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Discrete-Time Modeling and Input-Output Linearization of Current-Fed Induction Motors for Torque and Stator Flux Decoupled Control

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Abstract: *In this paper an exact discrete-time model of the induction motor in a current-fed mode, including stator flux components is derived and validated. The equations of the motor are written in a frame aligned with the rotor electrical position, which results in a linear, time-invariant system. Based on the derived exact discrete-time representation of the motor dynamics, an input-output linearizing control law is designed for decoupled torque and stator flux control. The applied design technique led to a non-trivial, still useful, definition of the electromagnetic output of the motor. Simulation results are presented showing that the aimed performance is obtained, that is, no coupling exists between the outputs, and the initial design problem of controlling a nonlinear interacting TITO system is reduced to a problem of controlling two linear and decoupled SISO systems with simple dynamics.*

Keywords: *Input-output linearization, induction motor, discrete-time nonlinear control.*

1. Introduction

The induction motor is probably the most widely used electric machine in industry. The related control problem, known for its difficulty, has received large attention in the scientific literature. Different solutions were found, the most renowned being the so-called "field-oriented control" [1, 2, 3]. Nowadays, in one of its many variants and modifications, it is in industrial practice where high dynamic performance is required. Another approach, subject of scientific research, allowing potentially for superior performance and absolute decoupling between the flux and torque, rather than only asymptotic (that is, under constant flux conditions), as in the field-oriented control case, is the input-output linearization based control [3-9]. Only few input-output linearization designs are based on a model of the motor containing stator flux components and defining the squared stator flux as an output [8, 9], while most of them define the rotor flux (squared) as the electromagnetic output of the motor and use the respective description of the machine. It is true however, that the main operational aspects of the motor are more clearly expressed in terms of the rotor flux magnitude, thus more easily related to control goals. For field-oriented designs though, despite these properties and even the additional advantages of the rotor flux oriented control over the stator flux orientation scheme from a control performance perspective, the latter scheme is probably more often used in industrial drives, because it possesses some implementation-related advantages, e.g., lower sensitivity of the flux estimation models. The input-output linearization approach can potentially eliminate some of the stator-flux orientation scheme disadvantages, such as the residual input coupling, while retaining its advantages.

In all cases the design is typically performed using continuous-time descriptions of the motor, while the control law is implemented using digital devices, being inherently a discrete-time process. This makes the task of proving and guaranteeing the stability of the overall system (interconnection of two nonlinear systems) very difficult, practically impossible. In this aspect, a control law designed on a discrete-time model will potentially eliminate this problem. The design of the control law directly in the discrete-time domain for voltage-command mode applications, i.e., using the complete models of the machine, relies also on approximations since the description does not allow exact discretization. For the current-fed modes of the operation however, exact discrete-time representations can be obtained when the equations of the motor are written in a frame aligned with the rotor electrical position.

For the rotor flux control case, a discrete-time field-oriented control law is proposed in [10, 11] and stability conditions are derived based on an exact discrete-time model of the motor dynamics. In [12, 13], an input-output linearization design based on the same discrete-time description is proposed and validated.

In this paper the stator flux control case is considered. First, an exact discrete-time model of the induction motor in a current-fed mode, including stator flux components is derived. Then, an input-output linearizing and decoupling control law is designed using this exact description. In Section 4 the proposed control law, along with the motor equations, is modeled in Matlab, Simulink environment and some simulation results are presented.

2. The stator-flux model

Under the common assumptions for symmetrical construction, sinusoidal distribution of the field in the air-gap and linearity of the magnetic circuits, the equivalent full-order two-phase model of the machine, expressed in the two-phase stator-fixed *α-β* frame with stator flux linkages and stator currents as states, is given as follows:

(1)
$$\begin{aligned}\dot{\varphi}_{s\alpha} &= -r_s i_{s\alpha} + u_{s\alpha}, \\ \dot{\varphi}_{s\beta} &= -r_s i_{s\beta} + u_{s\beta}, \\ \dot{i}_{s\alpha} &= -\gamma i_{s\alpha} - n_p \omega i_{s\beta} + \zeta \varphi_{s\alpha} + n_p (\sigma l_s)^{-1} \omega \varphi_{s\beta} + (\sigma l_s)^{-1} u_{s\alpha}, \\ \dot{i}_{s\beta} &= -\gamma i_{s\beta} + n_p \omega i_{s\alpha} + \zeta \varphi_{s\beta} - n_p (\sigma l_s)^{-1} \omega \varphi_{s\alpha} + (\sigma l_s)^{-1} u_{s\beta},\end{aligned}$$

(2)
$$\tau_m = n_p (\varphi_{s\alpha} i_{s\beta} - \varphi_{s\beta} i_{s\alpha}),$$

where $u_{s\alpha}(t),\ u_{s\beta}(t)$ are the stator voltages, $i_{s\alpha}(t),\ i_{s\beta}(t)$ – the stator currents, $\varphi_{s\alpha}(t),\ \varphi_{s\beta}(t)$ – the stator flux linkages, $\omega(t)$ – the rotor speed, $\tau_m(t)$ – the motor electromagnetic torque. The parameters in the model are defined as follows: $\sigma = 1 - m^2 (l_r l_s)^{-1}$, $\zeta = r_r (\sigma l_r l_s)^{-1}$, $\gamma = (l_r r_s + l_s r_r)(\sigma l_r l_s)^{-1}$, where l_s is the stator phase winding inductance, r_s – the stator phase winding resistance, l_r – the rotor phase winding inductance, r_r – the rotor phase winding resistance, m – the mutual inductance, n_p – the number of pole-pairs.

