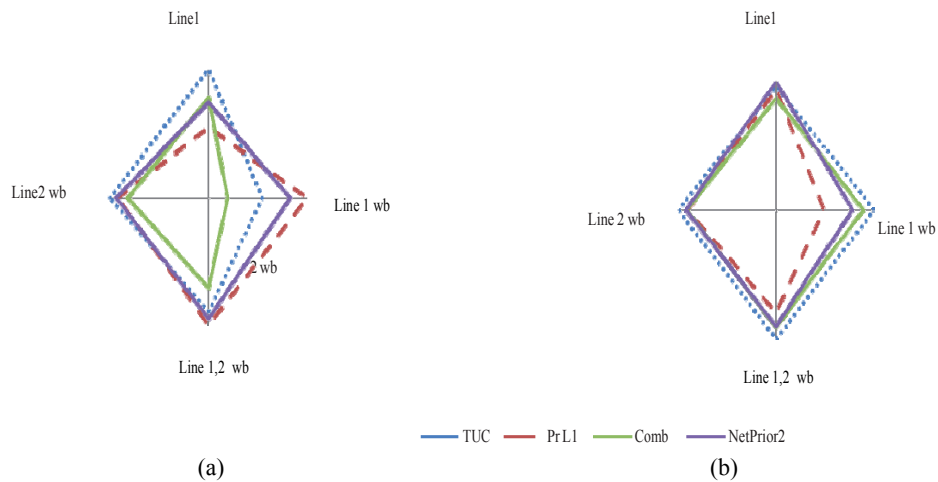Fig. 8. Strategies comparison for bus mean travel time in both cases

Fig. 9. Multicriteria graphics for the strategies in Case 1 (a) and Case 2 (b)

These results are compared together in Fig. 7, showing on the *x*-axis the strategies and the mean time, and on the *y*-axis the obtained congestion for the strategies and the values of the mean-time.

With respect to the bus travel time, the best strategy seems to be NetPrior2, because it reduces the bus travel time without increasing the total congestion. On the contrary, the strategy NetPrior reduces the travel time of Line 1 buses increasing the total congestion, but this one penalizes too much the arcs not traversed by the bus line.

6. Conclusions

In this paper traffic regulation strategies for urban networks have been presented. They were tested in simulation and compared to TUC strategy which does not include the PT in the command. The paper shows that the best way to favour the PT without deteriorating the general traffic conditions is to use a combined command: the green light duration of the intersections is calculated without taking into account the position of the PT for the arcs used by the PT. On the contrary, in the optimization criteria, a strong weight is given to the arcs which support the PT at instants when they are on these arcs. This strategy based on the LQ theory is realistic from the viewpoint of its implementation and the numerical results show its efficiency. More advanced tests and analysis of the networks with traffic real data will be necessary to validate it completely.

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