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ML-Structures in the Repetitive Robust Control Systems

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Abstract: *Repetitive control is an effective strategy for periodic perturbance suppression via filtering their influence onto the control system, assuming that the period of perturbances is known. In this present paper, modified Memory Loop (ML) structures are proposed and analyzed in the repetitive robust control systems. To achieve that purpose, different methods are used for the elaboration of configuration and functional capabilities of the ML-structures, in order to set repetitive control systems in the class of robust control systems.*

Key words: *Repetitive Robust Control, Robust Stability and Performance, Robust Stability Margin, Robust Performance Margin, Convergence.*

1. Introduction

Repetitive control is an effective strategy for automation of technological objects characterized by periodic signal perturbances. Repetitive control systems can be distinguished from the traditional feedback systems, due to the fact that they contain *ML*-filter [1-10]. The structure of a system with repetitive regulator R_{RC} (containg basic regulator *R* and memory loop MLM_L) and plant *G* is shown in Fig. 1. The internal parametric disturbances are denoted by ξ. It is assumed that the control signal y^0 or any other signal perturbances of the system (v, f) show periodic character with known constant value of the period T_p. The basic *ML* is a cut-off filter in the system for frequency $\omega_p = 2\pi/T_p$ of harmonic signals with a period T_p coinciding with the period of v, f or y^0. The efficiency of the repetitive system (Fig. 1) is manifested in its ability to filer out the influence of these perturbances via the *ML*-filter. It contains a model of a delay e^{-pT} and stores the cut-off frequency ω_p. Its function is realized through an additive component ε^0 over the error ε due to its specific structure as a dynamic system. Equations (1)-(3) are valid, where the input for R is ε^*,

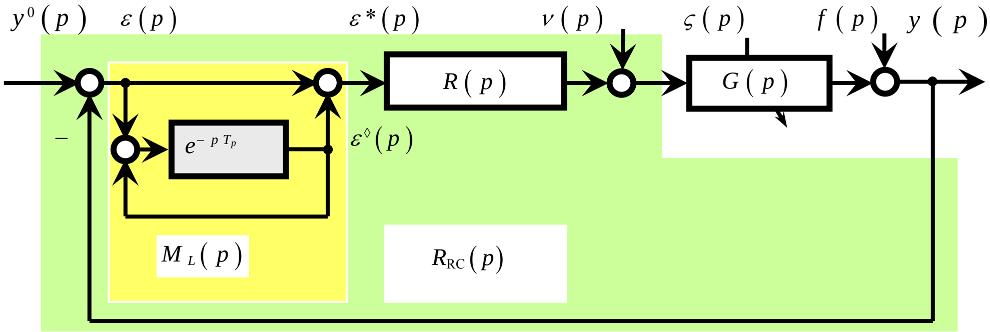


Fig. 1

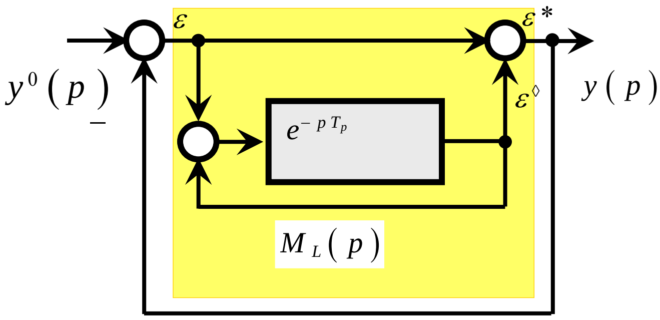


Fig. 2

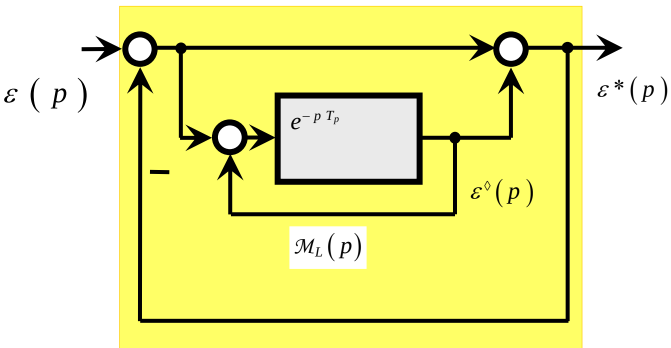


Fig. 3

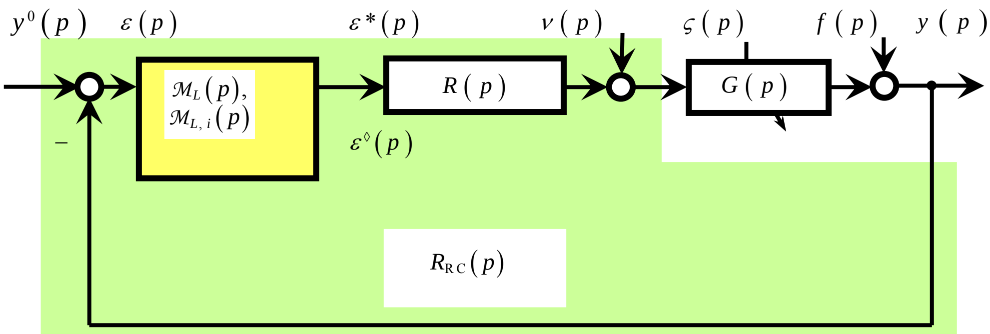


Fig. 4

(1) $$\varepsilon^*(p) = \varepsilon(p) + \varepsilon^{\Diamond}(p),$$

(2) $$\frac{\varepsilon^{\Diamond}(p)}{\varepsilon(p)} = \frac{e^{-pT_p}}{1 - e^{-pT_p}},$$

(3) $$M_L(p) = \frac{\varepsilon^*(p)}{\varepsilon(p)} = \frac{1}{1 - e^{-pT_p}}.$$

The properties of repetitive systems (Fig. 1) are based on *ML*-filter with memory M_L (3). This is obvious from the structure (Fig. 2) and the description of a single fictitious closed-loop repetitive system, from which, after equivalent transformation equations (4)-(5) are derived

(4) $$\Phi_{ML}(p) = \frac{y(p)}{y^0(p)} = \frac{1/\left(1 - e^{-pT_p}\right)}{1 + 1/\left(1 - e^{-pT_p}\right)},$$

(5) $$\lim_{\{p \to 0\}} \Phi_{ML}(p) = 1(p), \;\; \Phi_{ML}(p) \cong 1(p).$$

The properties of a single system with *ML*-structure (5) (Fig. 2) are similar to those of a standard repeater. Hence, the name of this class of systems is *repetitive systems or systems with "repetition"*. Their properties are preserved independently on the application points of the periodic disturbances (v, f, y^0). The characteristics of the single closed-loop system Φ_{ML} (5), which contains the basic MLM_L, are described further in this paper.

The disadvantages of the strategy for repetitive control are the requirements for:

- precise determination in advance of the period T_p of the signal disturbances v, f, y^0 and
- lack of fluctuations in the value of T_p.

In practice, the efficiency of repetitive control using basic *ML* can be achieved only if the period T_p of the signal disturbances is constant and known in advance. In [8-10] some systems are proposed for repetitive control based on real-time measurement of T_p and adjustment of the *ML* in respect to the fluctuations of the value of T_p.

In contrast to them, the present paper aims to propose efficient modifications of the basic *ML* towards $\mathcal{ML}$-structures, by means of which the repetitive systems with a fixed structure to be set in the class of robust control systems, and their disadvantages overcome. In order to achieve this goal several tasks are solved:

- modified functional and at fluctuations of the value of T_p *ML*-structures are created;
- the properties of the proposed new $\mathcal{ML}$-structures are analyzed;
- the design of $\mathcal{ML}$-filters with memory is analytically described;
- the efficiency of the proposed structures is assessed.

2. Improved $\mathcal{ML}$

In this paper elaborated $\mathcal{ML}M_L$ is proposed (Fig. 3) for the repetitive control systems (Fig. 4). It is distinguished from the basic ML-cut-off filter M_L [1-10] for the use of a model of a delay e^{-pT}, as the delay is a part of another structure (Fig. 1). The characteristics of the elaborated $\mathcal{ML}M_L$ as a bandwidth filter, are shown in Fig. 5. Equations (6) and (7) are valid and equations (8) and (9) are valid for the fictitious single closed-loop system that comprises the filter. The reason for the creating of $\mathcal{ML}$ (7) is to fulfill the requirements for:

- Stability of the $\mathcal{ML}$ as a component in the repetitive system;
- Use of $\mathcal{ML}$ bandwidth filter with memory allowing modifications needed to set repetitive systems in the class of robust systems.