In a current-fed mode, the stators currents are efficiently used as control inputs to the machine. In order to achieve such mode of operation, the introduction of a current control scheme is required. Several types exist, the main ones using high-gain, typically PI, current control loops [3, 14], feedforward schemes [3] and, most often, hysteresis relay loops [14]. In [14] a thorough overview of the current controllers for three-phase inverters can be found.

In order to introduce the current-fed mode of operation based on this description of the motor, the stator voltages are eliminated in the flux equations by expressing them from the current dynamics. Thus, we have:

(3)
$$\begin{aligned}\dot{\varphi}_{s\alpha} &= -\eta \varphi_{s\alpha} - n_p \omega \varphi_{s\beta} + \eta l_s i_{s\alpha} + \sigma l_s n_p \omega i_{s\beta} + \sigma l_s \dot{i}_{s\alpha} \\ \dot{\varphi}_{s\beta} &= -\eta \varphi_{s\beta} + n_p \omega \varphi_{s\alpha} + \eta l_s i_{s\beta} - \sigma l_s n_p \omega i_{s\alpha} + \sigma l_s \dot{i}_{s\beta}\end{aligned},$$

which is put in a matrix form as

(4)
$$\dot{\boldsymbol{\varphi}}_{s\alpha\beta} = -\boldsymbol{M}_\varphi \boldsymbol{\varphi}_{s\alpha\beta} + \boldsymbol{M}_i \boldsymbol{i}_{s\alpha\beta} + \sigma l_s \dot{\boldsymbol{i}}_{s\alpha\beta},$$

with $\boldsymbol{i}_{s\alpha\beta} = [i_{s\alpha},\ i_{s\beta}]^T$, $\boldsymbol{\varphi}_{s\alpha\beta} = [\varphi_{s\alpha},\ \varphi_{s\beta}]^T$, $\eta = r_r l_r^{-1}$.

The matrices in (4) can be decomposed as:

(5)
$$\begin{aligned}\boldsymbol{M}_\varphi &= \eta \boldsymbol{I} - n_p \omega \boldsymbol{J} \\ \boldsymbol{M}_i &= \eta l_s \boldsymbol{I} - \sigma l_s n_p \omega \boldsymbol{J}\end{aligned},$$

with $\boldsymbol{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $\boldsymbol{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

If the rotor speed is considered as a parameter in the description, it is seen that (4) represents a linear time-varying system, with stator currents as inputs and fluxes as states, and an additional output – the motor torque, being a nonlinear function of the inputs and states. As it can be seen, the current derivatives appear also in the flux derivative expression, i.e., a direct feed-through between the input and output exists.

3. Discrete-time representation of the stator-flux model

The exact discrete-time representation of the motor dynamics is obtained in the frame aligned with the electrical rotor position (and rotating with the electrical rotor speed).

Let us define the current and flux vectors in the considered frame (denoted by subscript indices $_{*A(B)}$)

$$\boldsymbol{i}_{sAB} = [i_{sA},\ i_{sB}]^{\mathrm{T}},\ \boldsymbol{\varphi}_{sAB} = [\varphi_{sA},\ \varphi_{sB}]^{\mathrm{T}},$$

and the transformation matrix between the stator-fixed and the rotating frame as

$$\boldsymbol{T} \equiv \boldsymbol{T}_{AB\to\alpha\beta} = \begin{bmatrix} \cos(n_{\mathrm{p}}\theta) & -\sin(n_{\mathrm{p}}\theta) \\ \sin(n_{\mathrm{p}}\theta) & \cos(n_{\mathrm{p}}\theta) \end{bmatrix}, \tag{6}$$

with θ being the rotor angular position.

For the transformation matrix we have:

$$\boldsymbol{T}_{\alpha\beta\to AB} = \boldsymbol{T}^{-1} = \boldsymbol{T}^{\mathrm{T}}. \tag{7}$$

It is also noted that the vector magnitudes are preserved by the transformation.

By substituting the current and the flux vectors in (4) as $\boldsymbol{i}_{s\alpha\beta} = \boldsymbol{T}\boldsymbol{i}_{sAB}$, $\boldsymbol{\varphi}_{s\alpha\beta} = \boldsymbol{T}\boldsymbol{\varphi}_{sAB}$, we obtain:

$$\frac{d}{dt}(\boldsymbol{T}\boldsymbol{\varphi}_{sAB}) = -\boldsymbol{M}_{\varphi}\boldsymbol{T}\boldsymbol{\varphi}_{sAB} + \boldsymbol{M}_{i}\boldsymbol{T}\boldsymbol{i}_{sAB} + \sigma l_{\mathrm{s}}\frac{d}{dt}(\boldsymbol{T}\boldsymbol{i}_{sAB}),$$

which, with the developed derivatives is rewritten as

$$\boldsymbol{T}\dot{\boldsymbol{\varphi}}_{sAB} + \dot{\boldsymbol{T}}\boldsymbol{\varphi}_{sAB} = -\boldsymbol{M}_{\varphi}\boldsymbol{T}\boldsymbol{\varphi}_{sAB} + \boldsymbol{M}_{i}\boldsymbol{T}\boldsymbol{i}_{sAB} + \sigma l_{\mathrm{s}}\boldsymbol{T}\dot{\boldsymbol{i}}_{sAB} + \sigma l_{\mathrm{s}}\dot{\boldsymbol{T}}\boldsymbol{i}_{sAB}. \tag{8}$$

