BULGARIAN ACADEMY OF SCIENCES

CYBERNETICS AND INFORMATION TECHNOLOGIES · Volume **5**, No 1

Sofia . 2005

A Communication System with Wavelet Packet Division Multiplexing in an Environment of White Gaussian Noise and Narrow-Band Interferences*

Nikolai Nikolov, Zdravko Nikolov

Institute of Information Technologies, 1113 Sofia
E-mails: nknikolov@abv.bg   Znikolov@iit.bas.bg

Abstract: *The performance of a communication system with wavelet packet division multiplexing in an environment of white Gaussian noise and narrow-band interferences is studied. Wavelet packet transforms and their place for generation and detection of communication signals in this paper are examined. They are examined as an alternative of multi-frequency systems and particularly OFDM. The frequency efficiency of such systems is shown. The performance in an environment of additive white Gaussian noise and narrow-band interference is examined. A noise model of the narrow-band interferences is designed according to the ones observed in the power network of 220 V.*

Keywords: *communication system, wavelet packet division multiplexing.*

I. Introduction

The communication channel coding transforms digital symbols to continuous signals in order to ensure their transition through the real physical system – the communication channel, to enable simultaneous transfer of independent information streams, to improve resistance to a number of harmful impacts – noise, fading, linear and nonlinear distortions, inter-carrier and inter-symbol interferences. For this purpose are used minimum correlated one to the other, in the perfect case – orthogonal, signal sets. To transmit a large number of independent messages three forms of orthogonality are used – by frequency, where a number of band limited signals are translated in different non-overlaping frequency bands; by time – a number of messages are transmitted in

* This research work is sponsored by Investigation Project No И1003/2000 of the Ministry of Education and Science, Bulgaria.

different time intervals; and by code – signals overlap in time and frequency, but are different in form. Wavelet transforms use a large variety of different orthogonal bases, which may be successfully used for transmission of information. The fast algorithms developed for these transforms (forward and reverse) offer easy generation of carrier signals and efficient detection by using a matched filter, without involving complex arithmetic.

II. Shaping filters for channel signals.

In digital communications, each message is represented by a set of binary symbols. To make possible the transmission over a real physical medium, one defines in accordance to a set of real continuous, some easy differentiable signals. An usual approach to transform binary symbols to continuous signals is depicted in Fig.1. The binary sequence enters a filter usually known as Nyquist filter with frequency 1/T as Dirac pulses with positive sign when a "1" is transmitted and a negative sign when a "0" is transmitted. The filter impulse response is such that after the first maximum in the time t_0, it has zero values for $t=t_0 \pm kT$, k=1, 2… [6, 7, 13, 14]:

(1) $$p(t) = \begin{cases} 1, & t = 0; \\ 0, & t = nT, \quad n \neq 0, \quad n \in Z, \end{cases}$$

where T is the orthogonality interval. If *p(t)* is represented as

(2) $$p(t) = \int \phi(\tau)\phi(\tau - t)d\tau ,$$

the information sequence may be encoded by $\phi(t)$ and decoding may be done by means of a matched filter. If one designates the set of information symbols by $\{a_n\}$, then the transmitted signal gets the form:

(3) $$s(t) = \sum_n a_n \phi(t - nT) .$$

When detecting by a strobe in time $t=t_0+kT+\tau$, where τ is the time spread through the communication channel, the separate binary symbols are detected without inter-symbol interference. This process is represented graphically in Fig.1f.

Detection by correlation is also possible, where the signal energy is estimated and at the output of the correlator the bipolar signal is obtained according to the transmitted sequence:

(4) $$\begin{aligned} r(t = mT) &= \int_{(m-M)T}^{mT} \sum_n a_n \phi(\tau - nT)\phi(\tau - (m - M)T)d\tau = \\ &= \sum_n a_n \delta_{n-m+M} = a_{m-M} . \end{aligned}$$

The ratio of the interference between particular symbols depends on the coherence between the transmitter and the receiver when detection is made by a strobe and by the width of the autocorrelation function of $\phi(t)$ when the detection is made by correlation.