A comparison between the characteristics of a single system with ML (3) and the elaborated $\mathcal{ML}$ (8) is shown in Fig. 5. The properties of the fictitious single system with $\mathcal{M}_L$ coincide with those of a standard repeater as a dynamic system, due to the higher convergence rate of series (7) compared to series (5).

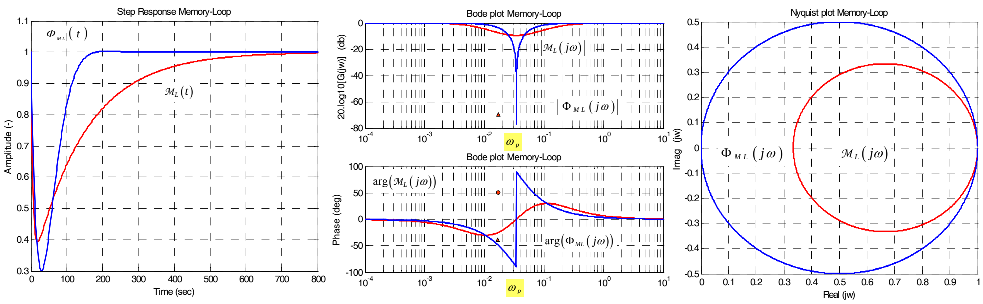
(6) $$\frac{\varepsilon^{\Diamond}(p)}{\varepsilon(p)}=\frac{e^{-pT_p}}{2-e^{-pT_p}},$$

(7) $$\mathcal{M}_L(p)=\frac{\varepsilon^*(p)}{\varepsilon(p)}=\frac{1}{2-e^{-pT_p}},$$

(8) $$\Phi_{\mathcal{ML}}(p)=\frac{y(p)}{y^0(p)}=\frac{1/\left(2-e^{-pT_p}\right)}{1+1/\left(2-e^{-pT_p}\right)},$$

(9) $$\Phi_{\mathcal{ML}}(p)=1(p),\quad \lim_{\{p\to 0\}}\mathcal{M}_L(p)=1(p).$$

In contrast to the basic ML cut-off filter M_L (3), the elaborated $\mathcal{ML}$ bandwidth filter with memory $\mathcal{M}_L$ (7) is a stable dynamic system. The quality control parameters of repetitive systems with $\mathcal{ML}$ are better than those of single systems with ML. The realization of the $\mathcal{ML}$ is possible when rational approximation of the delay e^{-pT} is applied using one of the well-known methods (one dimensional rows, chain polynomials, orthogonal or spherical polynomials, n-dimensional symmetric or asymmetric series) [11-12].



a) b) c)

Fig. 5

3. Modified $\mathcal{ML}_i$ -structures

On the basis of the $\mathcal{ML}$-bandwidth filter $\mathcal{M}_L$ (Fig. 3), 5 modified $\mathcal{ML}_i$-structures in the repetitive control systems are proposed in this paper (Fig. 4) – $\mathcal{M}_{L,1}$, $\mathcal{M}_{L,2}$, $\mathcal{M}_{L,3}$, $\mathcal{M}_{L,4}$, $\mathcal{M}_{L,5}$, where i is a structure index. The main reasons for their development are:

• to be achieved by $\mathcal{M}_{L,i}$ a typical characteristic (10) of a bandwidth filter in the presence of *horizontal profile in the module* | $\mathcal{M}_{L,i}$ $(j\omega)$| *of the angular frequency* $\Delta\omega_i$, symmetrical to the cut-off frequency ω_p with lower $\omega_{b,i}$ and upper $\omega_{h,i}$ limits ($\omega_{b,i} < \omega_p$; $\omega_{h,i} > \omega_p$),

$$(10)\quad \left\{ \begin{array}{c} \dfrac{d\,\left|\mathcal{M}_{L,i}(j\omega)\right|}{d\,\omega} = 0, \quad \forall\omega\in\Delta\omega_i; \\ \left|\mathcal{M}_{L,i}(j\omega)\right| = \text{const} \;<<< 1, \quad \forall\omega\in\Delta\omega_i; \\ \left|\mathcal{M}_{L,i}(j\omega)\right| \equiv 1, \quad \forall\omega\in[0,\omega_{b,i}],\; \Delta\omega\in[\omega_{b,i},\infty); \\ \Delta\,\omega_i = \left[\,\omega_{b,i}\,,\,\omega_{h,i}\right],\; \left(\omega_{b,i} < \;\omega_p < \;\omega_{h,i}\right) \end{array} \right\};$$

• to be determined the dynamical parameters of the adjustment of the $\mathcal{ML}_i$-structures, in order to prove the design of $\mathcal{ML}_i$ -structures and repetitive systems;

• to fulfill the requirements for: improving the quality and set repetitive systems in the class of robust control systems; achieving superior quantitative parameters of the quality of repetitive systems with $\mathcal{ML}_i$-structures in comparison with those with a single $\mathcal{ML}$.

3.1. *The first* structure $\mathcal{M}_{\mathcal{L},1}$ (11) is based on modifications of $\mathcal{M}_L$ (7), shown in Fig. 6. They represent augmentation of the cut-off frequency bandwidth of the $\mathcal{ML}$ (7) via consecutive connection of n dynamic links (n = 2, 3, 4, …) with a delay e^{-pT_p}.

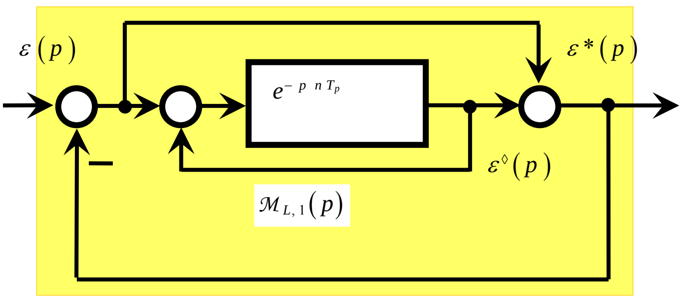
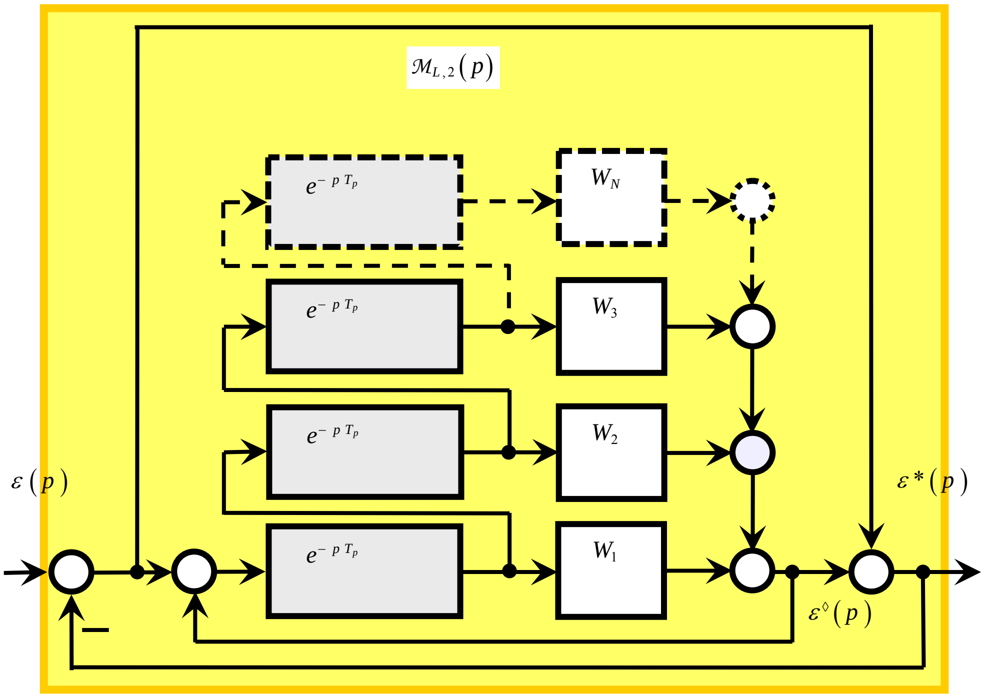
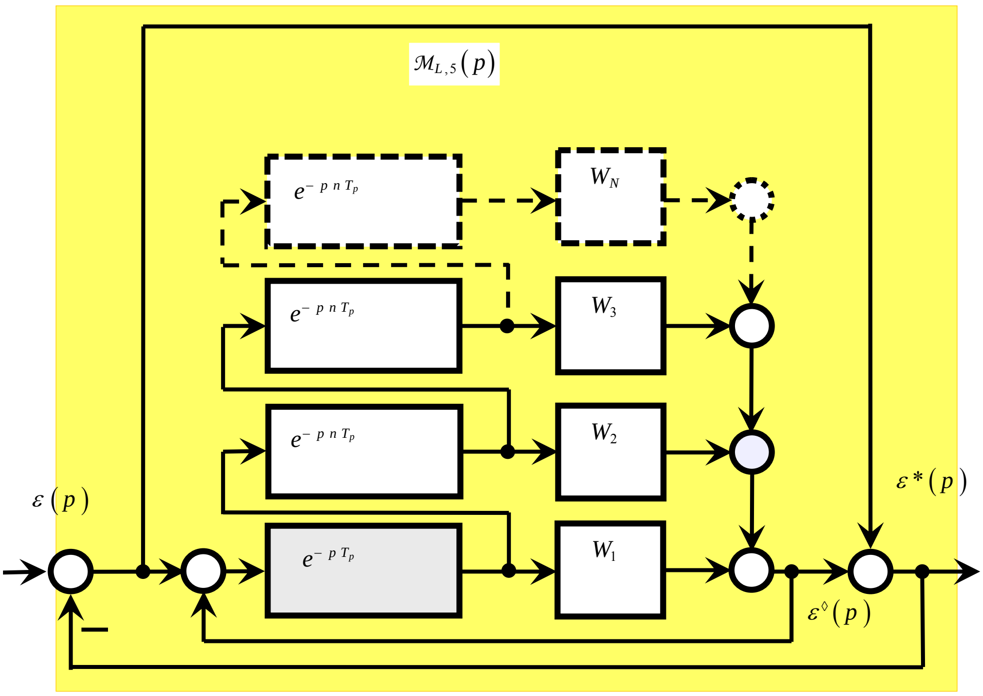