Given (7) and the following properties: $\boldsymbol{T} = e^{\theta\boldsymbol{J}}$, $\dot{\boldsymbol{T}} = n_{\mathrm{p}}\omega\boldsymbol{J}e^{\theta_e\boldsymbol{J}} = n_{\mathrm{p}}\omega\boldsymbol{J}\boldsymbol{T}$, ($\omega = \dot{\theta}$), equation (8) is reduced to

$$\dot{\boldsymbol{\varphi}}_{sAB} = -\eta\boldsymbol{\varphi}_{sAB} + \eta l_{\mathrm{s}}\boldsymbol{i}_{sAB} + \sigma l_{\mathrm{s}}\dot{\boldsymbol{i}}_{sAB}. \tag{9}$$

It is seen, that in this frame, the time-varying feature of the dynamics is eliminated and the system is linear time-invariant.

For the torque we have:

$$\begin{aligned} \tau_{\mathrm{m}}(t) &= n_{\mathrm{p}}(\varphi_{s\alpha}i_{s\beta} - \varphi_{s\beta}i_{s\alpha}) = n_{\mathrm{p}}\boldsymbol{i}^{\mathrm{T}}_{s\alpha\beta}(t)\boldsymbol{J}\boldsymbol{\varphi}_{s\alpha\beta}(t) = \\ &= n_{\mathrm{p}}\boldsymbol{i}^{\mathrm{T}}_{sAB}(t)\boldsymbol{T}^{\mathrm{T}}\boldsymbol{J}\boldsymbol{T}\boldsymbol{\varphi}_{sAB}(t) = n_{\mathrm{p}}\boldsymbol{i}^{\mathrm{T}}_{sAB}(t)\boldsymbol{J}\boldsymbol{\varphi}_{sAB}(t). \end{aligned} \tag{10}$$

Exact discrete-time description of the motor dynamics is obtained from (9), assuming that the stator currents are held constant during the sampling periods, i.e., assuming zero-order holds at the inputs:

$\boldsymbol{i}_{sAB}(t) = \boldsymbol{i}_{sAB}(t_k) = \mathrm{const}$ for $t_k \le t < t_{k+1}$ $t_k \equiv kT_0$, T_0 is the sampling period.

In order to obtain the discrete-time description, the solution of (9) is first considered. For this purpose, the flux equation is decomposed as:

(11) $$\dot{\boldsymbol{\psi}}_{sAB} = -\eta \boldsymbol{\psi}_{sAB} + \eta l_s (1-\sigma)\boldsymbol{i}_{sAB}, \quad \boldsymbol{\varphi}_{sAB} = \boldsymbol{\psi}_{sAB} + l_s \sigma \boldsymbol{i}_{sAB}.$$

For $t \geq t_k$, we have

$$\boldsymbol{\psi}_{sAB}(t) = \boldsymbol{\psi}_{sAB}(t_k)e^{-\eta(t-t_k)} + \eta l_s (1-\sigma)\int_{t_k}^{t} e^{-\eta(t-\upsilon)}\boldsymbol{i}_{sAB}(t_k)d\upsilon,$$

and ultimately

(12) $$\boldsymbol{\psi}_{sAB}(t) = \boldsymbol{\psi}_{sAB}(t_k)e^{-\eta(t-t_k)} + l_s(1-\sigma)(1-e^{-\eta(t-t_k)})\boldsymbol{i}_{sAB}.$$

Thus, from (11) it is obtained for the flux:

(13) $$\boldsymbol{\varphi}_{sAB}(t) = \boldsymbol{\varphi}_{sAB}(t_k)e^{-\eta(t-t_k)} + l_s(1-\sigma - e^{-\eta(t-t_k)})\boldsymbol{i}_{sAB}(t_k) + l_s\sigma\boldsymbol{i}_{sAB}(t).$$

For the torque we have

$$\tau_m(t) = n_p \boldsymbol{i}^{T}_{sAB}(t)\boldsymbol{J}\boldsymbol{\varphi}_{sAB}(t) = \tau(t_k)e^{-\eta(t-t_k)},$$

with

(14) $$\tau_m(t_k) = n_p(\varphi_{sA}(t_k)i_{sB}(t_k) - \varphi_{sB}(t_k)i_{sA}(t_k)).$$

Finally, for $t = t_{k+1}$, the flux is given by

(15) $$\boldsymbol{\varphi}_{sAB}(t_{k+1}) = \boldsymbol{\varphi}_{sAB}(t_k)e^{-\eta T_0} + l_s(1-\sigma - e^{-\eta T_0})\boldsymbol{i}_{sAB}(t_k) + l_s\sigma\boldsymbol{i}_{sAB}(t_{k+1}).$$

Though the motor torque is a nonlinear function of the states, the particular function structure, along with the considered input excitation (zero-order holds on the stator currents in the rotating frame) enables its exact reconstruction from its samples by a form of the exponential hold.

This, in turn gives the possibility to obtain an exact discrete-time representation of the speed dynamics, assuming it linear, as follows:

(16) $$J\dot{\omega} = \tau_m - c\omega - \tau_L$$

with J – moment of inertia, referred to the rotor; c – viscous friction coefficient.

