BULGARIAN ACADEMY OF SCIENCES

CYBERNETICS AND INFORMATION TECHNOLOGIES • Volume **13**, No 3

Sofia • 2013 Print ISSN: 1311-9702; Online ISSN: 1314-4081
DOI: 10.2478/cait-2013-0022

Fixed Point Theorem for Converse Commuting Mapping in Symmetric Spaces

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Abstract: *Using the concept of converse commuting mappings, our target is to prove some common fixed point theorems with respect to a contractive condition under implicit function relations and an integral type contractive condition in fuzzy symmetric spaces.*

Keywords: *Commuting point, coincidence point, commuting mappings, conversely commuting mappings, occasionally converse commuting (occ) mappings, converse commuting multivalued mappings, fuzzy symmetric space, implicit relation.*

2010 Mathematics Subject Classification: 03E72, 47H10, 54H25.

1. Introduction

In 2002 the concept of converse commuting maps was introduced by L ü [15], as a reverse process of weakly compatible mappings. Using the concept of converse commuting mappings, our target is to prove some common fixed point results under different contractive conditions in fuzzy symmetric spaces.

Symmetric spaces were introduced in 1931 by W i l s o n [14], as metric-like spaces without having the triangle inequality. Several fixed point results in such spaces were obtained. H i c k s and R h o a d e s [12] established some common fixed point theorems in symmetric spaces using the fact that some of the properties of metrics are not required in the proofs of certain metric theorems. The study of symmetric spaces was obtained in connection with some measurements in physics.

Over the years, the theory has found several important applications in the investigation of physical quantities in quantum particle physics and string theory.

Most of the existing mathematical tools for formal modeling, reasoning and computing are crisp, deterministic and precise in character. But in real life situations, the problems in economics, engineering, environment, social science, medical science, etc., do not always involve crisp data. Consequently, we cannot successfully use the traditional classical methods because of various types of uncertainties presented in the problem. To deal with the uncertainties, the fuzzy set theory [16] can be considered as one of the mathematical tools. That is why so many researchers are trying to fuzzify different classical mathematical concepts.

A new vista was opened for further progress following the initiation of fuzzy metric spaces and consequently several authors [1, 7, 8, 9] engaged themselves to make a headway upon it for the last four decades. But in this paper we have considered a fuzzy symmetric space, which is a more general concept than the fuzzy metric space.

To prepare this article, we have followed mainly the results of [4, 5, 13] and it is divided into six sections as follows. In Section 1 there is a brief introduction, in Section 2 a few definitions and basic concepts are given. In Section 3 a fixed point theorem for single valued maps is proved with the help of a contractive condition under implicit function relations, occasionally converse commuting (occ) and occasionally weakly compatible (owc). In Section 4 a fixed point theorem for multivalued maps is proved with the help of a contractive condition under implicit function relations, occasionally converse commuting (occ) and occasionally weakly compatible (owc). In Section 5 a fixed point theorem for single valued maps satisfying the integral type contractive condition and converse commuting condition is proved. In Section 6 a conclusion is given which briefly summarizes the main achievements and the possible directions for future work.

2. Preliminaries

We quote some definitions and statements of a few theorems which will be needed in the sequel.

Definition 2.1. The pair (X, μ) is called a fuzzy symmetric space if X is an arbitrary non-empty set and μ is a fuzzy set in $X^2 \times (0, \infty)$ satisfying the following conditions:

(i) $\mu(x, y, t) > 0$;

(ii) $\mu(x, y, t) = 1$ if and only if $x = y$;

(iii) $\mu(x, y, t) = \mu(y, x, t)$;

$(iv)\ \mu(x, y, t) : (0, \infty) \to (0, 1]$ is continuous for all $x, y \in X$ and $t > 0$.

If (X, μ) be a fuzzy symmetric space then μ is called a fuzzy symmetric for X.

Note that $\mu(x, y, t)$ can be thought of as the degree of nearness between x and y with respect to t. Let μ be a symmetric on X and D be the fuzzy metric on 2^X defined by the symmetric μ as follows:

$D(A, B, t) = \max\{\mu(a, b, t) : a \in A, b \in B\}$ for all $A, B \in 2^X$, and

$\mu(x, A, t) = \max\{\mu(x, y, t) : y \in A\}$ for all $x \in X$ and $A \in 2^X$.