Fig. 6



$\mathcal{M}_{L,2}(p)$
e^{-pT_p}
e^{-pT_p}
e^{-pT_p}
e^{-pT_p}
W_N
W_3
W_2
W_1
$\varepsilon(p)$
$\varepsilon^*(p)$
$\varepsilon^\diamond(p)$

Fig. 7



$\mathcal{M}_{L,5}(p)$
e^{-pnT_p}
e^{-pnT_p}
e^{-pnT_p}
e^{-pT_p}
W_N
W_3
W_2
W_1
$\varepsilon(p)$
$\varepsilon^*(p)$
$\varepsilon^\diamond(p)$

Fig. 8

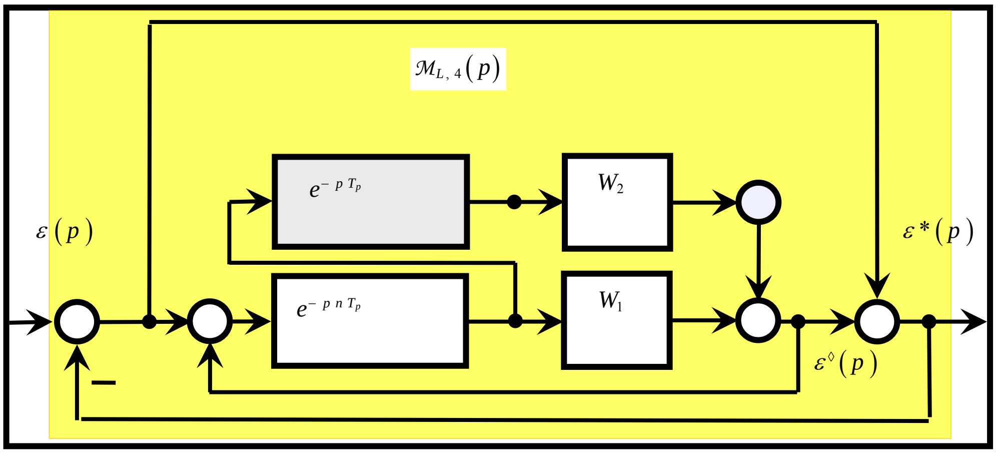


Fig. 9

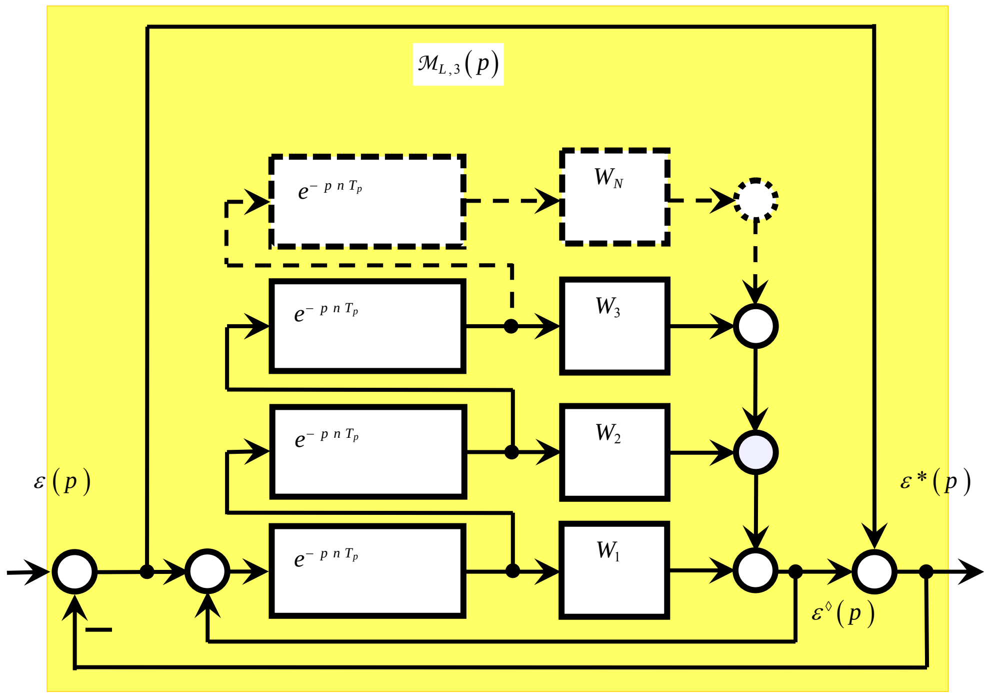


Fig. 10

(11) $$\mathcal{M}_{\mathrm{L},1}(p) = \frac{\varepsilon^*(p)}{\varepsilon(p)} = \left(2 - \prod_{q=2}^{n} \left(e^{-pT_p}\right)_q\right)^{-1} = \left(2 - e^{-pqT_p}\right)^{-1}.$$

3.2. ***The second*** structure $\mathcal{M}_{L,2}$ (12) is based on modifications of $\mathcal{M}_L$ (7), shown in Fig. 7, consisting of augmentation of the cut-off bandwidth frequency via combination of series and parallel connections of m links ($m = 2, 3, 4, \ldots$) with a delay e^{-pT_p}. This structure imposes requirement (13) on the sum of modules $|W_k|$ of the equivalent inertial links in the scheme;

(12) $$\mathcal{M}_{L,2}(p) = \frac{\varepsilon^*(p)}{\varepsilon(p)} = \left(2 - \sum_{k=1}^{m} W_k(p)\, e^{-pkT_p}\right)^{-1},$$

(13) $$\sum_{k=1}^{m} \left|W_k(j\omega)\right| \equiv 1,\ W_k(j\omega) = \kappa_k\,(j\omega T_k + 1)^{-1}.$$

3.3. ***The third*** structure $\mathcal{M}_{L,2}$ (14) is based on modifications of $\mathcal{M}_L$ (7), shown in Fig. 8, consisting of augmentation of the cut-off bandwidth frequency via parallel connection of *m* groups (*m* = 2, 3, 4, …) with *n* (*n* = 2, 3, 4, …) connected in series links with a delay e^{-pT_p}. This structure imposes requirement (13) on the sum of modules $|W_k|$ of the equivalent inertial links in the scheme;

(14) $$\mathcal{M}_{L,3}(p) = \frac{\varepsilon^*(p)}{\varepsilon(p)} = \left(2 - \sum_{k=1}^{m} W_k(p) \prod_{q=2}^{n} \left(e^{-pT_p}\right)_q\right)^{-1} = \left(2 - \sum_{k=1}^{m} W_k(p) e^{-pqT_p}\right)^{-1}.$$

3.4. ***The fourth*** structure $\mathcal{M}_{L,4}$ (15) is based on modifications of $\mathcal{M}_L$ (7), shown in Fig. 9, consisting of augmentation of the cut-off frequency bandwidth via parallel connection of links with a delay e^{-pT_p} and another group of *n* links (*n* = 2, 3, 4, …) connected in series with a delay e^{-pT_p}. The structure imposes requirement (13) on the sum of modules $|W_k|$;