The transfer function of the hold is $\dfrac{1-e^{-(s+\eta)T_0}}{s+\eta}$.

The overall discretization scheme is illustrated in Fig. 1.

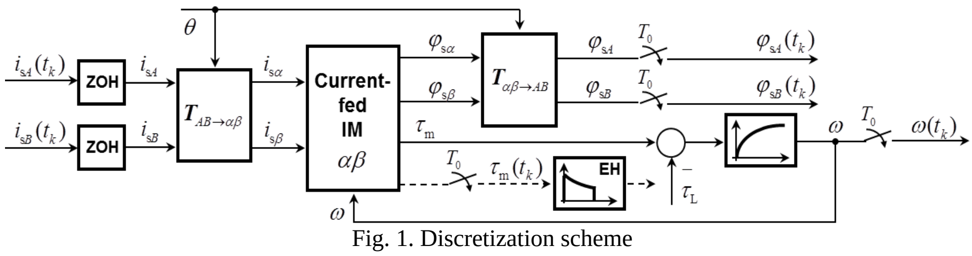

Fig. 1. Discretization scheme

The speed at the sampling instants is obtained as

(17) $$\omega(t_{k+1}) = e^{-cJ^{-1}T_0}\omega(t_k) + \frac{e^{-cJ^{-1}T_0} - e^{-\eta T_0}}{J\eta - c}\tau_m(t_k) - \frac{1}{J}\int_{t_k}^{t_{k+1}} e^{-cJ^{-1}(t_{k+1}-\upsilon)}\tau_1(\upsilon)d\upsilon.$$

In cases, when the load torque satisfies $\tau_1(t) = \tau_1(t_k)$ for $kT_0 \leq t < (k+1)T_0$, that is, it is constant during the sampling periods, we have:

(18) $$\frac{1}{J}\int_{t_k}^{t_{k+1}} e^{-cJ^{-1}(t_{k+1}-\upsilon)}\tau_1(\upsilon)d\upsilon = \frac{1-e^{-cJ^{-1}T_0}}{c}\tau_1(t_k)\,,$$

(for $c = 0$ we have $J^{-1}T_0\tau_1(t_k)$).

In these cases, a difference equation can be written for the motor speed. Equations (15), (14), (17), and (18) represent the exact discrete-time model of the system.

4. Input-output linearizing control law design

The theoretical foundations of the feedback linearization and the basic control design techniques extended for discrete-time systems can be found in [15, 16]. Here it will be only noted that the basic structure of a control system using such control laws consists of two loops – an inner one, in which the linearization is achieved, and an outer, linear loop, where the linear controller attributes the desired dynamics of the overall system.

The model used here as basis for the control law design is given by:

(19) $$\begin{aligned} x_1(t_{k+1}) &= e^{-\eta T_0}x_1(t_k) + l_\mathrm{s}(1-\sigma-e^{-\eta T_0})x_3(t_k) + l_\mathrm{s}\sigma u_1(t_k), \\ x_2(t_{k+1}) &= e^{-\eta T_0}x_2(t_k) + l_\mathrm{s}(1-\sigma-e^{-\eta T_0})x_4(t_k) + l_\mathrm{s}\sigma u_2(t_k), \\ x_3(t_{k+1}) &= u_1(t_k), \\ x_4(t_{k+1}) &= u_2(t_k), \end{aligned}$$

with $$[x_1(t_k),\ x_2(t_k),\ x_3(t_k),\ x_4(t_k)]^\mathrm{T} \equiv [\varphi_{sA}(t_k),\ \varphi_{sB}(t_k),\ i_{sA}(t_k),\ i_{sB}(t_k)]^\mathrm{T},$$
$$[u_1(t_k),\ u_2(t_k)]^\mathrm{T} \equiv [i_{sA}(t_{k+1}),\ i_{sB}(t_{k+1})]^\mathrm{T}.$$

As seen, model (15) is augmented by two additional state variables – $x_3(t_k)$ and $x_4(t_k)$, and their respective equations by adding a delay of one sampling period at each input. Thus, a description in a state-space form is obtained. Also, in this way, one sampling period is allowed for calculations, which renders the control law realizable since the control design technique applied will result in a static state feedback. For the control design, the induction motor is normally considered as a TITO-system, with the torque, rotor speed or position being the main output of mechanical nature and the flux magnitude (squared) as the second output of electromagnetic nature. Here, the controlled quantities are defined as follows:

(20) $$\begin{aligned} y_1(t_k) &\equiv \tau_\mathrm{m}(t_k) = n_\mathrm{p}(x_1(t_k)x_4(t_k) - x_2(t_k)x_3(t_k)), \\ y_2(t_k) &= x_1(t_k)x_1(t_{k-1}) + x_2(t_k)x_2(t_{k-1}) - e^{-\eta T_0}(x_1^2(t_{k-1}) + x_2^2(t_{k-1})). \end{aligned}$$

The first output $y_1(t_k)$ is defined as the motor torque. Thus, the speed dynamics (being linear) is not accounted for in the decoupling control, which makes it simpler and more robust because it does not include the mechanical parameters.