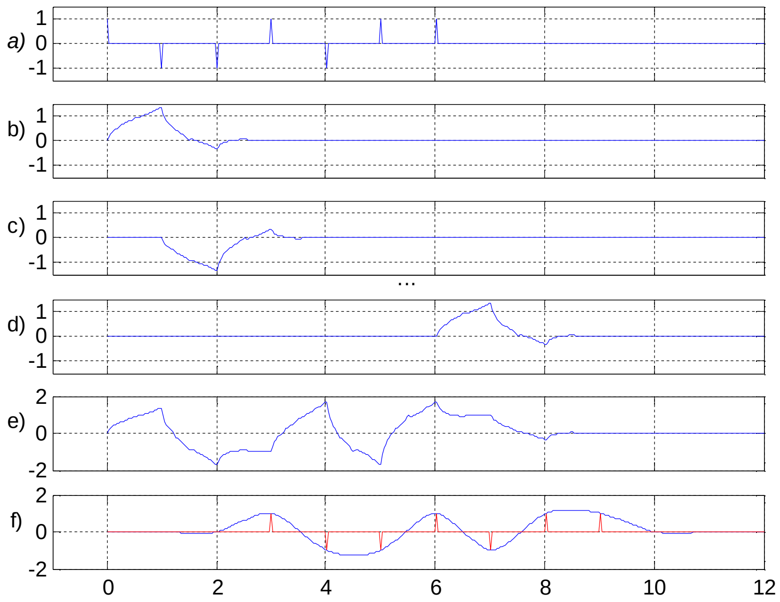

Fig. 1

A more common formulation of the problem for easy differentiation between signals is the use of orthogonal signals, which is considered in more details in the next section.

III. Wavelet functions and wavelet transforms for information exchange

The wavelet functions offer the opportunity of representation of stationary signals that are in principle all communications signals. A wavelet function possesses the important property to be good localized in two comparatively independent variables – frequency and time. This permits for a large variety of orthogonal functions to be generated by a simple translation of a single "mother" function in time as well as scaling this function and both [1-4].

In frequency domain the wavelet function has the form of a band-pass filter and in the perfect case it is a window:

$$\hat{\Psi}(\varpi) = \begin{cases} 1, & \pi < |\varpi| \le 2\pi, \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

The scaling function is associated with the wavelet and has the form of a low-pass filter and in the perfect case it is:

$$\hat{\Phi}(\varpi) = \begin{cases} 1, & |\varpi| \le \pi, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

By tradition the wavelet functions are related to multi-resolution analysis. In such analysis the analyzed function is represented as a sum of particular representations with different details. More strictly it is represented as sequential, embedded one in another subspaces:

(7) $$\ldots \subset V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \ldots$$

Each subspace V_l is composed by an orthonormal basis comprised by the same in form uniformly translated functions, with the same step onto one variable – the time, as V_l = span $\phi_{l,j}$ and $\phi_{l,j}(t)=\phi_l(t-jT_l)_{|l,j\in Z}$. The basis functions for the different subspaces are obtained from each other by scaling (shrinkage or stretching), most often dyadic:

(8) $$\phi_{l,0}(t) = 2^{l/2}\phi_0(2^l t),\ \phi_{l,k} = 2^{l/2}\phi_0(2^l t - kT_0),$$

and $T_l=2^{-l}T_0$. From the requirements that each subspace of a given level of resolution belongs to all subspaces of a higher level of resolution and the orthonormality of all base functions of the same level of resolution, some important relations are obtained, which allow the forward and reverse transforms to be performed as routine convolutional procedures. From the condition $V_0 \subset V_1$ it follows:

(9) $$\phi_0(t) = \sum_k h_k \phi_1(t-kT_1) = \sqrt{2}\sum_k h_k \phi_0(2t-kT_0) = \sum_k h_k \phi_1(t-kT_1),$$

where

(10) $$h_k = \langle \phi_0(t), \phi_1(t-kT_1) \rangle.$$

The orthonormality condition additionally puts the restriction

(11) $$\sum_k h_k h_{k-2m} = \delta_m.$$

An important property of the wavelet representation is that the reminder in going from higher to lower level of resolution is orthogonal to that lower level. The reminder is comprised by orthonormal base of the same functions uniformly translated and functionally related to scaling functions ϕ_l. Analytically this may be represented as

(12) $$V_l = V_{l-1} \oplus W_{l-1};$$

where V_l is the subspace with l-th level of resolution and W_{l-1} is the reminder of V_l when represented with one level lower of resolution. The subspace of a given level of resolution together with its orthogonal complement constitutes the next higher level of resolution. These properties relate analytically bases V_l and W_l and allow forward and reverse transforms to be accomplished by fast uniform pyramidal procedures:

(13) $$\psi_{l,k}(t) = 2^{l/2}\psi_0(2^l t - kT_0),$$

(14) $$\psi_0(t) = \sqrt{2}\sum_k g_k \phi_0(2t-kT_0) = \sum_k g_k \phi_1(t-kT_1).$$

The coefficients $\{h_k\}$ are $\{g_k\}$ related by expression [1]:

(15) $$g_k = (-1)^k h_{K-1-k}, \qquad k = 0,1,\ldots,K-1.$$

The equations (9) and (14) describe a discrete convolution and the obtained spectral components may be considered as a result of filtering. The decomposition and the perfect synthesis, i.e. forward and inverse wavelet transform is accomplished by equations [1, 5]:

(16a) $$a_{l-1}[k] = \sum_n h[n-2k]a_l[n],$$

(16b) $$d_{l-1}[k] = \sum_n g[n-2k]a_l[n],$$

(17) $$a_l[n] = \sum_k \{h[n-2k]a_{l-1}[k] + g[n-2k]d_{l-1}[k]\}.$$

IV. Wavelet packet functions

The wavelet packet functions were synthesized in the further development of the multi-resolution analysis. They constitute an orthonormal basis for every level of resolution, additionally dividing wavelet subspace W_l into two newer subspaces, which then may also be divided. The basis functions for the particular subspace are obtained by an iterative procedure (18) [8-11]. If one denotes the subspace V_l by $W_{l,0}$ and W_l by $W_{l,1}$, in an analogous way to equations (9) and (14), it is possible recursively to decompose each subspace $W_{l,m}$ $(0 \le m \le 2^{-l}-1)$ to two subspaces $W_{l-1,\,2m}$ and $W_{l-1,\,2m+1}$ by dividing the corresponding orthonormal basis $\{ w_{l,m}(t - nT_l) \}_{n \in Z}$:

(18a) $$w_{l-1,2m}(t) = \sum_n h_n w_{l,m}(t - nT_l),$$

(18b) $$w_{l-1,2m+1}(t) = \sum_n g_n w_{l,m}(t - nT_l).$$

Here the secondary indices describe the position of a given function in the tree structure of the wavelet packet as depicted in Fig.2.

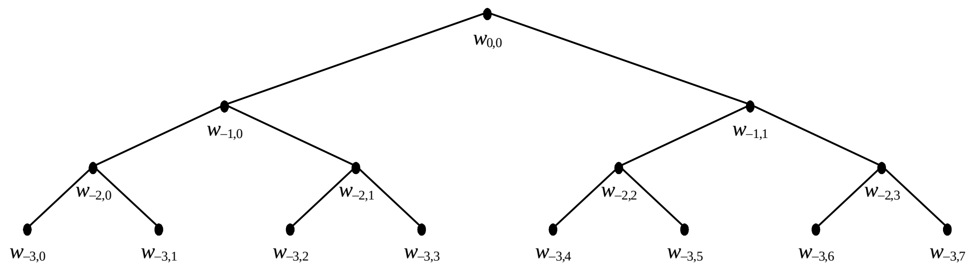

Fig. 2

Carrying out the iterative procedure from the highest node (0, 0) along the branches of the structure it is possible to construct basis functions $w_{l,m}(t)$ of arbitrary level.

The synthesis of a channel signal modulated by an information sequence is carried out by a procedure analogous to equation (17):

(19) $$s_{l,m}[n] = \sum_k \{h[n-2k]s_{l-1,2m}[k] + g[n-2k]s_{l-1,2m+1}[k]\};$$

where: $l = 1 - L, \ldots, 0$, $m = 0, \ldots, 2^{-l} - 1$,

l is the current level,

L is the deepness of the tree,

m is the number of the node for the given level,

k – the time index for the signal at level $l - 1$,

n – the time index at level l.

The equation (17) describes an over-sampling by two, followed by a filtering of the two "daughter" nodes and summing of both produced signals. It is done for each node in the binary tree above, starting from level $1 - L$ to zero. The information sequence is put as wavelet coefficients in the terminal nodes (these which have no "daughter" nodes). This procedure represents inverse discrete wavelet packet transform (iDWPT). The signal transform in continuous domain is given by the equation [11, 12]:

$$s(t) = \sum_{n} s_{0,0}[n] w_{0,0}(t - nT_0), \quad (20)$$

where $s_{0,0}[n]$ is the equivalent sequence synthesized by iterative use of (19).

V. Information signals

The wavelet packet functions shown constitute an orthonormal system, which permits their direct usage as carrier signals in a multi-carrier system for information exchange. Each particular basis function may be modulated with an information symbol for the particular channel and after summation of the signals for all channels, the final multi-carrier signal is obtained.

An information system based on this approach is analogous to OFDM systems (Fig. 3). To some extent it is related also to the generation and detection of the channel signals – for both approaches fast procedures exist. Essential difference is the absence of complex numbers when wavelet packets are used.