(15) $$\begin{aligned}\mathcal{M}_{L,4}(p) &= \frac{\varepsilon^*(p)}{\varepsilon(p)} = \\ &= \left(2 - \left(W_0(p)e^{-pT_p} + W_1(p) \prod_{q=2}^{n} \left(e^{-pT_p}\right)_q\right)\right)^{-1} = \\ &= \left(2 - \left(W_0(p)e^{-pT_p} + W_1(p)e^{-pqT_p}\right)\right)^{-1}.\end{aligned}$$

3.5. ***The fifth*** structure $\mathcal{M}_{L,5}$ (16) is based on modifications of $\mathcal{M}_L$ (7), shown in Fig. 10, consisting of augmentation of the cut-off frequency bandwidth via parallel connection of links with a delay e^{-pT_p} and *m* groups (*m* = 2, 3, 4, …) of *n* links (*n* = 2, 3, 4, …) connected in series with a delay e^{-pT_p}. The structure imposes requirement (13) on the sum of modules $|W_k|$:

(16) $$\begin{aligned}\mathcal{M}_{L,5}(p) &= \frac{\varepsilon^*(p)}{\varepsilon(p)} = \\ &= \left(2 - \left(W_0(p)e^{-pT_p} + \sum_{k=1}^{m} W_k(p) \prod_{q=2}^{n} \left(e^{-pT_p}\right)_q\right)\right)^{-1} = \\ &= \left(2 - \left(W_0(p)e^{-pT_p} + \sum_{k=1}^{m} W_k(p)e^{-pqkT_p}\right)\right)^{-1}.\end{aligned}$$

4. Analysis of $\mathcal{ML}_i$-structure properties

The properties of the modified $\mathcal{ML}_i$-structures $\mathcal{M}_{L,\,1}$, $\mathcal{M}_{L,\,2}$, $\mathcal{M}_{L,\,3}$, $\mathcal{M}_{L,\,4}$, $\mathcal{M}_{L,\,5}$, designed for repetitive systems (Fig. 4), are determined by the dynamic parameters (Figs. 6-10)**:**

- $\Delta\omega_i$ – frequency bandwidth of the horizontal profile of the characteristic;
- $|\mathcal{M}_{\mathcal{L},i}|$ – value of the characteristic module for the bandwidth $\Delta\omega_i$ of the horizontal profile, which values are generally a function of the corresponding $\mathcal{ML}_i$ -structure, with a number of links or groups of links l with a delay and the value of cut-off frequency ω_i.

The structures $\mathcal{M}_{L,\,1}$, $\mathcal{M}_{L,\,2}$, $\mathcal{M}_{L,\,3}$, $\mathcal{M}_{L,\,4}$, $\mathcal{M}_{L,\,5}$ (Figs. 6-10) are simulated for a period T_p = 100 s. The links with a delay are approximated by symmetrical n-dimensional series of Padé [11-12**]**. The results of the model simulation of the $\mathcal{ML}_i$ -structures and the $\mathcal{ML}$-loop are shown in Fig. 11.

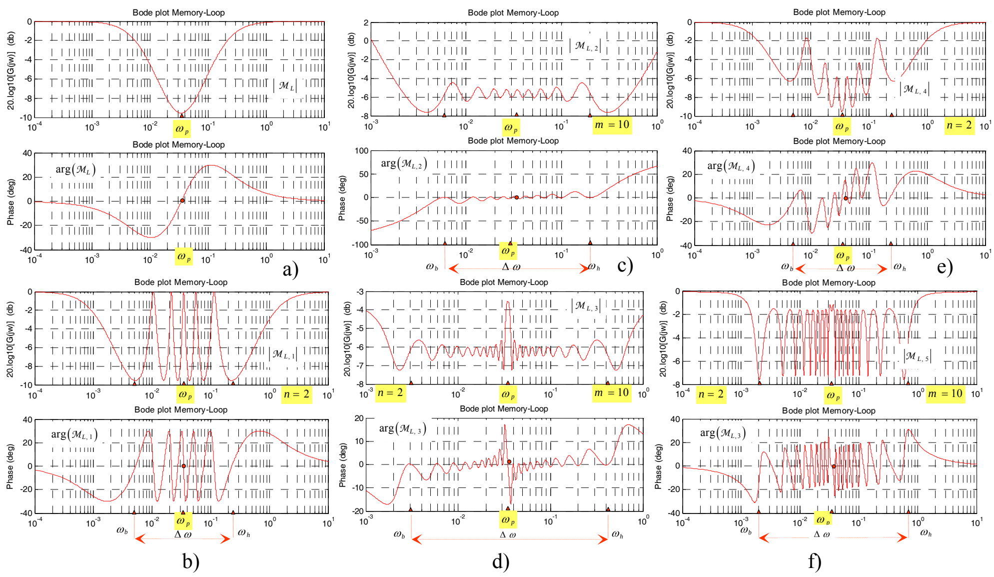


Fig. 11

The structures (Figs. 6-10) are modifications of $\mathcal{ML}$ (7), shown in Fig. 3. Via auxiliary links (regarding the $\mathcal{ML}$-structure) or groups of links with a delay e^{-pT_p}, the characteristics (Fig. 11) of the $\mathcal{ML}_i$-structures ($|\mathcal{M}_{L,\,i}|$) are profiled as band-pass filters (10). The number of links l being used or groups of links adds l-order of auxiliary harmonics (which are multiples of ω_p for $\omega = l\omega_p = 2l\pi/T_p$, l = 2, 3, 4, …) in the characteristic of the $\mathcal{ML}_i$-structure. These harmonics are placed symmetrically regarding ω_p in the characteristic. Thus, through the introduced l auxiliary harmonics, the *point* at $\omega = \omega_p$ of the $\mathcal{ML}$-structure characteristic *is transformed in a horizontal profile* for the frequency bandwidth $\Delta\omega_i$, $\Delta\omega_i = f(\omega_p, l_i)$ of the corresponding $\mathcal{ML}_i$-structure characteristic (Fig. 11). In logarithmic representation the bandwidth $\Delta\omega$ has a central point ω_p. Its width is proportional to

the number of links l being used or groups of links in the $\mathcal{M}$
horizontal profile of the corresponding $\mathcal{ML}_i$-structure augments its
(at the point ($\omega = \omega_p$) toward bandwidth cut-off properties in $\Delta\omega_i$.
even multiplier is chosen (l = 2, 4, 6, …), the characteristic of t
$\mathcal{ML}_i$-structure is closer to the desired form of standard bandwidth
shaping of the characteristic via auxiliary harmonics is poss
dynamics of the links or groups of links with a delay in the
approximated by n-dimensional series of Padé [11-12]. The realiz
or groups of links via approximation of the delay by one dim
Taylor, *Maclaurin*, *Butterworth* or by *chain* approximation, or by
approximation, or by *Chebyshev*-orthogonal polynomial approx
offer possibilities for shaping of the characteristics of the $\mathcal{M}$
auxiliary harmonics.

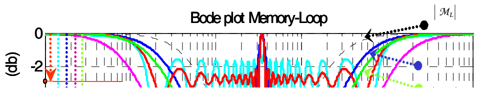


Fig. 12

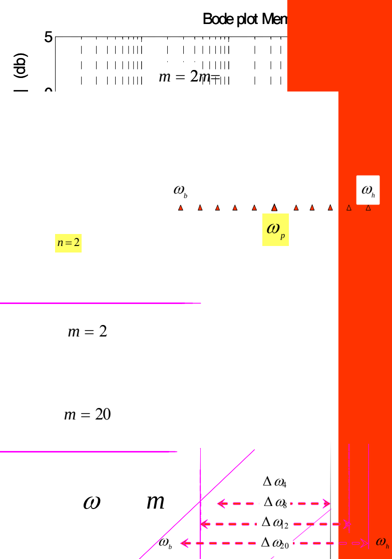


Fig. 13

A comparison between the characteristics of the proposed $\mathcal{ML}_i$-structures and $\mathcal{ML}$ (dashed line) for period T_p = 100 s and for n = 2, m = 10 (l = 10) is shown in Fig. 12. In Fig. 13 the characteristics are illustrated of parallel simulated $\mathcal{M}_{L,\,2}$ structures, modeled via m = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 groups of links with a delay. Their results (Figs. 12, 13) show that:

● the size of the frequency bandwidth $\Delta\omega$ is a function of the corresponding modified structural solution i and the number l_i of links with a delay being used, $\Delta\omega_i = f(\mathcal{M}_{L,\,i}, l_i)$; the size of $\Delta\omega_i$ grows when increasing l_i; for these particular examples, it is 0.035-0.375 rad/s;