The second output $y_2(t_k)$ is defined after the following modifications, starting from the expression for the squared stator flux $x_1^2(t_k) + x_2^2(t_k)$. First, the previous values of each component are introduced, so that the resulting control law becomes an affine function of the new external input variables. Thus, the left term in the expression is obtained. Then, a correction term, proportional to the previous value of the flux square, is added, so that the stabilization of $y_2(t_k)$ guarantees physically acceptable regimes of the motor operation and ultimate stabilization of the stator flux. It should be noted that the minus sign is important, the value of the coefficient – $e^{-\eta T_0}$ is chosen so that the resulting expressions for the control law are simplified. In the steady-state (at a constant speed), $y_2(t_k)$ is related to the squared stator flux in the following way [12]:

$$y_2(t_k) = \left|\boldsymbol{\varphi}_s(t_k)\right|^2 (\cos(\omega_{sl} T_0) - e^{-\eta T_0}),$$

where ω_{sl} is the electrical slip speed. Since the sampling period is typically atmost in the millisecond range and the slip speed is generally low (typically a single-digit percentage of the rotor speed), it can be assumed with satisfactory precision that $\cos(\omega_{sl} T_0) \approx 1$ and

$$y_2(t_k) \approx \left|\boldsymbol{\varphi}_s(t_k)\right|^2 (1 - e^{-\eta T_0}) \; . \tag{21}$$

The expressions for both outputs are written for $t = t_{k+1}$ as (22) and the input-output description is put in the vector-matrix form (23):

$$\begin{aligned} y_1(t_{k+1}) &= -n_p(e^{-\eta T_0} x_2(t_k) + l_s(1-\sigma - e^{-\eta T_0}) x_4(t_k)) u_1(t_k) + \\ &+ n_p(e^{-\eta T_0} x_1(t_k) + l_s(1-\sigma - e^{-\eta T_0}) x_3(t_k)) u_2(t_k), \\ y_2(t_{k+1}) &= x_1(t_{k+1}) x_1(t_k) + x_2(t_{k+1}) x_2(t_k) - e^{-\eta T_0}(x_1^2(t_k) + x_2^2(t_k)) = \\ &= l_s(1-\sigma - e^{-\eta T_0})(x_1(t_k)x_3(t_k) + x_2(t_k)x_4(t_k)) + l_s\sigma x_1(t_k) u_1(t_k) + l_s \sigma x_2(t_k) u_2(t_k); \end{aligned} \tag{22}$$

$$\begin{bmatrix} y_1(t_{k+1}) \\ y_2(t_{k+1}) \end{bmatrix} = \begin{bmatrix} 0 \\ a(t_k) \end{bmatrix} + \begin{bmatrix} b_{11}(t_k) & b_{12}(t_k) \\ b_{21}(t_k) & b_{22}(t_k) \end{bmatrix} \begin{bmatrix} u_1(t_k) \\ u_2(t_k) \end{bmatrix}, \tag{23}$$

with $\boldsymbol{B}(t_k) \equiv \begin{bmatrix} b_{11}(t_k) & b_{12}(t_k) \\ b_{21}(t_k) & b_{22}(t_k) \end{bmatrix}$ – the decoupling matrix and

$$\begin{aligned} a(t_k) &= l_s(1-\sigma - e^{-\eta T_0})(x_1(t_k)x_3(t_k) + x_2(t_k)x_4(t_k)), \\ b_{11}(t_k) &= -n_p(e^{-\eta T_0} x_2(t_k) + l_s(1-\sigma - e^{-\eta T_0}) x_4(t_k)), \\ b_{12}(t_k) &= n_p(e^{-\eta T_0} x_1(t_k) + l_s(1-\sigma - e^{-\eta T_0}) x_3(t_k)), \\ b_{21}(t_k) &= l_s \sigma x_1(t_k), \\ b_{22}(t_k) &= l_s \sigma x_2(t_k). \end{aligned}$$

By introducing the control inputs as

$$\begin{bmatrix} u_1(t_k) \\ u_2(t_k) \end{bmatrix} = \mathbf{B}^{-1}(t_k) \begin{bmatrix} v_1(t_k) \\ v_2(t_k) - a(t_k) \end{bmatrix}, \tag{24}$$

with $v_1(t_k)$, $v_2(t_k)$ being the new input signals, the input-output relations for the system obtained are given by

$$\begin{bmatrix} y_1(t_{k+1}) \\ y_2(t_{k+1}) \end{bmatrix} = \begin{bmatrix} v_1(t_k) \\ v_2(t_k) \end{bmatrix}. \tag{25}$$

As seen from the equations, no coupling exists between the two outputs which delays their respective input signals by one sampling period. It is also noticed, that the obtained input-output dynamics is of second order, while the initial description is of fourth, so that second order internal dynamics, unobservable from the outputs, exists. It must be such that the system states remain bounded while controlling the outputs, otherwise, the control law would be unuseful. Here the non-trivial definition of the second output prevents the direct defining of a complete novel set of coordinates and the respective transfomation, so that the internal stability analysis can be performed. In order to do so, the model must be extended by two additional variables, so that the second output becomes a static function of states. No internal instabilities were observed during simulations of the system behaviour, the formal analysis however will be a subject of a future research. It must be noted that in the continuous-time case, with the stator flux square as a system output, the internal dynamics can be expressed in terms of the rotor flux components, which guarantees internal stability when input-output stability is achieved.

In order to introduce the control law, the matrix inversion in (24) must be possible, i.e., the decoupling matrix must be non-singular. A discussion on the way to ensure it is given in the following lines.