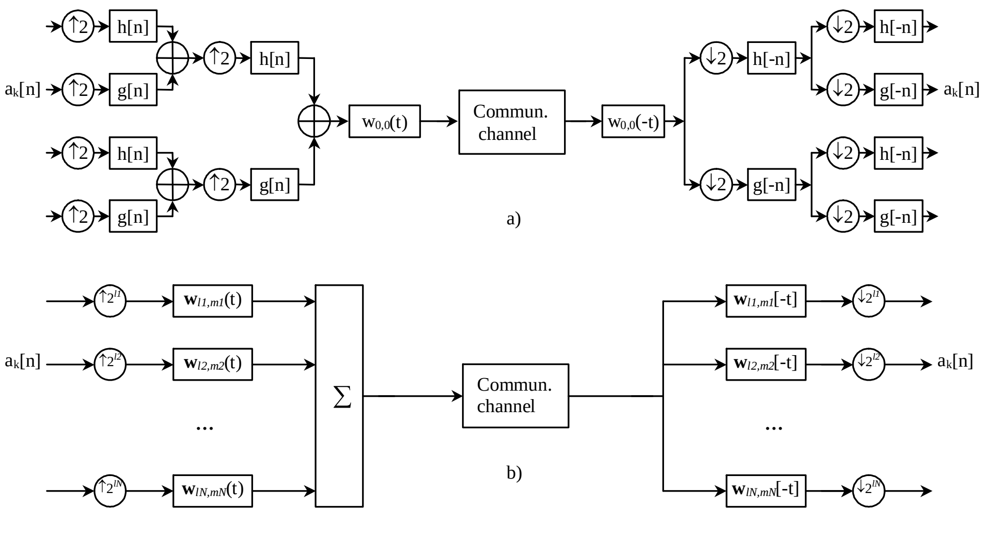

Fig. 3

The modulated signal in a multi-carrier wavelet packet system takes the form:

(21) $$s(t) = \sum_{l=\Lambda, m=M_\lambda} \sum_k a_{l,m}[k] w_{l,m}(t - kT_l).$$

Here Λ is the set of the levels from which terminal nodes are used, M_λ is the set of used terminal nodes of a given level λ and $a_{l,m}[k]$ is the information sequence.

A realization of (19) and (20) is shown in Fig. 3a for level $L = 2$, and realization of (21) for an arbitrary level – in Fig. 3b.

The recovery of the information sequence is accomplished by forward wavelet packet transform (DWPT). It may be carried out by the equations for decomposition, directly derived from (16):

(22a) $$s_{l-1,2m}[k] = \sum_n h[n - 2k] s_{l,m}[n],$$

(22b) $$s_{l-1,2m+1}[k] = \sum_n g[n - 2k] s_{l,m}[n],$$

while the procedure is started from node (0, 0) along the branches of the whole tree reaching the terminal nodes.

An alternative for recovery of the information symbols is

(23) $$\tilde{a}_{l,m}[k] = \int_{kT_l}^{kT_l + T_l'} s(t) w_{l,m}(t - kT_l) dt,$$

where T_l' is the support (duration) of $w_{l,m}(t)$. Both alternatives are illustrated in the right most part of Fig. 3a and Fig. 3b respectively.

It should be noted that when using the second alternative – equations (21) and (23) – and when using efficient multi-rate filters [15], the procedures are more efficient in a number of operations, but it is necessary to hold in memory all the coefficients of the equivalent filters $\{w_{l,m}[n]\}$, while the alternative with original iDWPT and DWPT requires just holding the coefficients $\{h_n\}$and $\{g_n\}$, but is less efficient in number of operations aspect.

Wavelet packet division multiplexing leads to an increase of symbol duration, which in turn decreases the impact of the used channel disperse. The relative impact of the necessary time guard interval to avoid inter-symbol interference highly decreases, proportional to the number of the wavelet packet signals used. The modulation and demodulation are accomplished in an integral process for all the channels.

Compared to the conventional frequency division multiplexing, wavelet packet may be about 30% more efficient in the occupied frequency band. Because of the strong orthogonality wavelet packet signals may overlap in frequency preserving the full separability. In frequency division case overlapping in frequency is not admissible, which requires guard bands, usually 30% of the pass band of a particular channel. Additional losses are a result of the finite roll-off in the transition band of the band pass filters.

For illustration purposes the use of a given frequency band by the conventional frequency division is shown in Fig. 4a and Fig. 4b by wavelet packet respectively. The used wavelets are of the class of Daubechies with 14 coefficients. In both cases a two-side spectrum is used. The more efficient usage of the frequency band in the

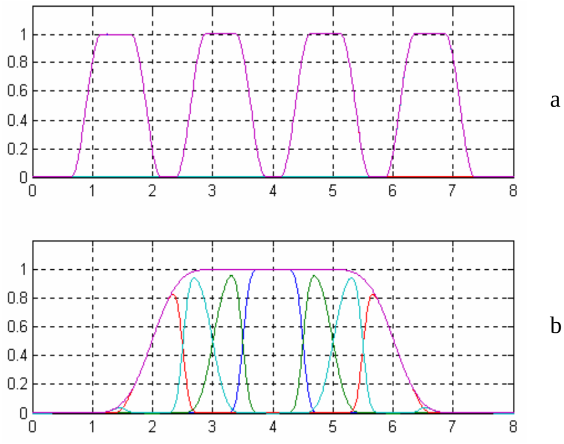

Fig. 4

wavelet packet case is noticed when the efficiently occupied frequency band containing 99% of the signal energy is 30% narrower.

VI. Experimental results

A. Influence of additive white Gaussian noise

In this paper the probability of erroneous detection of a particular bit is examined for a 16-channel wavelet packet system under the influence of additive white Gaussian noise. The wavelet packet is generated by Daubechies wavelets with 14 coefficients. The reception is optimal – by a correlation or matched filtering and decision is taken according to the maximum likelihood criteria.