Definition 2.2. A sequence $\{x_n\}_n$ in a fuzzy symmetric space is said to converge to $x \in X$ if and only if $\lim_{n\to\infty} \mu(x_n, x, t) = 1$.

A sequence $\{x_n\}_n$ in the fuzzy symmetric space is said to be a **Cauchy sequence** if and only if $\lim_{n\to\infty} \mu(x_n, x_{n+p}, t) = 1$.

A fuzzy symmetric space (X, μ) is said to be **complete** if and only if every Cauchy sequence in X is convergent in X.

Let f, g be single valued mappings from X into itself and $F : X \to 2^X$ be a multivalued mappings.

Definition 2.3 [14]. Two self-maps f and g are called **converse commuting**, if for all x in X the $fgx = gfx$ implies $fx = gx$.

Definition 2.4 [14]. $x \in X$ is said to be a **commuting point** of f and g if $fgx = gfx$.

Definition 2.5 [6]. The mappings g and F are called **converse commuting**, if for all $x \in X$, $gFx = Fgx$ implies $gx \in Fx$.

Definition 2.6 [6]. x is said to be a **commuting point** of g and F if $Fgx = gFx$.

Definition 2.7 [4]. Two self-mappings f and g are called **occasionally converse commuting** (occ), if for some x in X the $fgx = gfx$ implies $fx = gx$.

It is verified by an example in [6] that every conversely commuting mapping is occ, but the reverse needs not to be true.

Definition 2.8 [2]. Two self-maps f and g are called **weakly compatible**, if $fgx = gfx$ whenever $fx = gx$, for all x in X.

The concept of a weakly compatible mapping [2] was generalized to occasionally weakly compatible [3].

Definition 2.9 [3]. Two self-mappings f and g are called **occasionally weakly compatible** (owc), if $f\,g\,x = g\,f\,x$ whenever $f\,x = g\,x$, for some x in X.

Implicit relations:

Let F_6 be the set of all real-valued functions $F(t_1, \cdots, t_6) : \mathbb{R}^6 \to \mathbb{R}$ be a continuous function satisfying the property

$$(F_1) : F(t, t, 1, 1, t, t) > 1 \text{ for all } 0 \le t < 1.$$

Example 2.10. Let

$$F(t_1, \cdots, t_6) = 1 + \frac{t_1 + t_2 + t_3}{\max\{t_4, t_5, t_6\}}.$$

Then (F_1) is obvious and $F \in F_6$.

Example 2.11. Let

$$F(t_1, \cdots, t_6) = t_1 + t_2 + t_3 + t_4 + t_5 + t_6 \text{ or,}$$

$$F(t_1, \cdots, t_6) = 1 + \frac{t_1 + t_2}{2} + \frac{t_3 + t_4}{2} + \frac{t_5 + t_6}{2}.$$

Then (F_1) is obvious and $F \in F_6$.

3. Fixed point theorem with occasionally converse commuting mapping

Theorem 3.1. Let $A, B, S, T : X \to X$ be four self-mappings satisfying:

(1) $$\begin{gathered}\phi\big(\mu(Ax, By, t), \mu(Sx, Ty, t), \mu(Ax, Sx, t),\\ \mu(By, Ty, t), \mu(By, Sx, t), \mu(Ax, Ty, t)\big) \le 1,\end{gathered}$$

where $x, y \in X$ and $\phi \in F_6$. If one of the following conditions holds:

(i) the pair (A, S) is occ and the pair (B, T) is owc,

(ii) the pair (B, T) is occ and the pair (A, S) is owc,

then A, B, S and T have unique common fixed point in X.

