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Height Target Estimation in a Three Positioned Radar System

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Abstract: *The target height error estimation is evaluated in the present paper. The principles of three-positioned radiolocation are used. The three-positioned radar system sensibility versus radar accuracy or versus the geometrical parameters of the target is investigated. The mathematical dependences of the mean and standard deviation values of the target coordinates are obtained. These dependences allow the preliminary finding of some parameters of the three-positioned radar in order to increase the measurement accuracy.*

Keywords: *height target estimation, measurement accuracy, radar system sensibility*

1. Introduction

The first experimental radar systems developed in the USA and Great Britain are two-positioned. The transmitter and receiver are positioned separately at a range as long as the target range. In 1936 an antenna switch is developed and the co-work of receiver and transmitter becomes possible. The two-positioned radars have almost not been used since then. In the beginning of the 50's the interest in the three-positioned radar systems is raised again. This interest grows rapidly in recent years [1-3].

The advantage of the three-positioned radiolocation, if compared to the classical one-positioned radiolocation [4-6], is the high efficiency of using the information, contained in the electromagnetic space structure. It is well known that the reflected signal diffracts in the whole space around (excluding the shielded areas). In contrast to this, the one-positioned radars receive target information only from a small part of the electromagnetic field, which corresponds to the aperture of the receiver antenna. In three-positioned radars, information is gathered from some different parts of the target reflected electromagnetic field. Therefore, the information and the interference robust of the radar system are increased [7].

Most of the civil and military radars detect only azimuth and the target range. Different methods are used for target height detection; phased antenna arrays or multi element antennas [5].

The use of the three-positioned radiolocation increases the information that could be gathered from the target. This information could be used for target height detection [8]. Caspers obtains mathematical dependencies for target height estimation by using two or three antennas [3].

Our task in this paper is to find sufficiently cheap and simple for technical realization system for determination of the three co-ordinates of a flying target. The review of similar research results and our research findings reveal that the use of a three-positioned system of two-coordinate radars is such a decision.

We use these dependencies for obtaining the target height error estimation by means of standard detection with two-coordinate synchronized radars. The task is solved for the case when the three radars detect and estimated the target parameters simultaneously and deliver them to a center for data unification. The simulation results are obtained by MATLAB modeling, when the projection of the target is inside and outside the triangle formed by the radars. The mean square error of the target height is lower in the cases when the projection of the target is located into the triangle. We receive new analytical results for the mean value and the standard deviation of a three-positioned radar system.

The achieved results could be applied for height target estimation in multi-static radars using existing communication and radar networks.

2. Principles of the three-positioned radiolocation

The use of a three-positioned radar system allows target coordinate detection by only measuring the three target distances. Complete synchronization of work and the distances between the radars (the particular radar coordinates) are needed.

For coordinate target estimation in a three-positioned system, the distances between the radars and the three-target ranges should be known (r_1, r_2 and r_3). The target coordinates when using a three-positioned radar system can be detected with [3]. The coordinates (x, y, z) are obtained according to the diagram presented in Fig.1:

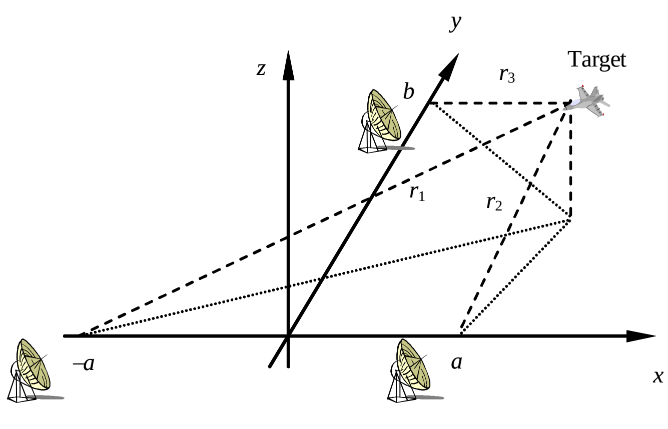

Fig. 1. Diagram of a three-positioned radar system

(1) $$x = \frac{r_1^2 - r_2^2}{4a},$$

(2) $$y = \frac{r_1^2 + r_2^2 - 2r_3^2 + 2(b^2 - a^2)}{4b},$$

(3) $$z = \pm\sqrt{r_1^2 - (x+a)^2 - y^2}.$$

3. Numerical characteristics of target coordinates in a three-positioned radar system

3.1. Mean value

In this paper the mean values of the target coordinates (x, y and z) are obtained as follows:

mean value of the x coordinate

(4) $$M[x] = \frac{1}{4a}\left(M[r_1]^2 - M[r_2]^2 + D[r_1] - D[r_2]\right);$$

mean value of the y coordinate

(5) $$M[y] = \frac{1}{4b}\left(M[r_1]^2 + M[r_2]^2 - 2M[r_3]^2 + D[r_1] + D[r_2] - 2D[r_3] + 2\left(b^2 - a^2\right)\right);$$

mean value of the z coordinate (target height)

(6) $$M[z] = \sqrt{M[r_1]^2 - M[x]^2 - 2aM[x] - a^2 - M[y]^2 + D[r_1] - D[x] - D[y] - D[z]}.$$

3.2. Standard deviation

The standard deviations of the target coordinates (x, y and z) are also obtained:

standard deviation of the x coordinate

(7) $$D[x] = \frac{1}{16a^2}\left(D\left[r_1^2\right] + D\left[r_2^2\right] - 2K_{r_1^2 r_2^2}\right);$$

standard deviation of the y coordinate

(8) $$D[y] = \frac{1}{16b^2}\left(D\left[r_1^2\right] + D\left[r_2^2\right] + 4D\left[r_3^2\right] + 2\left(K_{r_1^2 r_2^2} - K_{r_1^2 r_3^2} - K_{r_2^2 r_3^2}\right)\right);$$

standard deviation of the z coordinate (target height)

(9) $$D[z] = M[r_1]^2 - M[x]^2 - 2aM[x] - a^2 - M[y]^2 - M[z]^2 + D[r_1] - D[x] - D[y].$$

It can be seen that the standard deviation of x and y strongly depends on the mean values of the three target ranges. Consequently, the greater the target range is, the greater the error of coordinates detection is. This error can be decreased with increasing the distances between the three radars (the bases a and b).

4. Simulation results

The target height error, estimated by using a three-radar system, for different values of the target height, is shown in Figs. 2 and 3. The presented results are achieved for the following input parameters: target height z (1, 5, 10, 15 and 20 km); bases $a = b = 2$ km (Fig. 2) and $a = b = 50$ km (Fig.3); azimuth of the first antenna 30°; range mean-square error of 23 m.

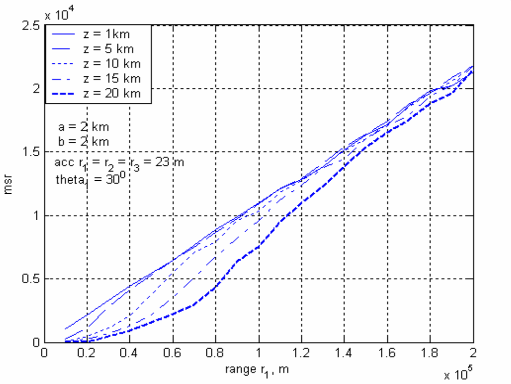

Fig. 2

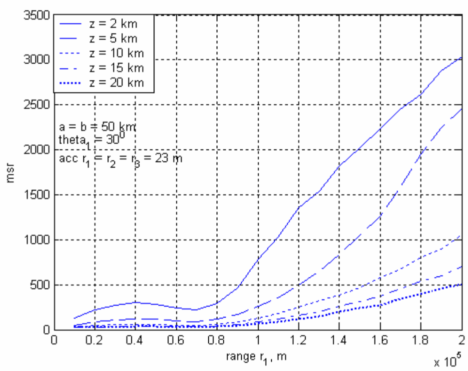

Fig. 3

If the bases and the target height are with higher values, the mean square error of the target height measuring is lower than in other situations. If we consider a target range of 200 m, bases $a = b = 50$ km and height of 20 km, the mean square error is less than 500 m (Fig. 3).