● the dependence (17) approximates with high precision the relation between the used number l_i of links with a delay in the corresponding $\mathcal{ML}_i$-structure and the size of $\Delta\omega_i$, $\Delta\omega_i = \omega_{h,i} - \omega_{b,i}$, as it is used when designing the corresponding $\mathcal{ML}_i$-filter with memory in the regulators R_{RC} of the repetitive control system (Fig. 4):

$$l_i\left(\omega_p,\Omega_i\right)=\frac{\log_{10}\left(\omega_p-\omega_{b,\,i}\right)}{\log_{10}\left(\omega_{h,\,i}-\omega_p\right)}=\frac{\log_{10}\left(\omega_p-\omega_p\Omega_i^{-1}\right)}{\log_{10}\left(\omega_p\Omega_i-\omega_p\right)},$$

(17)

$$\left(\begin{array}{c}\omega_{b\,,\,i}<\omega_p<\omega_{h,\,i};\ \omega_{h,\,i}-\omega_{b,\,i}=\Delta\omega_i>0;\ 2\le l_i\le 20;\\ \Omega_i=\left(\omega_p/\omega_{b,\,i}\right)=\left(\omega_{h,\,i}/\omega_p\right);\ 1.5\le\Omega\le 3.0\end{array}\right);$$

● the cut-off properties of $\mathcal{ML}_i$-structures as bandwidth filters are determined by *the module of the horizontal profile* $|\mathcal{M}_{L,\,i}(j\omega)|$ (10) *for the frequency bandwidth* $\Delta\omega_i$, which is a function of the structure *i*; depending on the structure *i* in the region of the frequency bandwidth $\Delta\omega_i$, the module is the range from $|\mathcal{M}_{L,\,3}| = -3$ dB to $|\mathcal{M}_{L,\,3}| = -7$ dB; for each structure *i* the module of the horizontal profile in the region of $\Delta\omega_i$ is $|\mathcal{M}_{L,\,i}|$ = const, independent on the number of the links with a delay being used;

● each of the proposed modifications is a feasible solution of the assigned task to work out $\mathcal{ML}_i$-structures, which are efficient even at fluctuations of the value of T_p in the repetitive control systems (Fig. 4);

● under one and the same conditions, the properties of the $\mathcal{ML}_i$-structures as cut-off bandwidth filters differ; their properties are a function of**:** the structural solution for shaping the characteristic (Figs. 11, 12, 13); the used number *l* of auxiliary links or groups of links with a delay; the used method and order of the approximation of the delay; the value of the cut-off frequency ω_p.

5. Design of $\mathcal{ML}_i$-filters

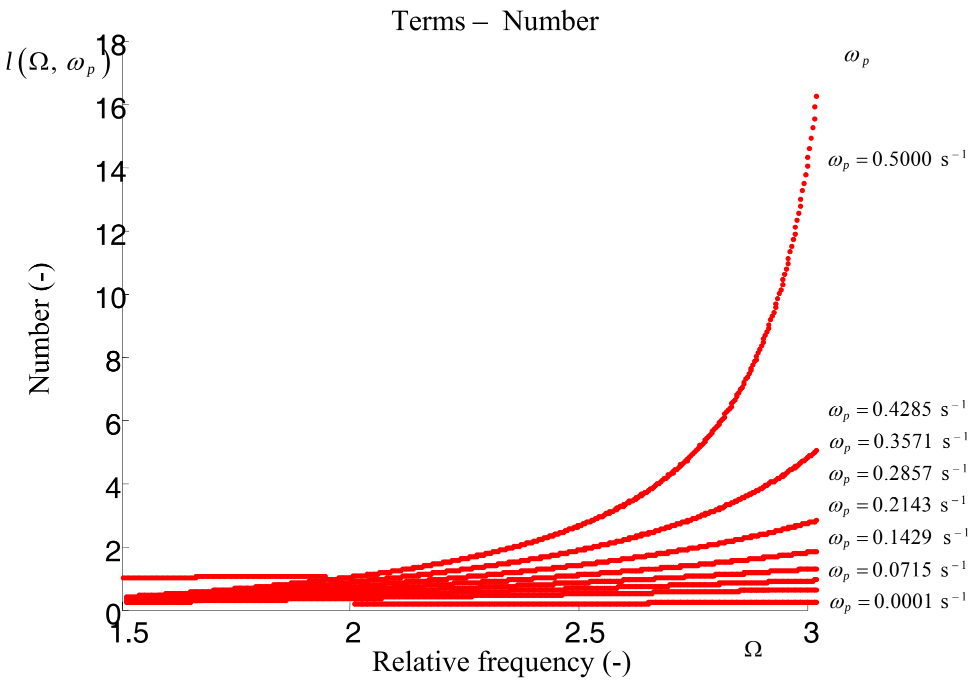
The dynamic properties and the values of the parameters for adjustment of the regulator *R* should obey the aim and criteria, presented to the system (Fig. 4) during its synthesis. They are not a function of the corresponding $\mathcal{ML}_i$-filter with memory in the regulator R_{RC}. In this respect, the design of the $\mathcal{ML}_i$-filter with memory in repetitive control systems (Fig. 4) is autonomous and it is not related to the synthesis of *R*. Taking into account the results of the analysis of the $\mathcal{ML}$-structure properties in the preceding section, visualized by $l(\Omega,\,\omega_p)$ (17) in Fig. 14, it follows that the *design method* of the $\mathcal{ML}_i$-filters determines the type of the structure *i* and the number *l* of links with a delay (17), and the design *algorithm* consists in:

● the choice of the most suitable of the modified structures (Figs. 6-10), according to the desired value of the module $|\mathcal{M}_{L,\,i}(j\omega)|$ of the characteristic for the bandwidth $\Delta\omega_i$ of the horizontal profile (Fig. 12);

● the determination of the number *l* of the links or groups of links with a delay, according to the desired size of the frequency bandwidth $\Delta\omega$ – analytically with given values of ω_p and Ω using equation (17) or graphically using Fig. 14 ($0.0001\ \mathrm{s}^{-1} \le \omega_p \le 0.5\ \mathrm{s}^{-1}$, $1.5\ \mathrm{s}^{-1} \le \Omega \le 3.0$);

● the choice of the method for approximation of the delay;

● the analytical design or the corresponding pre-programming in digital technical tools for automation of the components of the chosen $\mathcal{ML}_i$-filter structure.



ω_p	ω_p	T_p	T_p
rad/s	s^{-1}	s	min
0.000628	0.0001	10000.0	166.66
0.449110	0.0715	13.9832	0.2330
0.897591	0.1429	6.9965	0.1166
1.346073	0.2143	4.6654	0.0777
1.794555	0.2857	3.4994	0.0583
2.243037	0.3571	2.7997	0.0466
2.691518	0.4285	2.3332	0.0388
3.140000	0.5000	2.0000	0.0333

Fig. 14

6. Efficiency of $\mathcal{ML}_i$-structures

In the present paper the efficiency of the proposed modified structures (Figs. 6-10) is determined by the fact to what extent each of them under one and the same altered conditions meets better the requirements for:

- improving the quality parameters of the repetitive control systems (Fig. 4);
- setting repetitive systems in the class of robust control systems;

● supremacy of the quantitative quality parameters of the repetitive systems with $\mathcal{ML}_i$-structures over those with $\mathcal{ML}$ and over the classical systems with a standard regulator and feedback.

In order to assess the quality of the systems (including those for repetitive control with $\mathcal{ML}_i$-structures) on the basis of a particular numerical example of an industrial object (defined by a nominal model *G** (18) and perturbed on the upper limit $G^{\blacklozenge}$ (19) model), as a criterion a critical aperiodic transient process with period T_p = 400 s of the perturbations, the following are designed:

●*a*● a system comprising a standard $\mathcal{PID}$-controller (20);

●*b*● repetitive (Fig. 4) systems (21)-(27) with a $\mathcal{PID}$-controller (20) with $\mathcal{ML}$ and $\mathcal{ML}_i$-structures (Figs. 6-10), for T_p = 400 s and with approximation of the delay by *n*-dimensional symmetric series (28) for *n* = 2, *m* = 4;

●*c*● repetitive (Fig. 4) systems (21)-(27) with a $\mathcal{PID}$-controller (20) with $\mathcal{ML}_2$-structure (Fig. 7), for T_p = 400 s and with approximation of the delay: by *n*-dimensional symmetric series (28), by chain polynomials (29), by one-dimensional *Butterworth* series (30), by spherical *Bessel* polynomials (31); by orthogonal *Chebychev* polynomials (32) for *n* = 2, *m* = 10;

●***d***● repetitive (Fig. 4) systems (21)-(27) with a $\mathcal{PID}$-controller (20) with $\mathcal{ML}$-and $\mathcal{ML}_2$-structure (Fig. 7), for T_p = 400 s and with approximation of the delay by *n*-dimensional symmetric series (28) for *n* = 2, *m* = 2, 4, 6, 8, 10:

(18) $$G^*(p) = 0.15\,(1+4p)^{-1}e^{-10p},$$

(19) $$G^{\blacklozenge}(p) = 0.24(1+3p)^{-1}e^{-10p},$$

(20) $$R(p) = 2.35\,(1+8p)(2p+1)\big(8p(0.4p+1)\big)^{-1},$$

(21) $$\mathcal{M}_L(p) = \left(2-e^{-pT_p}\right)^{-1} = \left(2-R_{2,2}^{(400)}(p)\right)^{-1},$$

(22) $$\mathcal{M}_{L,1}(p) = \left(2-e^{-piT_p}\right)^{-1} = \left(2-\left(R_{2,2}^{(400)}(p)\right)^{i}\right)^{-1},$$

(23) $$\mathcal{M}_{L,2}(p) = \left(2-\sum_{k=1}^{10}W_k(p)R_{2,2}^{(400)}(p)\right)^{-1},$$

(24) $$\mathcal{M}_{L,3}(p) = \left(2-\sum_{k=1}^{10}W_k(p)\left(R_{2,2}^{(400)}(p)\right)^{10}\right)^{-1},$$

(25) $$\mathcal{M}_{L,4}(p) = \left(2-\left(W_0(p)R_{2,2}^{(400)}(p)+W_1(p)\left(R_{2,2}^{(400)}(p)\right)^{10}\right)\right)^{-1},$$

(26) $$\mathcal{M}_{L,5}(p) = \left(2-\left(W_0(p)R_{2,2}^{(400)}(p)+\sum_{k=1}^{10}W_k(p)\left(R_{2,2}^{(400)}(p)\right)^{10}\right)\right)^{-1},$$

(27) $$W_k(p) = \kappa_k\,(0.0001p+1)^{-1},$$

(28) $$e^{-p\tau} \mathrel{\hat{=}} R_{n,n}^{(\tau)}(p) = \frac{\displaystyle\sum_{k=0}^{n} \frac{(-1)^k\,\Gamma(2n-k+1)\,\Gamma(n+1)}{\Gamma(2n+1)\,\Gamma(k+1)\,\Gamma(n-k+1)}(p\tau)^k}{\displaystyle\sum_{k=0}^{n} \frac{\Gamma(2n-k+1)\,\Gamma(n+1)}{\Gamma(2n+1)\,\Gamma(k+1)\,\Gamma(n-k+1)}(p\tau)^k},$$

(29) $$e^{-p\tau} \mathrel{\hat{=}} R_{0,n}(p) \equiv \prod_{k=1}^{n} k^k\left(k+(p\tau)\right)^{-k},\ n \geq 1,$$

(30) $$e^{-p\tau} \mathrel{\hat{=}} R_{0,n}(p) = \frac{1}{B_n(p)} = \frac{1}{b_0 + b_1(p\tau) + b_2(p\tau)^2 + \,.\,.\, + b_n(p\tau)^n},$$

(31.a) $$R_{n,x}(x) = \sum_{k=0}^{n} \frac{\Gamma(n+k+1)(x/2)^k}{\Gamma(k+1)\Gamma(n-k+1)},\ n \geq 0,$$

(31.b) $$R_n(p) = H\{R_{n,x}(x)\} = H\left\{\left(\sum_{k=0}^{n} \frac{(\Gamma(n+k+1))\,(x/2)^k}{\Gamma(k+1)\,\Gamma(n-k+1)}\right)\right\} = \\ = \sum_{k=0}^{n} \frac{(\Gamma(n+k+1))\,(p\tau/2)^k}{\Gamma(k+1)\,\Gamma(n-k+1)},\ n \geq 0,$$

(31.c) $$e^{-p\tau} \mathrel{\hat{=}} \frac{R_0(p)}{R_n(p)} = \frac{R_0(p)}{\left(\displaystyle\sum_{k=0}^{n} \frac{(\Gamma(n+k+1))(p\tau/2)^k}{\Gamma(k+1)\Gamma(n-k+1)}\right)} = \\ = \frac{R_0(p)}{b_0 + b_1(p\tau) + b_2(p\tau)^2 + \ldots + b_n(p\tau)^n},\ n \geq 0,$$

(32.a) $$R_{n,x}(x) = 2^{(n-1)} \prod_{k=1}^{n}\left(x - \cos\left(\frac{(2k-1)\pi}{2n}\right)\right),\ n \geq 0,$$

(32.b) $$R_n(p) = H\{R_{n,x}(x)\} = H\left\{2^{(n-1)}\prod_{k=1}^{n}\left(x - \cos\left(\frac{(2k-1)\pi}{2n}\right)\right)\right\} = \\ = 2^{(n-1)}\prod_{k=1}^{n}\left((2p\tau+1) - \cos\left(\frac{(2k-1)\pi}{2n}\right)\right),$$

(32.c) $$e^{-p\tau} \mathrel{\hat{=}} \frac{R_0(p)}{R_n(p)} = \frac{R_0(p)}{2^{(n-1)}\displaystyle\prod_{k=1}^{n}\left((2p\tau+1) - \cos\left(\frac{(2k-1)\pi}{2n}\right)\right)} = \\ = \frac{R_0(p)}{b_0 + b_1(p\tau) + \ldots + b_n(p\tau)^n},\ n \geq 0.$$

Synthesized systems according to ***“a, b, c, d”*** are modeled. These models are simulated in parallel, and the results are presented as follows. For each system, considering equations (18)-(19), the following is done:

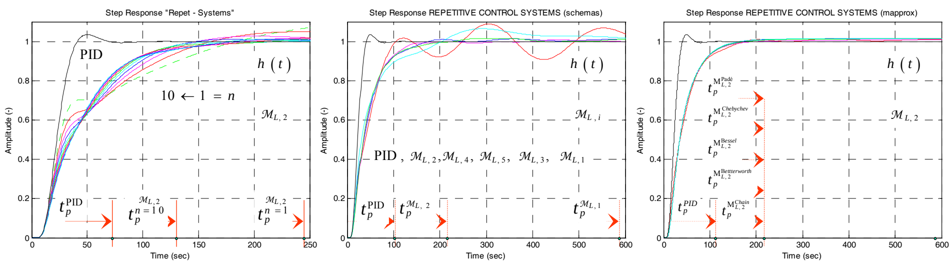
● ***determined*** time for regulation t_p (Fig. 15) in respect to the transfer function $h(t)$ in the closed-loop systems ***“a, b, c, d”*** using the nominal model G^* (18);

● ***assessed*** stability (Fig. 16) by means of the reserve of stability in module GM (33.a) and in phase PM (33.b) using G^* (18) of the systems ***“a, b, c, d”***,

$$\text{(33.a)}\quad \mathrm{GM} = 20\,\log_{10}\left|W^*\left(j\omega_\pi\right)\right|\ \mathrm{dB},\ \ \omega_\pi : \arg W^*\left(j\omega_\pi\right) \equiv \pi,$$

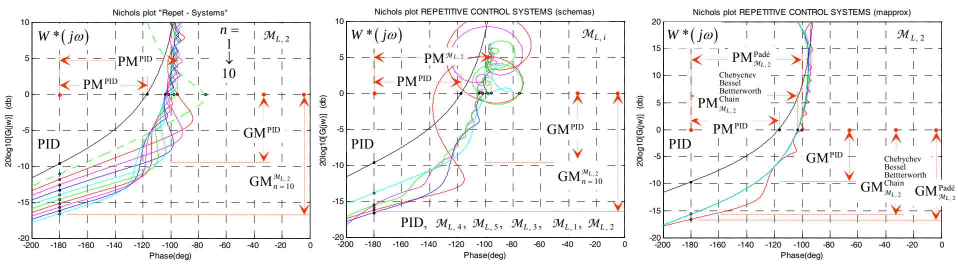
$$\text{(33.b)}\quad \mathrm{PM} = -\left(\arg\left(W^*\left(j\omega_0\right)\right) + 180^\circ\right)\mathrm{deg},\ \ \omega_0 : \left|W^*\left(j\omega_0\right)\right| = 1,$$

where: ω_π-value of the frequency ω, for which the argument of the open-loop system has a value of 180° (ω_π: $\arg(W^*(j\omega_\pi)) \equiv \pi$ – the first intersection point of the hodograph $W^*(j\omega)$ with the negative part of the real axis when increasing the value of frequency in the interval $\omega\in[0, \infty)$ in polar coordinate representation); ω_0 – the value of the frequency ω, for which $|W^*(j\omega)|$ obtains the value of one ($\omega_0|W^*(j\omega_0)|=1$ – the first intersection point of the hodograph $|W^*(j\omega_0)|$ with the unit circle when increasing the value of the frequency in the interval $\omega\in[0, \infty)$ and $W^*(j\omega)$ enters the unit circle in polar coordinate representation).