Proof: Suppose that the pair (B, T) is occasionally weakly compatible owc. Then, by definition, there exists a coincidence point $\lambda \in C(B, T)$ such that $BT\lambda = TB\lambda$ whenever $B\lambda = T\lambda = z$ (say), where $C(B, T)$ denotes the set of the coincidence points of B and T. So that for a given λ

(2) $$Bz = Tz \text{ whenever } B\lambda = T\lambda = z.$$

Next, since (A, S) is occasionally converse commuting occ then, by definition, there exists any $u \in X$ such that $ASu = SAu$ implies $Au = Su = w$ (say). So that for a given u

(3) $$Aw = Sw \;\Rightarrow\; Au = Su = w.$$

We claim that $AAu = Bz$. If not, then putting $x = Au$ and $y = z$ in (1), and using $ASu = SAu = AAu$ and $Tz = Bz$, we obtain

$$\phi\big(\mu(AAu, Bz, t), \mu(AAu, Bz, t), 1, 1, \mu(AAu, Bz, t), \mu(AAu, Bz, t)\big) \le 1,$$

a contradiction of (F_1). Thus $AAu = Bz$. Therefore $Aw = Bz = Sw = Tz$. We claim $Au = Bz$. If not, then putting $x = u$ and $y = z$ in (1) and using (2) and (3), we get

$$\phi\big(\mu(Au, Bz, t), \mu(Au, Bz, t), 1, 1, \mu(Au, Bz, t), \mu(Au, Bz, t)\big) \le 1,$$

a contradiction of (F_1). Thus $Au = Bz$. Therefore

(4) $$Au = Bz = Tz = Su = AAu = SAu.$$

It follows that Au is a common fixed point of A and S. Next, we claim that $Bz = z$. If not, then putting $x = u$ (given) and $y = \lambda$ (given) in (1) and using (4) we obtain

$$\phi\big(\mu(Bz, z, t), \mu(Bz, z, t), 1, 1, \mu(z, Bz, t), \mu(Bz, z, t)\big) \le 1,$$

a contradiction of (F_1) . Thus $Bz = z$. Therefore

$$Bz = z = Tz = Au = Su = AAu = SAu.$$

Hence, z is a common fixed point of A, B, S and T. For uniqueness, let z_0 be another common fixed point of A, B, S, T. Then by putting $x = z$ and $y = z_0$ in (1), we obtain a contradiction of (F_1). Thus A, B, S and T have a unique common fixed point.

The proof for the alternative case is similar. This completes the proof.

Theorem 3.2. Let $A, B, S, T : X \to X$ be four self-maps satisfying condition (1) for each $x, y \in X$ and $\phi \in F_6$. If both (A, S) and (B, T) are occ, then A, B, S and T have a unique common fixed point in X.

Proof: Let occ(A, S) denotes the set of occasionally converse commuting points of A and S. Since the pair (A, S) is occasionally converse commuting, by definition, there exists any $u \in \text{occ}(A, S)$ (subset of X), such that $ASu = SAu \;\Rightarrow\; Au = Su$. Hence, $\mu(Au, Su, t) = 1$.

It follows that

$$AAu = ASu = SAu.$$

Similarly, the occasionally converse commuting points for the pair (B, T) implies that there exists $v \in \text{occ}(B, T)$, such that $BTv = TBv \;\Rightarrow\; Bv = Tv$.

Hence,

$$\mu(Bv, Tv, t) = 1 \text{ and so } BBv = BTv = TBv.$$

Let us show that $Au = Bv$. If not, then putting $x = u$ and $y = v$ and using $\mu(Au, Su, t) = 1$ and $\mu(Bv, Tv, t) = 1$ in (1) we obtain

$$\phi(\mu(Au, Bv, t), \mu(Au, Bv, t), 1, 1, \mu(Au, Bv, t), \mu(Au, Bv, t)) \leq 1,$$

a contradiction of (F_1). Thus $Au = Bv$. Next, we claim that Au is a fixed point of A. Suppose not , then by putting $x = Au$ and $y = v$ in (1) we obtain

$$\phi(\mu(AAu, Au, t), \mu(AAu, Au, t), 1, 1, \mu(AAu, Au, t), \mu(AAu, Au, t)) \leq 1,$$

a contradiction of (F_1). Thus $Au = AAu$. Similarly $Bv = BBv$. Since $Au = Bv$, we have

$$Au = Bv = AAu = ASu = SAu = BBv = BTv = TBv.$$

Therefore $Au = w$ (say), is a common fixed point of A, B, S and T. For uniqueness, let $w' \neq w$ be another common fixed point of A, B, S and T, then by (1) we have