The transmitted signal is defined by (21) and the received one is:

(24) $$r(t) = s(t) + \eta(t)\,,$$

where $\eta(t)$ is white Gaussian noise with spectral density N_0. The signal after the correlation becomes:

(25) $$\tilde{a}_{l,m}[k] = \int_{kT_l}^{kT_l+T_l'} w_{l,m}(t-kT)\left[\sum_{l\in\Lambda, m\in M_\lambda}\sum_{n=-\infty}^{\infty} a_{l,m}[n]\,w_{l,m}(t-nT_l)+\eta(t)\right]dt =$$

$$= a_{l,m}[k] + \int_{kT_l}^{kT_l+T_l'} w_{l,m}(t-kT_l)\eta(t)dt = a_{l,m}[k] + \eta_{l,m}[k].$$

Here $\eta_{l,m}[k]$ is the noise component at the output of the correlator – a random variable with Gaussian distribution, obtained as a result of a linear transform over random Gaussian process [16]. For the variance one obtains:

(26) $$\sigma_{l,m}^2 = E[\eta_{l,m}^2[j]] = \iint E[\eta(t)\eta(\tau)]w_{l,m}(t-jT_l)w_{l,m}(\tau-jT_l)dtd\tau =$$

$$= N_0\iint \delta(t-\tau)w_{l,m}(t-jT_l)w_{l,m}(\tau-jT_l)dtd\tau =$$

$$= N_0\int w_{l,m}(t-jT_l)w_{l,m}(t-jT_l)dt = N_0\,.$$

The analytically determined probability of the error is

(27) $$\mathrm{Pe} = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi N_0}} e^{-\frac{(x-1)^2}{2N_0}} dx. .$$

The signal energy is normalized according to N_0. Such determined probability of error is shown graphically in Fig. 5a for particular discrete values of the signal/noise ratio, marked off by points. Experimentally the error rate is determined for 10^5 symbols. In Fig.5a the probability of error for the different 16 channels is shown and in Fig.5b the averaged error rate for the whole 16-channel system is shown. Simulative examinations are done by using also other wavelet packets but the results are not shown here because they do not indicate any significant differences from these shown for Daubechies wavelets.

B. Influence of narrowband interferers

For the current examination the narrow band (frequency-impulse) disturbances are of particular interest, predominant type of disturbances in the power supply network of