The influence of the size of the bases (a and b) over the target mean square error is shown in Figs. 4, 5 and 6.

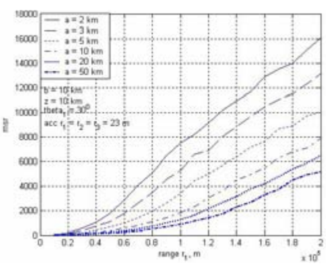

Fig. 4

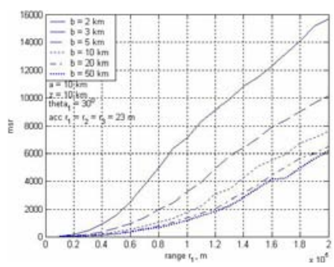

Fig. 5

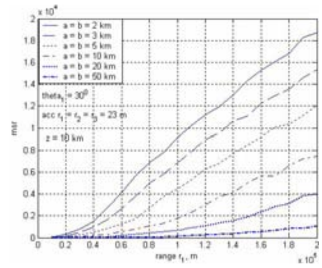

Fig. 6

In target range estimation, the higher the values of the bases are, the lower mean square error can be obtained. This can also be seen from the numerical expressions for the standard deviation of target coordinates, in which the bases are present in the denominator and therefore decrease the error.

When three radars are used, the target coordinates estimation is independent on the azimuth. However, if the range to the first radar r_1 is constant, the ranges to the other radars will change accordingly the azimuth of both the first radar and the bases. This range change versus the azimuth leads to the dependence of the msr on the azimuth, because the target numerical expressions depend on the numeral expressions of r_1, r_2, r_3 (Fig. 11).

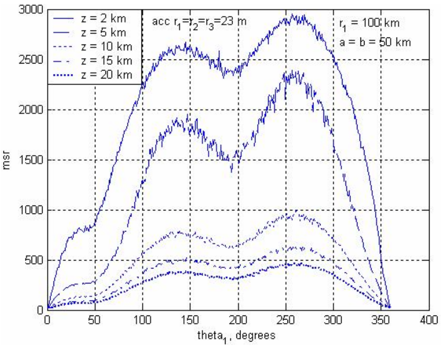

Fig. 7

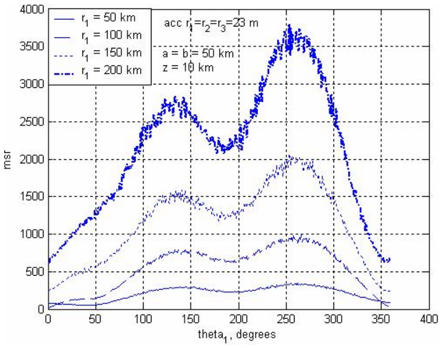

Fig. 8

The mean square error of the target height estimation versus the azimuth of the first radar, for constant target range (r_1 = 100 km) and differing target height z (2, 5, 10, 15 and 20 km), is shown in Fig. 7. The target height results for a target height of 10 km and for a variety of target ranges r_1 (50, 100, 150 and 200 km) are shown in Fig. 8. If the bases of the radar system are with higher values, the mean square error strongly depends on the azimuth. In the interval 90° – 300° the msr is higher because the two target ranges r_2 and r_3 increase much more than r_1.

The mean square error of the target height estimation versus different accuracies of the target ranges is shown in Fig. 9. The results are obtained for the target height 10 km, distances among the three radars 200 km, azimuth of the first radar 30° and accuracies of the target measuring (23, 50, 100, 200, 300, 400 m). We consider also a target circulating around the first radar.

If the target range inaccuracy increases, the msr of the target range estimation rises as well. Moreover, if the target range increases, the msr of the target height estimation rises and after target range 200 km the rising is faster. A dashed line marks the border of 800 msr for the target height estimation.