$$\phi(\mu(Aw, Bw', l), \mu(Sw, Tw', t), \mu(Aw, Sw, t),$$

$$\mu(Bw', Tw', t), \mu(Bw', Sw, t), \mu(Aw, Tw', t)) \leq 1 \Rightarrow$$

$$\Rightarrow \phi(\mu(w, w', t), \mu(w, w', t), 1, 1, \mu(w', w, t), \mu(w, w', t)) \leq 1,$$

a contradiction. Thus $w = w' = Au$ and Au is a unique common fixed point of A, B, S and T. This completes the proof.

4. Fixed point theorem with converse commuting multi valued mapping

In this section we further assume that any $F \in F_6$ also satisfies the following condition:

(F_2): F is non-increasing in variables t_2, t_5, t_6.

Theorem 4.1. Assume that four mappings $f, g : X \to X$ and $F, G : X \to 2^x$ satisfy the inequality

(5)
$$\phi(\mu(fx, gy, t), D(Fx, Gx, t), \mu(fx, Fx, t),$$
$$\mu(gy, Gy, t), \mu(fx, Gy, t), \mu(gy, Fx, t)) \leq 1,$$

for each $x, y \in X$, where $\phi \in F_6$. If (f, F) and (g, G) are converse commuting mappings, f and F have a commuting point, g and G have a commuting point, then there exists a common fixed point of f, g, F and G.

Proof: Let u be the commuting point of f, F and v be the commuting point of g and G. Since f and F are converse commuting, we have $fFu = Ffu$ and

$fu \in Fu$, hence $\mu(fu, Fu, t) = 1$. It follows that $ffu \in fFu = Ffu$, hence $\mu(ffu, Ffu, t) = 1$.

Similarly, we have $gv \in Gv$, $\mu(gv, Gv, t) = 1$ and $ggv \in gGv = Ggv$, hence $\mu(ggv, Ggv, t) = 1$.

Let us show that $fu = gv$. If not, since

$$D(Fu, Gv, t) \geq \mu(fu, gv, t),\ \mu(Fu, gv, t) \geq \mu(fu, gv, t)$$

and $\mu(fu, Gv, t) \geq \mu(fu, gv, t)$ by (5) and (F_2) we have successively

$$\phi(\mu(fu, gv, t), D(Fu, Gv, t), \mu(fu, Fu, t),$$
$$\mu(gv, Gv, t), \mu(fu, Gv, t), \mu(gv, Fu, t)) \leq 1 \Rightarrow$$
$$\Rightarrow (\mu(fu, gv, t), \mu(fu, gv, t), 1, 1, \mu(fu, gv, t), \mu(fu, gv, t)) \leq 1,$$

a contradiction of (F_1). Hence, $fu = gv$. We claim that fu is a fixed point of f. Suppose that $fu \neq ffu$. Then

$$\mu(fu, ffu, t) = \mu(ffu, fu, t) = \mu(ffu, gv, t)$$

and by (5) and (F_2) we have successively

$$\phi(\mu(ffu, gv, t), D(Ffu, Gv, t), \mu(ffu, Ffu, t),$$
$$\mu(gv, Gv, t), \mu(ffu, Gv, t), \mu(gv, Ffu, t)) \leq 1 \Rightarrow$$
$$\Rightarrow \phi(\mu(ffu, gv, t), \mu(ffu, fu, t), 1, 1, \mu(ffu, gv, t), \mu(gv, ffu, t)) \leq 1 \Rightarrow$$
$$\Rightarrow \phi(\mu(ffu, fu, t), \mu(ffu, fu, t), 1, 1, \mu(ffu, fu, t), \mu(ffu, fu, t)) \leq 1,$$

a contradiction of (F_1). Therefore, $fu = ffu$. Similarly we have $gv = ggv$. Since $fu = gv$, we have $fu = gv = ggv = gfu$ and fu is a fixed point of g.

On the other hand, $fu = gv \in Ggv = Gfu$ and $ffu \in fFu = Ffu$. Hence, fu is a common fixed point of f, g, F and G.