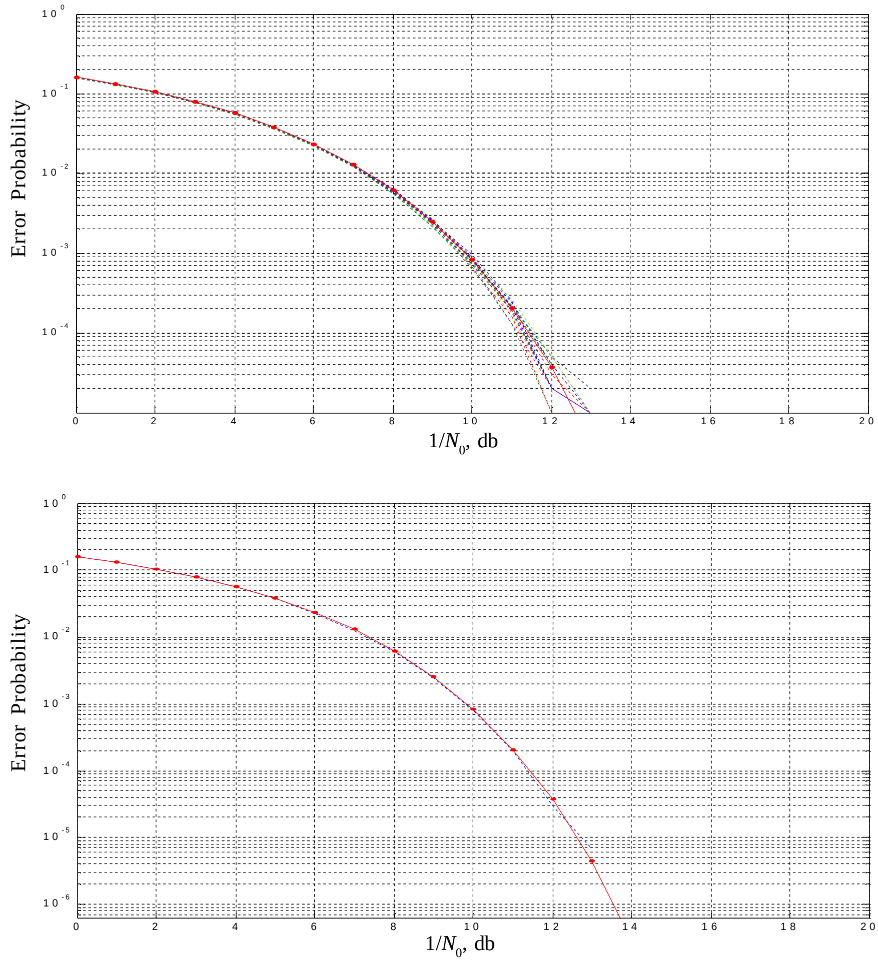

Fig. 5

220 V. Some investigations [17] show that more than 50 % of the noise power in that network are a result of such type of disturbances in the range 0.3-20 MHz, while the efficiently occupied by them bandwith is less than 5%. Here a model of this type of noise is synthesized to examine its influence on the communication by wavelet packets with wavelets as those shown in the previous section and by OFDM. It is assumed that the interferers to have pure harmonic description are

$$\eta(t) = u\sum_{k=1}^{K} a_k \sin(2\pi f_k t)\,, \tag{28}$$

where $\{f_k\}$ are randomly generated with uniform distribution in the frequency band frequencies used; $\{a_k\}$ are amplitudes for each frequency and are also generated by uniform distribution and further normalized in a way that $\{a_k\}$ correspond to the condition

$$\sqrt{\sum_{k=1}^{K} a_k^2} = \sqrt{2}\ .$$

The signal/noise ratio is given by the coefficient u. The number of the narrow band interferes is selected in the following way –frequency resolution is chosen for processing in the frequency domain ΔF=1/2048 (for normalized sampling frequency F_s=1) which gives 1024 frequency bands. In [17] it is shown that narrow band interferers occupy about 5% of the whole frequency band and as we have chosen the frequency band occupied by the signal to be 3/4 of the whole, it is accepted K = 23.

The examination is done for 128 nominal channels from which for transmission 75% are used, i.e. 96 channels. It is assumed that 25% of the channels are not usable because of high interferences, optimizing the noise protection or some other reason. In wavelet packet division this requires the tree structure to be developed up to level 7. The Fourier power spectral density of the information signal, constituted by the compactly packed 96 wavelet packet functions and placed in the middle of the given frequency band, is shown in Fig. 6a. It is seen that the internal structure of the power distribution over different subcarriers of the wavelet packet in Fig. 6a is uneven but the envelope of the power density of the whole signal is kept constant.

For OFDM communication with the same number of channels and the same occupied frequency band the subcarrier signals are:

$$v_{2n}(t) = \begin{cases} \sqrt{\dfrac{2}{T}}\cos(2\pi(n+0.5)t/T), & 0 \le t < T, \\ 0, & \text{otherwise;} \end{cases} \tag{29}$$

$$v_{2n+1}(t) = \begin{cases} \sqrt{\dfrac{2}{T}}\sin(2\pi(n+0.5)t/T), & 0 \le t < T, \\ 0, & \text{otherwise} \quad n = 0, 1, ..., 63. \end{cases}$$

The included coefficient 0.5 has the role to place the frequency band of the transmitted signal in the middle of the used band and to match that for the wavelet packet. Actually used signals have n = 8, 9, …, 55 and in this way their number is also 96.

The signal Fourier power spectral density of the so generated subcarriers is shown in Fig. 6b. Comparison of Fig. 6a and Fig. 6b shows lower steepness of the

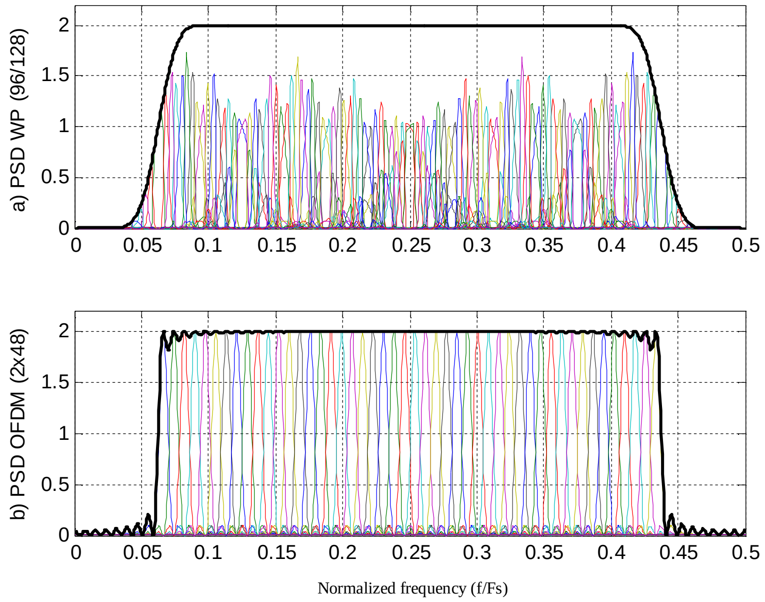

Fig. 6

transition band for the wavelet packet, which is expected because of lower power concentration in frequency domain.