The msr of the target height estimation versus the azimuth is shown in Fig. 10. An assumption that the target circulates around the first radar is made and different accuracies of the target range are considered. The results are obtained for a target height of 10 km and for target ranges to the three radars 23, 50, and 100 m respectively. We assume that if the target does not circulate around the first radar, but around the co-ordinate center, then the msr target error estimation will not be so dependent on the target azimuth. We suppose that in real conditions the pulse package will allow increasing the target range measuring by using specific processing. As a result, the msr of the target height measuring will be increased.

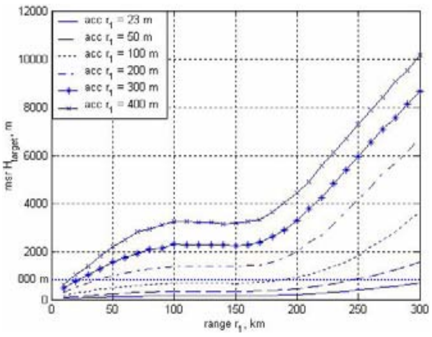

Fig. 9

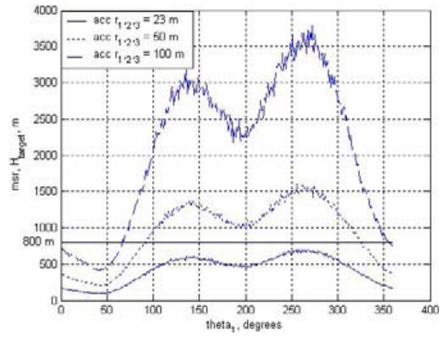

Fig. 10

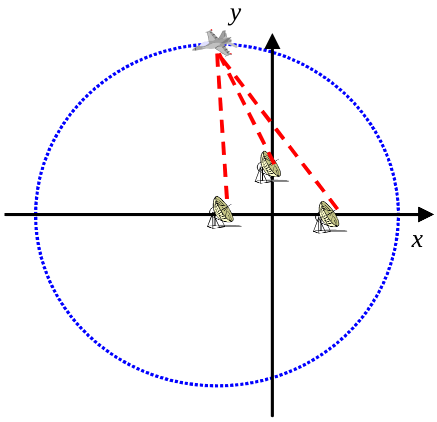

Fig. 11

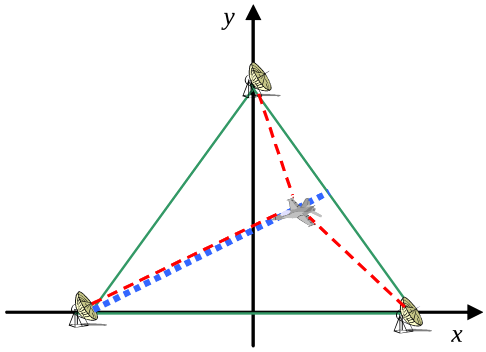

Fig. 12

The absolute value of the mean square error of the target height estimation, the percentage error of the real target height and the percentage error of the distance to the target are shown in Figs. 13, 14 and 15. The experimental results are obtained for the following input parameters: target height z of 1, 3, 5 km, target range accuracy from the three radar censors acc = 23 m. We analyze the situation when the three radar censors form an equilateral triangle and the target projection moves along the bisector of this triangle (Fig.12). The distance between the censors is chosen to be 100 km because the simultaneous automatic discovery of the estimated target is with detection probability P_d=0.9 and false alarm probability P_{fa}=10^{-6}.

When the height target increases, then the error of height measuring decreases (Figs.13 and 14). The error is smallest (lowest) in case of a disposed target close to the radar censor. When the target moves away from the first radar, the error increases. The mean square error of height measuring in percentage from the real target height is about 2–10% from flying targets at 3-5 km height. According to the distance to the target, the percentage error of height measuring is about 0.2–1% from the distance to the target.

The absolute value of the mean square error of the target height estimation, the percentage error from the real target height and the percentage error from the target distance are presented in Figs.16, 17 and 18. The experimental results are obtained for the following input parameters: height target z = 5 km, accuracy of estimation of the target distances to the three radar censors acc of 23, 50, 70 m.