5. Integral type fixed point theorem with converse commuting mapping

In this section we suppose that H is the collection of all functions $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ such that φ is summable, Lebesgue integrable, non-negative and satisfies

$$\int_0^{\varepsilon} \varphi(t)\, dt > 0 \text{ for each } \varepsilon > 0.$$

Theorem 5.1. Let A, B, S and T be self-maps of a symmetric space (X, μ) such that

(i) the pairs (A, S) and (B, T) are conversely commuting and

(ii) the contraction condition

(6) $$\int_0^{(\mu(Ax,By,t))^p} \varphi(t)\,dt > \int_0^{M_p(x,y)} \varphi(t)\,dt$$

holds for all $x, y \in X$ and

$$M_p(x,y) = \alpha(\mu(Sx,Ty,t))^p + (1-\alpha)\min\Big\{(\mu(Ax,Sx,t))^p, (\mu(By,Ty,t))^p,$$
$$\left(\mu(Ax,Sx,t)^{\frac{p}{2}}\mu(Ax,Ty,t)^{\frac{p}{2}}\right), \left(\mu(Ax,Ty,t)^{\frac{p}{2}}\mu(Sx,By,t)^{\frac{p}{2}}\right)\Big\}$$

and $0 < \alpha \le 1$, $p \ge 1$. If A and S have a commuting point and B and T have a commuting point, then A, B, S and T have a unique common fixed point.

Proof: Let $C(A,S)$ denotes the set of converse commuting points of A and S and $C(B,T)$ denotes the set of coincidence points of B and T. Suppose A and S have a commuting point u, B and T have a commuting point v, then from condition (i), $u \in C(A,S)$ and $v \in C(B,T)$. That is ,

$$ASu = SAu \Rightarrow Au = Su \text{ and } BTv = TBv \Rightarrow Bv = Tv.$$

Hence, $ASu = SAu = AAu$, $\mu(Au, Su, t) = 1$, $BTv = TBv = BBv$ and $\mu(Bv, Tv, t) = 1$.

If possible, suppose that $AAu \ne BTv$. Now

$$M_p(Au,Bv,t) = \alpha(\mu(SAu,TBv,t))^p +$$
$$+(1-\alpha)\min\Big\{(\mu(AAu,SAu,t))^p, (\mu(BBv,TBv,t))^p,$$
$$\left(\mu(AAu,SAu,t)^{\frac{p}{2}}\mu(AAu,TBv,t)^{\frac{p}{2}}\right), \mu(AAu,TBv,t)^{\frac{p}{2}}\mu(SAu,BBv,t)^{\frac{p}{2}}\Big\} =$$
$$= \alpha(\mu(AAu,BTv,t))^p + (1-\alpha)\min\Big\{1, 1, \mu(AAu,BTv,t)^{\frac{p}{2}},$$
$$\mu(AAu,BTv,t)^{\frac{p}{2}}\mu(AAu,BTv,t)^{\frac{p}{2}}\Big\} = \alpha(\mu(AAu,BTv,t))^p +$$
$$+(1-\alpha)\min\left\{1, 1, \mu(AAu,BTv,t)^{\frac{p}{2}}, \mu(AAu,BTv,t)^p\right\} \Rightarrow$$
$$\Rightarrow M_p(Au,Bv,t) = \mu(AAu,BTv,t)^p$$

and condition (6) yields

$$\int_0^{\mu(AAu,BBv,t)^p} \varphi(t)\,dt > \int_0^{M_p(Au,Bv,t)} \varphi(t)\,dt = \int_0^{\mu(AAu,BBv,t)^p} \varphi(t)\,dt$$

a contradiction. Thus $AAu = BTv$. Next we claim that $Au = Bv$. If not, we have

$$M_p(u,v) = \alpha(\mu(Su,Tv,t))^p + (1-\alpha)\min\Big\{(\mu(Au,Su,t))^p, (\mu(Bv,Tv,t))^p,$$
$$\left(\mu(Au,Su,t)^{\frac{p}{2}}\mu(Au,Tv,t)^{\frac{p}{2}}\right), \left(\mu(Au,Tv,t)^{\frac{p}{2}}\mu(Su,Bv,t)^{\frac{p}{2}}\right)\Big\} = \alpha(\mu(Au,Bv,t))^p +$$