The transmitted signal for OFDM communication is represented by

(30) $$s(t) = B^{\mathrm{T}}(k)V(t-kT) = s_k(t),$$

where $B(k)=[b_1(k), \ldots, b_{96}(k)]$ and $V(t)=[V_1(t), \ldots, V_{96}(t)]$ are the vector of the k-th transmitted symbol, compound of particular binary symbols ±1 and the vector of the subcarrier signals. After a matched filtering, the signal present at the output of the correlator after k-th successive received symbol has the form:

(31) $$\hat{B}_m[k] = \int_{kT}^{(k+1)T} s(t)V(t)dt + \int_{kT}^{(k+1)T} \eta_k(t)V(t)dt = \left[b_1[k] + r_{\eta_1}[k] + \ldots + b_{96}[k] + r_{\eta_{96}}[k]\right]^{\mathrm{T}}.$$

Here $\eta_k(t)$ is the noise signal during the time when k-th symbol is received and $r_{\eta_i}[k] = \langle r_k(t), V_i(t) \rangle$ is correlation of the same signal with i-th subcarrier. Fig. 7 shows a particular realization of the spectrums of a wavelet packet channel signal, an OFDM signal and a noise signal. It is noticeable that the transition band for OFDM signal is steeper but there are also noticeable out of band frequency components.

In Fig. 8 a realistic picture of the spectrum is shown of the sum of the transmitted signal and narrow band interferers. The power spectrum density of the interferers is many times higher than that of the transmitted signal.

For the channel signals so described and noise characteristics by simulation the probability of erroneously received bit (BER) is determined for different signal/noise ratios. The results obtained are shown in Fig. 9a and Fig. 9b for particular subchannels, for wavelet packet carriers and OFDM respectively. Probabilities are determined over transmission of 10^5 symbols. The signal/noise ratio (SNR) is determined as the ratio of the total energies of the transmitted signal and the noise. The figure shows large differences for the probability of error for particular subchannels and taking in