a) b) c)

Fig. 15



a) b) c)

Fig. 16

Table.1

System	GM	PM	t_p	System	GM	PM	t_p	System	GM	PM	t_p
	dB	deg	s		dB	deg	s		dB	deg	s
PID	–8	–62	76	PID	–8	–62	76	PID	–8	–62	76
$\mathcal{M}^{n=2}_{\mathcal{L},2}$	–12	–75	238	$\mathcal{M}_{\mathcal{L},2}$	–17	–105	200	$\mathcal{M}^{\text{Padé}}_{\mathcal{L},2}$	–17	–105	200
$\mathcal{M}^{n=4}_{\mathcal{L},2}$	–14	–82	232	$\mathcal{M}_{\mathcal{L},4}$	–16	–96	358	$\mathcal{M}^{\text{Chain}}_{\mathcal{L},2}$	–16	–98	200
$\mathcal{M}^{n=6}_{\mathcal{L},2}$	–15	88	225	$\mathcal{M}_{\mathcal{L},5}$	–15	–91	426	$\mathcal{M}^{\text{Betterworth}}_{\mathcal{L},2}$	–16	–98	200
$\mathcal{M}^{n=8}_{\mathcal{L},2}$	16	96	220	$\mathcal{M}_{\mathcal{L},3}$	–14	–88	580	$\mathcal{M}^{\text{Bessel}}_{\mathcal{L},2}$	–16	–98	200
$\mathcal{M}^{n=10}_{\mathcal{L},2}$	–17	–105	200	$\mathcal{M}_{\mathcal{L},1}$	–13	–82	650	$\mathcal{M}^{\text{Chebychev}}_{\mathcal{L},2}$	–16	–98	200
Number of the links or groups of links with a delay				Type of the $\mathcal{ML}$-structure				Method for delay approximation			

Regarding the reserves of stability, the *results* (Figs. 15, 16), summarized in Table 1, *confirm* the advantage of the repetitive systems (21)-(27) over the standard systems (20), but it is at the expense of increased time for adjustment t_p. Its magnitude depends on the structure *i* and the number of used links *l* in the structure. The time for adjustment t_p is not a function of the method used for approximation of the delay.

● *Nyquist*-robust analysis *is performed* of the characteristics of the open-loop systems ***“a, b, c, d”*** (34)-(35) and the robust stability and robust quality of the repetitive systems with $\mathcal{ML}_i$-structures are proven. The functional set Π (34) models the uncertainty in the real object under control, where $\Pi(j\omega) \in \mathrm{G}(j\omega)$. It is defined by the variations Δ*G* of the characteristic of the real plant *G* around its *nominal model G**. The maximum value of this re-parametrization and/or restructuring $\overline{\ell}_a$ (respectively $\overline{\ell}_m$) determines the so called *“perturbed on upper limit”* model of the plant $G^{\blacklozenge}$. The variations of *G* are the reason for changes in the characteristic of the system, which are modeled by a functional set π (35). The *Nyquist*-analysis method represents graphically the form of π through a family of circles $\pi(j\omega_i)$. The centers of $\pi(j\omega_i)$ are the featuring points ω_i of the hodograph of the *nominal open-loop system* $W^*(j\omega_i) = R(j\omega_i)G^*(j\omega_i)$. For each value ω_i of the frequency ω, the corresponding circle $\pi(j\omega_i)$ is the locus, which can be occupied by the featuring point $\omega = \omega_i$ as a result of the variations of the real system $W(j\omega_i) = R(j\omega_i)G(j\omega_i)$, from $W^*(j\omega_i)$ to *“the pertubed on upper limit”* system $W^{\blacklozenge}(j\omega_i) = R(j\omega_i)G^{\blacklozenge}(j\omega_i)$. The radius $r^0(\omega_i)$ of the circle $\pi(j\omega_i)$, corresponding to each value of ω_i, is determined by (36), and the parametric equation of the circumference $\pi^0(jw_i)$, which describes the circle $\pi(jw_i)$, is (37):

$$(34)\quad \Pi(j\omega) = \left\{ \begin{aligned} &\Delta G(j\omega) : \left|G(j\omega) - G^*(j\omega)\right| \le \overline{\ell}_a(\omega),\ \omega \in [0;\infty) \\ &\Delta G(j\omega) : \frac{\left|G(j\omega) - G^*(j\omega)\right|}{\left|G^*(j\omega)\right|} \le \overline{\ell}_m(\omega),\ \overline{\ell}_m(\omega) = \frac{\overline{\ell}_a(\omega)}{\left|G^*(j\omega)\right|} \end{aligned} \right\},$$

(35) $$\pi(j\omega) \in W(j\omega),\ \omega \in [0;\infty),$$

(36) $$r^0(\omega_i) = |l_a(\omega_i)R(\omega_i)| = |l_m(\omega_i)R(\omega_i)G^*(\omega_i)|,$$

(37) $$\pi^0(j\omega_i) = \begin{cases} \mathrm{Re}^0(\omega_i) = \mathrm{Re}^*(\omega_i) + r(\omega_i)\cos\Omega,\ \ \Omega \in [0,\infty) \\ \mathrm{Im}^0(\omega_i) = \mathrm{Im}^*(\omega_i) + r(\omega_i)\sin\Omega,\ \ \Omega \in [0,\infty) \end{cases}.$$

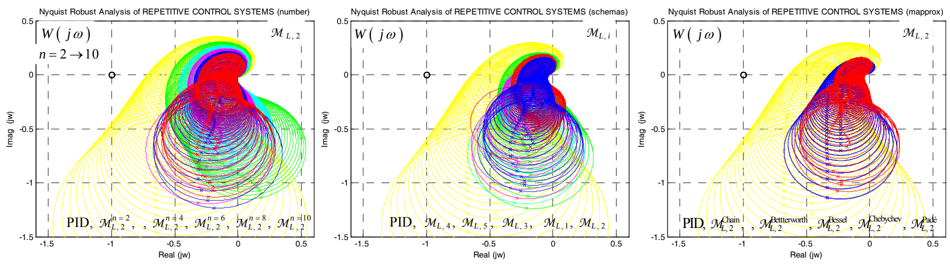
The system is stable for the whole range Π of the variations ΔG (in this respect, robustly stable), if the set $\pi(j\omega)$**,** which corresponds to Π, does not contain the point $(-1, j_0)$ for any of the values of the frequency ω in the range $\omega\in[0, \infty)$**.** This is possible only in the cases, when the distance between any point $\omega = \omega_i$ of $W^*(j\omega)$**,** determined by the value of the module $|1 + G^*(\omega_i)R(\omega_i)|$ and the point $(-1, j_0)$ is greater than the radius $r^0(\omega_i)$:

(38) $$r^0(\omega_i) = |G^*(\omega_i)R(\omega_i)|\,\overline{\ell}_m(\omega_i).$$

The requirement for achieving robust stability of the system toward all points from $\pi(j\omega)$ (35) in these cases is affected by (39), (40) (Fig. 17) for the variations (18)-(19);

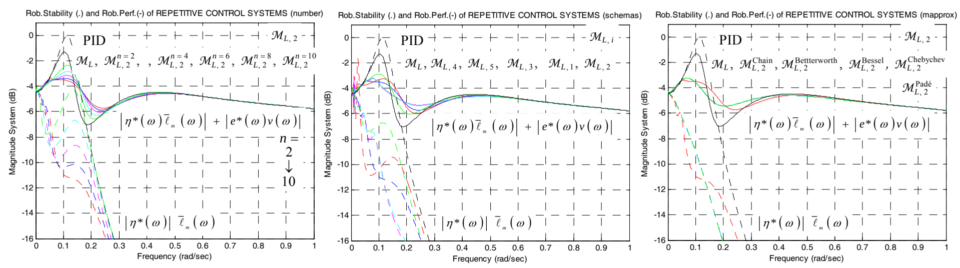
(39) $$|1 + G^*(\omega)R(\omega)| > r^0(\omega),\ \forall\omega,\ ,$$

(40) $$|1 + G^*(\omega)R(\omega)| > |G^*(\omega)R(\omega)|\,\overline{\ell}_m(\omega),\ \forall\omega.$$



a) b) c)

Fig. 17



a) b) c)

Fig. 18

● Robust analysis *is done* on the characteristics of the response of the closed-loop systems ***"a, b, c, d"*** and for the variations (18)-(19) the robust stability and the robust quality of the repetitive systems with $\mathcal{ML}_i$-structures are proven (Fig. 18). The closed-loop systems are robustly stable and with robust performance if the requirements toward the sensitivity functions e^* and the complementary sensitivity functions η^* are fulfilled:

(41) $$\left|\eta^*(\omega)\right|\overline{\ell}_m(\omega) < 1,\ \forall\omega,$$

(42) $$\left|\eta^*(\omega)\overline{\ell}_m(\omega)\right| + \left|e^*(\omega)v(\omega)\right| < 1,\ \forall\omega.$$

The results (Figs. 17, 18) *prove* that in the assigned by (18)-(19) region, the repetitive systems (Fig. 4) with $\mathcal{ML}$ and with $\mathcal{ML}_i$-structures ***"b, c, d"*** fulfill requirements (39)-(42) and they are robustly stable and with robust quality. For the system ***"a"*** with a standard $\mathcal{PID}$-controller (20) is proven that it does not fulfill requirements (39)-(42). It is an essential advantage of the repetitive systems and confirms the fact that by means of the proposed $\mathcal{ML}_i$-structures the repetitive systems ***"b, c, d"*** (Fig. 4) are set in the class of the robust control systems.