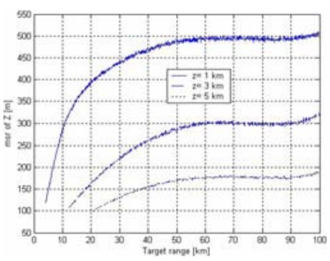

Fig. 13

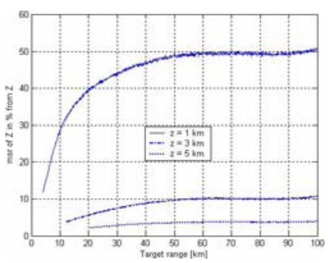

Fig. 14

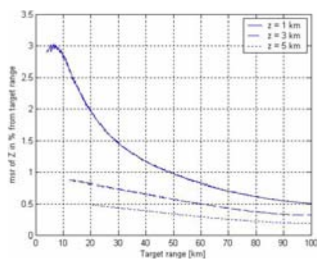

Fig. 15

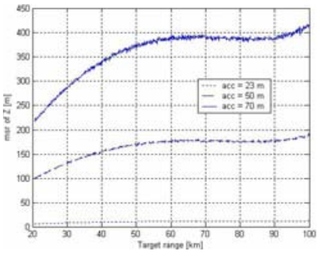

Fig. 16

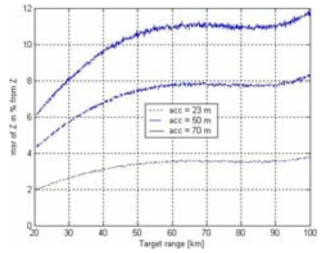

Fig. 17

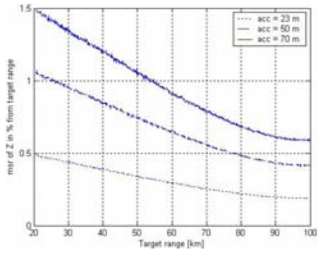

Fig. 18

When the accuracy of measuring of the distance to the target increases, the error of height estimation decreases, as it is shown in Figs.16 and 17. The error is smallest (lowest) in case of a disposed target near the radar censor. When the target moves away from the first radar, the error increases. The mean square error of height measuring in percentages from the real target height is approximately 2–4% from flying targets at 5 km height and an accuracy of measuring the distance to the target 23 m. Depending on the distance to the target, the percentage error of height measuring is less than 0.5% when acc = 23 m.

The absolute value of the mean square error of target height estimation and the percentage errors from the distance to the target are shown in Figs.19, 20 and 21. The results are obtained for the following input parameters: target height z= 5 km, accuracy of estimation of the target distance to the three radar censors acc of 23, 50, 70 m, angle theta of the first radar from 0 to 60° and target range r_1 of 20, 50, 80, 100 km. When the target distance to the first radar increases, the msr increases as well. The MSR is minimum for angle theta 30° and r_l=2b/3, after that the error increases. The variation range of the error is a very large when the angle theta changes.

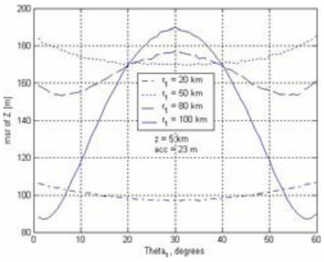

Fig. 19

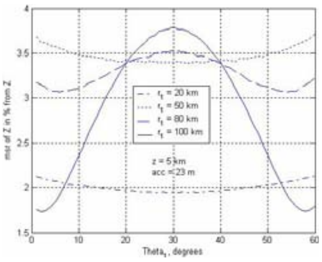

Fig. 20

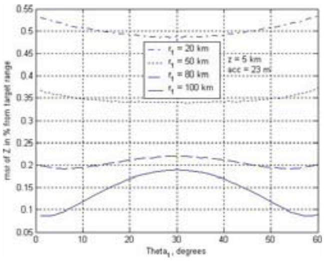

Fig. 21

Numerical characteristics of target coordinates in a three-positioned radar system. The numerical characteristics of the target coordinates (mean value and standard deviation) are studied in order to clarify the nature of the mean square error. The experimental results are obtained for the following input parameters: target range to the first radar r_1 = 200 km, target height z = 10 km, bases a = b = 50 km and range measuring msr r_1, r_2 and r_3 – 23 m.