$$+(1-\alpha)\min\left\{1, 1, \mu(Au,Bv,t)^{\frac{p}{2}}, \mu(Au,Bv,t)^{\frac{p}{2}} \mu(Au,Bv,t)^{\frac{p}{2}}\right\} \Rightarrow$$

$$\Rightarrow M_p(u,v,t) = \mu(Au,Bv,t)^p$$

and the condition (1) yields

$$\int_0^{\mu(Au,Bv,t)^p} \varphi(t)\,dt > \int_0^{M_p(u,v,t)} \varphi(t)\,dt = \int_0^{\mu(Au,Bv,t)^p} \varphi(t)\,dt,$$

a contradiction . Thus $Au = Bv$. Therefore $Au = Su = Bv = Tv$.

Further, we show that $AAu = Au$. Suppose not, then, since $M_p(Au,v,t) = [\mu(AAu,Bv,t)]^p$, the condition (6) yields

$$\int_0^{(\mu(AAu,Bv,t))^p} \varphi(t)\,dt > \int_0^{M_p(Au,v,t)} \varphi(t)\,dt = \int_0^{(\mu(AAu,Bv,t))^p} \varphi(t)\,dt$$

a contradiction. Thus $AAu = Au$. Hence,

$$AAu = ASu = SAu = BBv = BTv = TBv = Au = Su = Bv = Tv.$$

It shows that Au is a common fixed point of A, B, S and T . Uniqueness follows easily. This completes the proof.

Theorem 5.2. Let A, B, S and T be self-maps of a fuzzy symmetric space (X, μ), such that

(i) the pairs (A,S) and (B,T) are conversely commuting and

(ii) the contractive condition:

(7) $$\int_0^{\mu(Ax,By,t)} \varphi(t)\,dt > \int_0^{M_p(x,y,t)} \varphi(t)\,dt$$

holds for all $x, y \in X$ and

$$M_p(x,y) =$$

$$= \min\{\mu(Sx,Ty,t), \mu(Ax,Sx,t), \mu(By,Ty,t), \max\{\mu(By,sx,t), \mu(Ax,Ty,t)\}\}.$$

If A and S have a commuting point and B and T have a commuting point, then A, B, S and T have a unique common fixed point.

Proof: If $u \in C(A,S)$ and $v \in C(B,T)$ then

$$ASu = SAu = AAu, \ \mu(Au,Su,t) = 1,$$

$$BTv = TBv = BBv \ \text{ and } \ \mu(Bv,Tv,t) = 1.$$

Now we show that $AAu = BBv$. If not, then, since $M_p(Au,Bv,t) = \mu(AAu,BBv,t)$ and

$$\int_0^{\mu(AAu,BBv,t)} \varphi(t)\,dt > \int_0^{M_p(Au,Bv,t)} \varphi(t)\,dt = \int_0^{\mu(AAu,BBv,t)} \varphi(t)\,dt$$

a contradiction appears. Thus $AAu = BBv$. Next we claim that $AAu = Au$. If not, then, since $M_p(u,Au,t) = \mu(Au,AAu,t)$, and condition (7) yields

$$\int_0^{\mu(Au,AAu,t)} \varphi(t)\,dt > \int_0^{M_p(u,Au,t)} \varphi(t)\,dt = \int_0^{\mu(Au,AAu,t)} \varphi(t)\,dt$$

i.e., a contradiction. Thus $Au = AAu$. So that

$$AAu = ASu = SAu = Au = Bv = BBv = BTv = TBv.$$

Hence, Au is a common fixed point of A, B, S and T. Uniqueness of the common fixed point follows easily. This completes the proof.

Note 5.3. None of the above two theorems is a special case of each other.

6. Conclusion

Many research works have been done in symmetric spaces and fuzzy symmetric spaces. To establish the results in such spaces, we usually assume different properties like (W_3) , (W_4) , (H_E), (W^*), etc. For reference see [10, 11]. But this paper is realized with the help of implicit function relations and without considering such type of assumptions. As a result, to prepare this article, we do not need to consider the sequential approach. One can try to apply this approach to establish Hyers-Ulam stability for different types of functional equations.

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