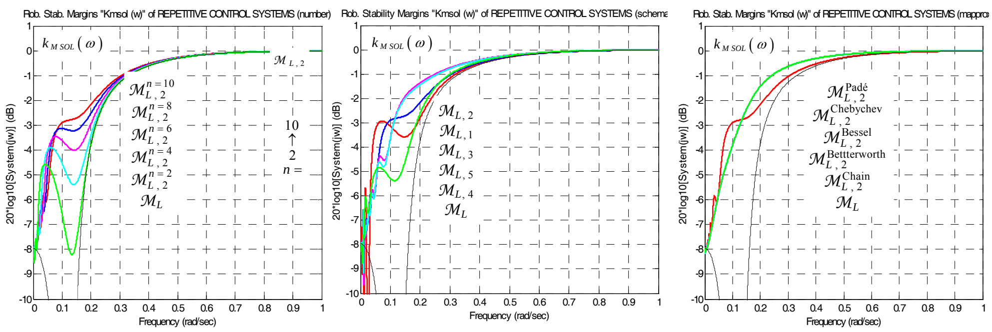
● the reserve of robust stability is *determined* [11] on the characteristics of the open-loop k_{MSOL} (43) repetitive systems ***"b, c, d"*** (Fig. 19), and on the characteristics of the closed-loop k_{MSCL} (44) repetitive systems ***"b, c, d"*** (21)-(27) for the variations (18)-(19):

(43) $$k_{\mathrm{MSOL}}(\omega) = \frac{r(j\omega)}{\left|1 + R(j\omega)G^*(j\omega)\right|} \le 1,\ \forall\omega,\ \omega\in[0,\infty),$$

(44) $$k_{\mathrm{MSCL}}(\omega) = 1 - \left|\eta(\omega)\ell_m(\omega)\right| \ge 0,\ \forall\omega,\ \omega\in[0,\infty).$$

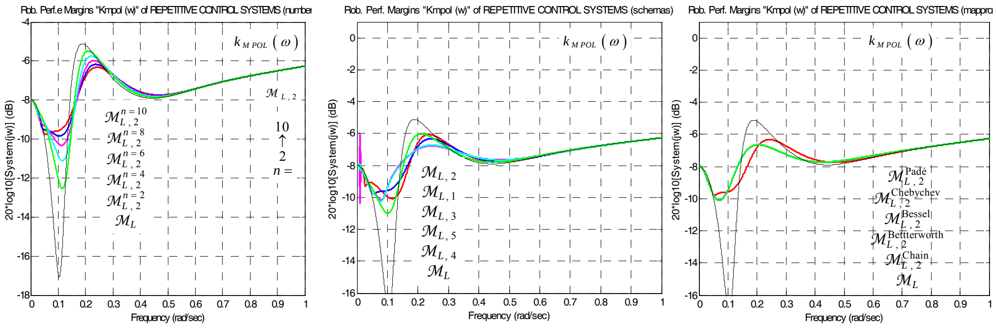
● the reserve [11] of robust performance k_{MPOL} (45) (Fig. 20) of the repetitive systems ***"b, c, d"*** (Fig. 4) (21)-(27) *is determined* for the variations (18)-(19);

(45) $$k_{\mathrm{MPOL}}(\omega) = \frac{\left|1 + R(j\omega)G^*(j\omega)\right| - r(j\omega)}{\left|1 + R(j\omega)G(j\omega)\right|} = \frac{\left|1 + R(j\omega)G^*(j\omega)\right| - r(j\omega)}{\left|1 + R(j\omega)G^{\blacklozenge}(j\omega)\right|} \le 1,\ \forall\omega,\ \omega\in[0,\infty).$$



a) b) c)

Fig. 19



a) b) c)

Fig. 20

The reserves of robustness (43)-(45) are quantitative assessment of the the capability of the synthesized system, preserving its robust properties to counteract efficiently the parametric or structural disturbances outside the range of re-parametrization and restructuring (18)-(19), specified during the design process. The greater the value of this quantitative assessment for a particular system is, thw greater its capabilities will be to counteract efficiently the disturbances outside the designed norms.

In contrast to the reserves *GM* and *PM* (33) of stability of the systems with fixed structure and parameters (quantitatively determined as scalars), the reserves of robustness (43)-(45) are determined as *functions of the frequency and they are not scalars quantities*. They are quantitative assessment of the robust properties of control systems for industrial plants, which analytical model $\Pi(j\omega) \in \mathcal{G}(j\omega)$ (34) varies parametrically and structurally as a function of the perturbances ξ (Fig. 4) of apriori uncertainty.

The reserve of robust stability $k_{\mathrm{MSOL}}(\omega)$ (43) is determined either as a ratio for each value of the frequency $\omega = \omega_i$, of the radius $r^0(\omega_i)$ of the circles $\pi^0(j\omega_i)$, representing the apriori uncertainty, and the distance $|1+G^*(\omega_i)R(\omega_i)|$ from the corresponding point of the hodograph of the nominal open-loop system $W^*(j\omega_i)$ to the point $(-1, j_0)$, or as $k_{\mathrm{MSCL}}(\omega)$ (44) – functional dependence of the frequency of the positive, complementary to unity module $\left|\eta(\omega)\ell_m(\omega)\right|$ of the closed-loop system.

The reserve of robust quality $k_{\mathrm{MPOL}}(\omega)$ (45) is determined for each value of the frequency $\omega = \omega_i$ as a ratio of the difference between the distance from $W^*(j\omega)$ to the point $(-1, j_0)$ and the radius $r^0(\omega_i)$ of the circles, representing apriori uncertainty, and the distance from $W^{\blacklozenge}(j\omega)$ to the point $(-1, j_0)$, defined by the value of the module $\left|1+G^{\blacklozenge}(\omega_i)\, R(\omega_i)\right|$.

The reserves of robustness (41) and (43) are shown in Figs. 19 and 20 for the analyzed repetitive systems ***"b, c, d"*** (Fig. 4). *The results confirm the advantages* of the systems with $\mathcal{ML}_r$-filters over the repetitive systems with $\mathcal{ML}$-filter.

7. Conclusion

The novelty of this work is in the achieved original and new results, methods and proves, giving rational solutions in the development and application of repetitive control systems**:**

● Elaborated $\mathcal{ML}$- and modified $\mathcal{ML}_r$-stable structures are proposed as bandwidth cut-off filters with memory and with horizontal profile in repetitive control systems, which are efficient even at variations of the period T_p of the disturbances; also a method for shaping the characteristics via additive harmonics is proposed; and it is proven that the proposed structures set repetitive systems in the class of robust control systems.

● The properties of these systems are analyzed and the dynamic parameters are determined for adjustment of $\mathcal{ML}_r$-filters, their dependence upon the parameters of the corresponding structures and cut-off frequency is defined and analyzed analytically;

● A method and an algorithm for the design of $\mathcal{ML}_r$-filters are developed for the synthesis of repetitive control systems;

● The efficiency of the proposed $\mathcal{ML}_r$-stable structures is proven through:

– Comparative assessment of the quantitative parameters of the quality (determined by the time for adjustment and the reserves of stability),

– Proof of the robust properties,

– Comparative assessment of the quantitative parameters of the robust properties (determined by the reserves of robust stability and the robust performance) of the standard and repetitive systems under one and the same conditions;

● The advantages of the systems with $\mathcal{ML}_r$-filters over repetitive systems with $\mathcal{ML}$-filter are confirmed;

● A number of numerical examples are solved, confirming the efficiency of the proposed methods in the design of $\mathcal{ML}_r$-filters and repetitive control systems.

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