The following substitutions are introduced for notational simplicity:

$$\bar{\varphi}_{\mathrm{r}1}(t_k) = e^{-\eta T_0} x_1(t_k) + l_{\mathrm{s}}(1-\sigma-e^{-\eta T_0}) x_3(t_k),$$
$$\bar{\varphi}_{\mathrm{r}2}(t_k) = e^{-\eta T_0} x_2(t_k) + l_{\mathrm{s}}(1-\sigma-e^{-\eta T_0}) x_4(t_k).$$

The determinant of the matrix is given by

(26) $$\det \boldsymbol{B}(t_k) = -n_{\mathrm{p}} l_{\mathrm{s}} \sigma (x_1(t_k)\bar{\varphi}_{\mathrm{r}1}(t_k) + x_2(t_k)\bar{\varphi}_{\mathrm{r}2}(t_k)) \neq 0.$$

The invertibility condition (requirement for a non-zero determinant of the decoupling matrix) represents the requirement for non-ortogonality of the stator flux vector $\boldsymbol{\varphi}_{\mathrm{s}AB}(t_k)$ and the vector $\bar{\boldsymbol{\varphi}}_{\mathrm{r}}(t_k) = [\bar{\varphi}_{\mathrm{r}1}(t_k),\ \bar{\varphi}_{\mathrm{r}2}(t_k)]^{\mathrm{T}}$.

Note that as $T_0 \to 0$, $\bar{\boldsymbol{\varphi}}_{\mathrm{r}}(t_k) \to m l_{\mathrm{r}}^{-1} \boldsymbol{\varphi}_{\mathrm{r}AB}(t_k) = m l_{\mathrm{r}}^{-1}(\boldsymbol{\varphi}_{\mathrm{s}AB}(t_k) - l_{\mathrm{s}}\sigma \boldsymbol{i}_{\mathrm{s}AB}(t_k))$. Thus, in the asymptotic continuous-time case, the condition is reduced to preventing the stator and rotor flux vectors from becoming orthogonal, which, as pointed out in [9], can be related to the maximal available torque of the machine for its current electromagnetic state. More strictly, if the maximal available motor torque is not required, that is, if the torque satisfies

(27) $$\tau_{\mathrm{m}} < \tau_{\mathrm{m}}^{\max} = n_{\mathrm{p}} m (\sigma l_{\mathrm{s}} l_{\mathrm{r}})^{-1} |\boldsymbol{\varphi}_{\mathrm{s}}| |\boldsymbol{\varphi}_{\mathrm{r}}|$$

with $\tau_{\mathrm{m}}^{\max}$ being the maximal torque, the realizability of the control law is guaranteed.

The motor torque, expressed in terms of the stator and rotor fluxes, is given by $\tau_{\mathrm{m}} = n_{\mathrm{p}} m (\sigma l_{\mathrm{s}} l_{\mathrm{r}})^{-1} \boldsymbol{\varphi}_{\mathrm{s}}^{\mathrm{T}} \boldsymbol{J} \boldsymbol{\varphi}_{\mathrm{r}}$ and $\tau_{\mathrm{m}}^{\max}$ is obtained when the two flux vectors are orthogonal.

It should be noted, that only the strict equality is not allowed. Also, normally $\tau_{\mathrm{m}}^{\max}$ is much higher than the rated machine torque for the nominal operation regimes, so that the invertibility condition does not impose substantial limitations on the achievable performance.

The insight gained from the continuous-time case can be applied for the discrete-time case considered here. An approach to ensure the realizability of the proposed control law may consist of a dynamically saturating signal $v_1(t_k)$ (it can be viewed as a torque reference) so that $\det \boldsymbol{B}(t_k)$ remains negative.

5. Simulation results

To validate the proposed discrete-time model of the induction motor and the linearizing control law, a set of simulations were conducted. The following model was implemented in Matlab, Simulink environment.

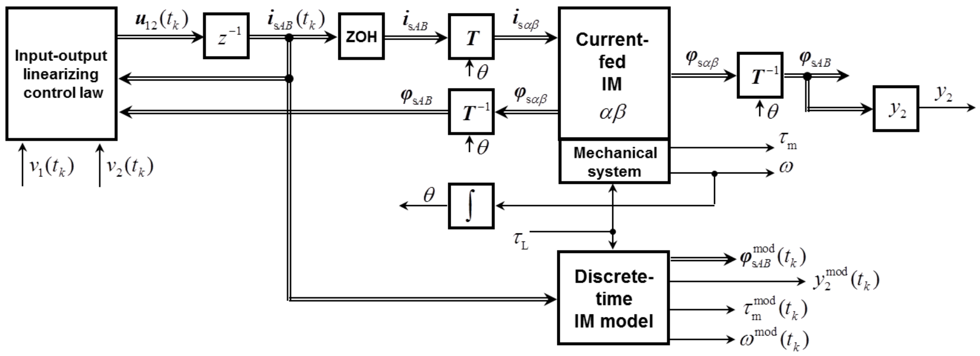

Fig. 2. Simulation model

The induction machine parameters used in the simulations are taken from [7] and are as follows: $r_s = 0.052\ \Omega$, $l_s = 0.03175$ H, $r_r = 0.07\ \Omega$, $l_r = 0.0323$ H, m = 0.031 H, $n_p = 2$, $J = 0.41\ \text{kg.m}^2$. The nominal power and the rated torque are $P_{nom} = 37$ kW and $\tau_{nom} = 240$ N.m. The value of the viscous friction coefficient is set to $c = 10^{-4}$ N.m.s.