The mean square errors of the target coordinates are shown in Fig. 23. For the given situation it can be observed that the x coordinate error estimation is 50-80 m, y is 90-150 m, and target height estimation is 1-2 km. The error depends on the target range, the bases size and the msr of the target range measuring.

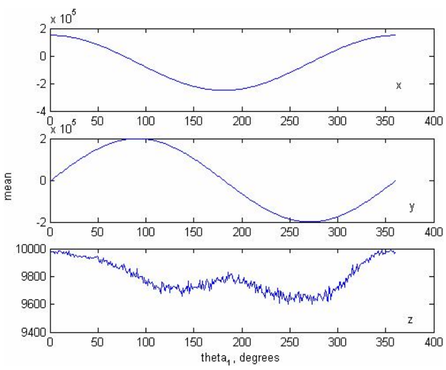

Fig. 22. Mean value of target coordinates

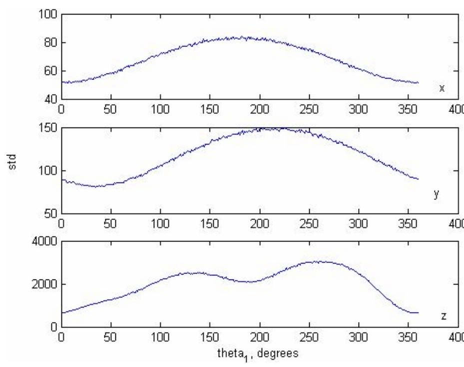

Fig. 23. Mean square error of target coordinates

Due to the fact that in this situation the bases are small, and the target range is great, the mean square error of coordinates estimation is high.

The target range change and the msr of target range estimation versus the first radar azimuth are shown in Figs. 24 and 25. The mean value of the target range to the first radar is constant – 200 km, but the other two ranges change according to the law shown in Fig. 24. The target range error is 23 m and does not depend on the radar azimuth.

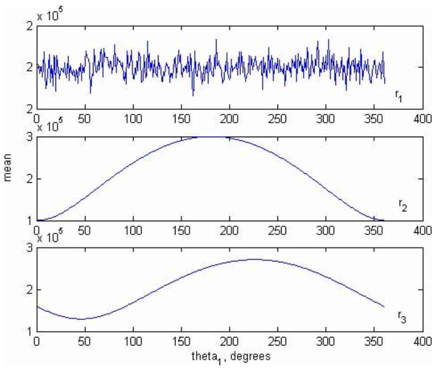

Fig. 24. Mean value of r_1, r_2, r_3

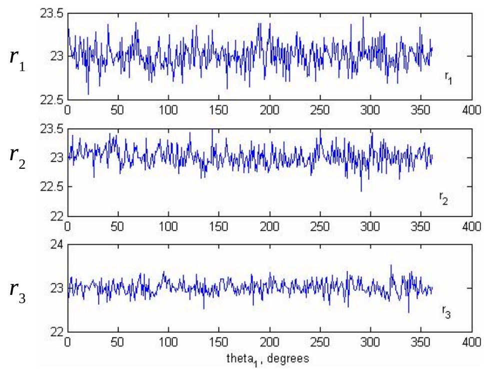

Fig. 25. Mean square error of r_1, r_2, r_3

5. Conclusions

We estimate the target height error by using standard review two-coordinate synchronized radars. The task is solved under the condition that the three radars detect and estimate the target parameters simultaneously and deliver them to a center for data unification. The research work is conducted by means of Monte Carlo simulation in MATLAB environment. We have studied the cases, when the target projection is inside and outside the triangle, formed by the radars. The mean square error of the target height is lower in cases, when the target projection is located into the triangle, compared to the cases, when it is outside the triangle.

The mean square error of the target height measuring is lowest in the following case: large distances among the radars (great bases), high accuracy of target range measuring; high-flying targets. New analytical results for the mean value and the standard deviation of a three-positioned radar system are also obtained and presented. They may be applied for height target estimation in multistatic radars using existing communication and radar networks.

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