The sampling period was set to $T_0 = 1$ ms. Some of the results are shown in Fig. 3.

All transients confirm the validity of the discrete-time description of the motor since all signals produced by the model (shown held during sampling periods), match exactly the motor fluxes and the speed (the details shown in the embedded graphs). Also, the expected control performance is confirmed since no coupling is observed between the outputs and each output tracks its respective input with a delay of one sampling period, as given by (25).

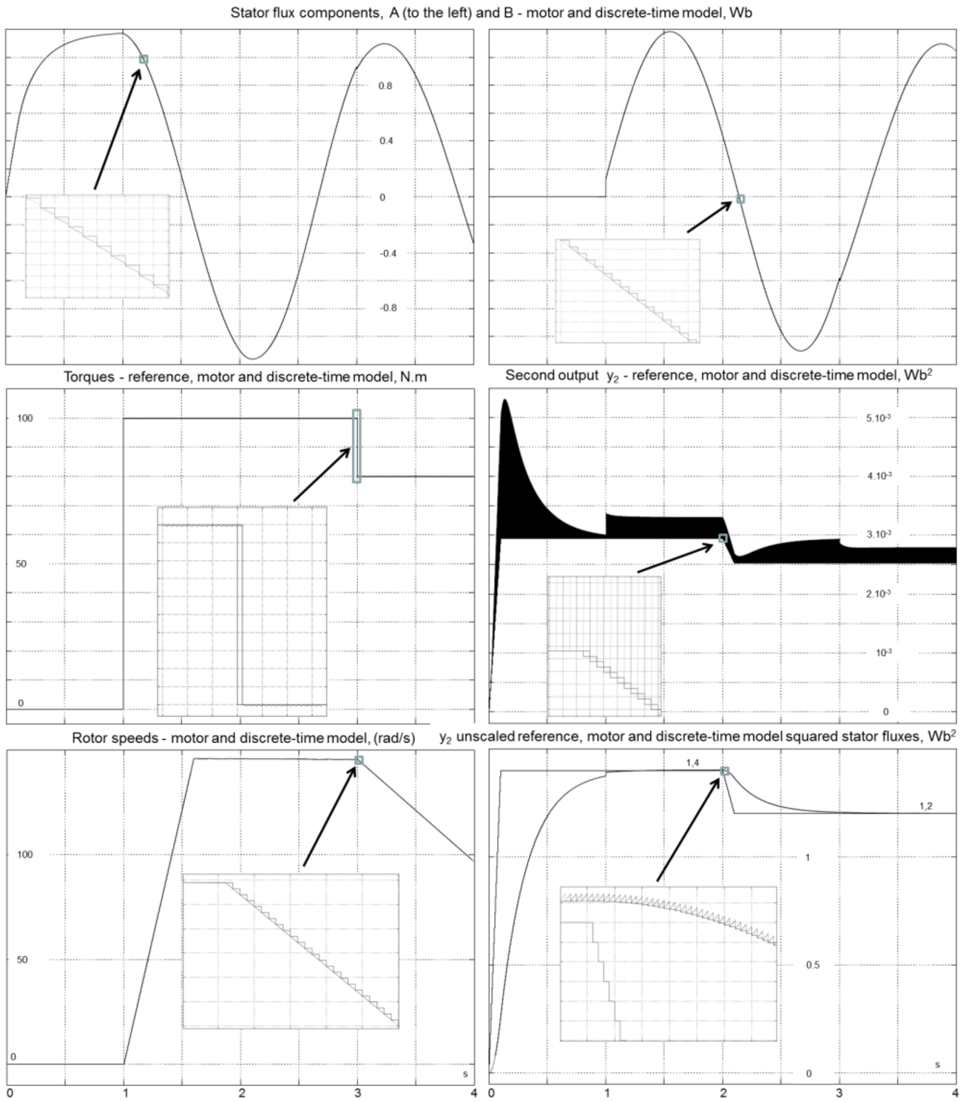

Fig. 3. Simulated transients

No outer loops were introduced and the two input variables are used as references for the respective outputs. The steady state values of $v_2(t_k)$ were generated from the desired magnitudes of the squared stator flux by using (21). As seen, the actual squared flux magnitude reaches the desired reference value, thus proving that the scale factor in (21) is correct. It should be noted that slight deviations above the obtained in this way reference value, will be observed since the slip speed is not zero. These, as well as the ripples observed in the flux magnitude are insignificant from a practical point of view. Insignificant ripples are observed also in the motor torque. On the other hand, large ripples are observed in the second output of the motor, though this is not relevant to the control or the operation regimes of the motor. As seen, a certain dynamic lag is observed between the actual squared flux value and the second output reference, the settling-time of the flux seeming to be practically unaffected by the reference rise-time. Finally, at $t = 1$ s, as the torque step reference is applied, a small deviation in the squared flux magnitude is seen, which shows coupling between these quantities, though again, as obvious, the amplitude is insignificant. A load torque of 100 N.m is applied at $t = 1.6$ s.

Fig. 4 shows transients with an outer control loop introduced in the flux subsystem. The controller is designed, based on the following assumption:

$$y_2(t_k) \approx x_1^2(t_k) + x_1^2(t_k) - e^{-\eta T_0}(x_1^2(t_{k-1}) + x_2^2(t_{k-1})) = \left|\boldsymbol{\varphi}_s(t_k)\right|^2 - e^{-\eta T_0}\left|\boldsymbol{\varphi}_s(t_{k-1})\right|^2.$$

Thus, a discrete-time transfer function can be defined and the outer loop is configured as given in Fig. 5. The respective variations of $v_2(t_k)$ and $y_2(t_k)$ are shown in the graph to the right. The torque and speed responses are the same as in Fig. 3.

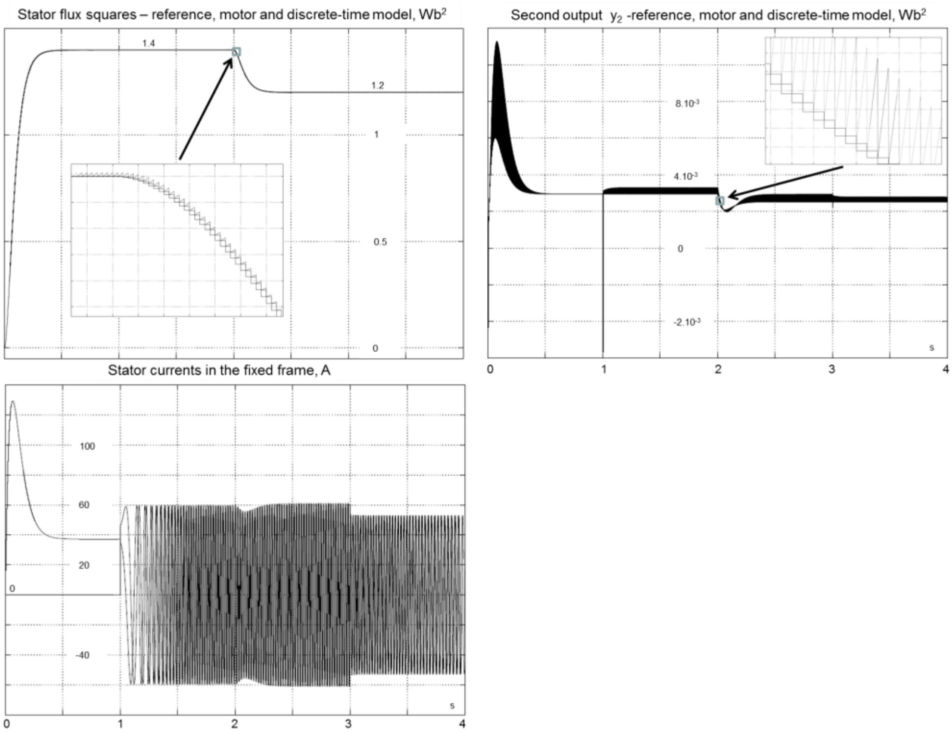

Fig. 4. Simulated transients

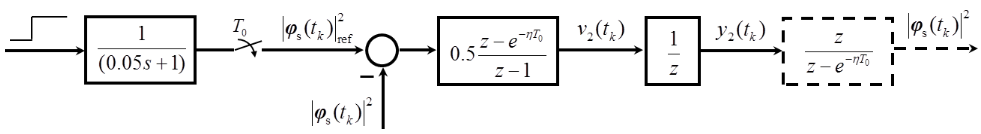

Fig. 5. Outer control loop configuration

6. Conclusion

In this paper an exact discrete-time model of the induction motor in a current-fed mode, including stator flux components, is derived and validated. As above mentioned, in the practical setup the current-fed mode is forced by additional current control loops. In order the discrete-time representation to hold exactly, the stator currrents applied to the motor (the currents in the α-β frame, or at least their reference values), must vary between the sampling instants, unless the rotor speed is zero, as seen by the coordinate transformation between the two frames. This will require a higher performance current control, which in turn will call for faster, and possibly more complex current control loops, as well as higher sampling rates in the position signal acquisition channel.

Based on the derived exact discrete-time representation of the motor dynamics, an input-output linearizing and decoupling control is designed for torque and stator flux magnitude control. The applied design technique requires a non-trivial definition of the electromagnetic output of the motor. However, a well specified modification results in a useful output definition, which is motivated by thorough analysis and discussion. The major benefit of the proposed scheme is that the stability analysis of the closed-loop system is trivial, since no approximations are done in any stage of the design. Of course, precise current control is assumed for a discrete-time model of the motor that holds exactly.

Some simulated transients are presented showing that the aimed performance is obtained, that is, no coupling exists between the outputs, and the initial design problem of controlling the nonlinear interacting TITO system is reduced to a problem of controlling two linear and decoupled SISO systems with simple dynamics.

The proposed control law calculation requires stator fluxes, which in a practical setup will require the implementation of a certain flux estimation scheme in the overall control system structure. This represents a whole separate research field. The voltage model is the trivial choice, though it is known for the pure integration-related drift problems. Here, a different model arises, not including pure integration, although the rotor resistance returns in the expressions, thus bringing back the related variability issues.

The future research may focus first on including the current deviations from their desired values in the formal setup and studying the induced effects. Of course, further simulation studies with more detailed models, accounting the different processes, present in a practical implementation, such as current control, noises, flux estimation, parameter variability are planned, in order to validate the applicability of the proposed control law in a practical setup and ultimately, lead to experimental validation on a physical testbed.

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