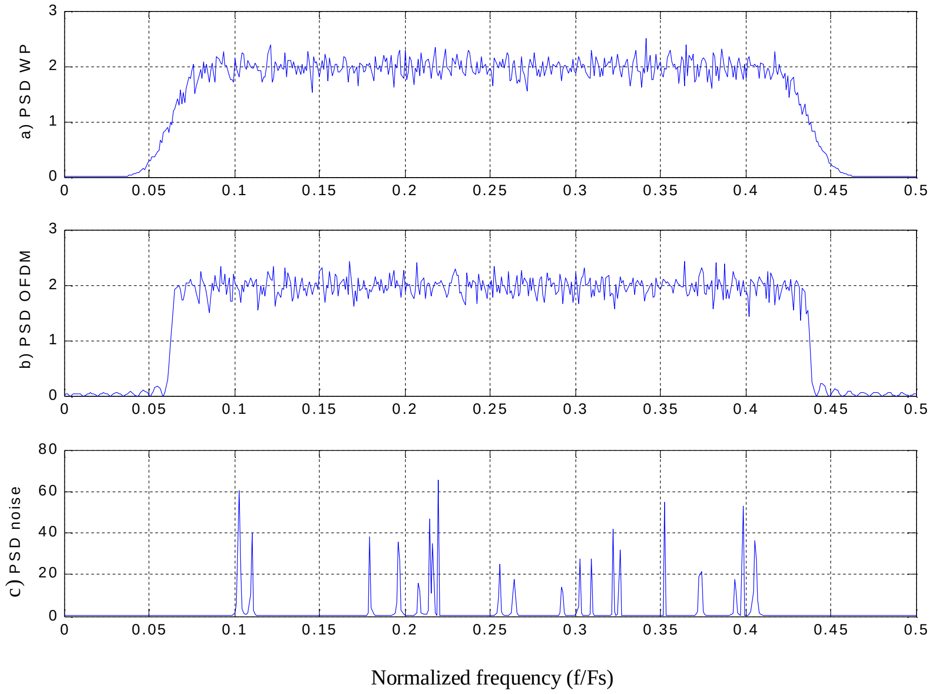

Fig. 7

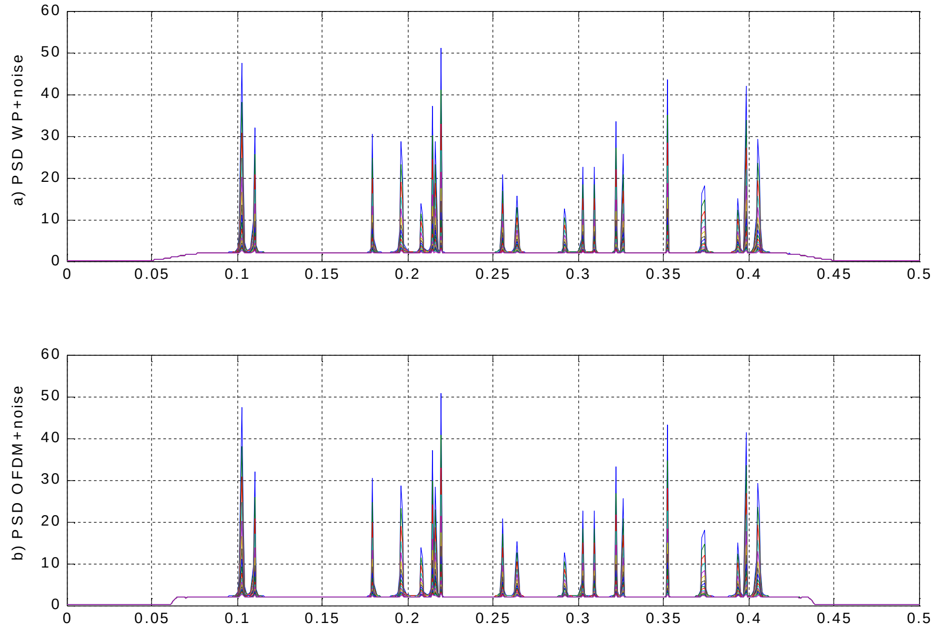

Fig. 8

account narrow band characteristic of the noise it is natural. The obtained experimental results show presence of a critical signal/noise ratio, beyond which BER decreases

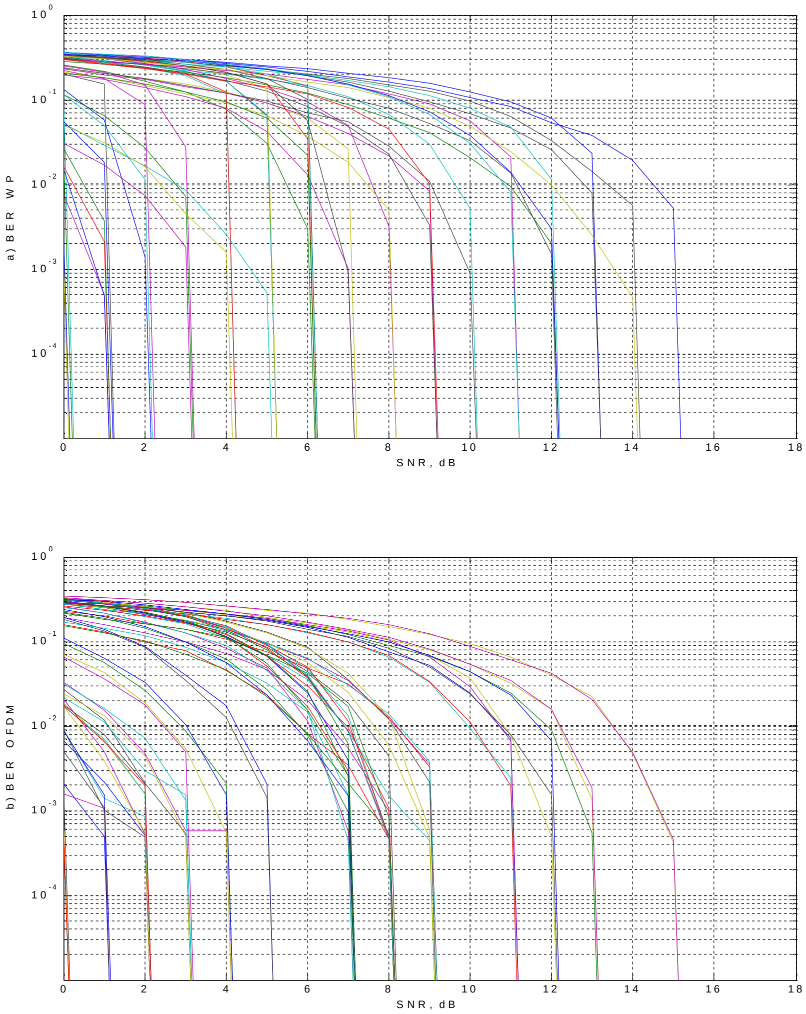

Fig. 9

dramatically. This critical ratio is also different for each particular subchannel. Obviously this is related to the remoteness in frequency of the channel and the narrow band interferer as well as its power.

In both described approaches for division of the channel the concept is intrinsic for excluding the subchannels with high noise level from the modulation scheme and sharing out the energy between remaining subchannels for maximizing the information capacity. In this paper the number of the usable and the unusable subchannels is determined for different SNR. As a criteria for usability of a particular channel threshold value of BER=10^{-3} is chosen. In this index both schemes for channel division exhibit almost the same properties. For achieving acceptable reliability of such system relatively high SNR>12 dB is necessary. Taking into account the frequency sporadic character of the noise (for the investigated noise model) an optimization procedure for energy reallocation will highly reduce the required SNR.

Table 1

SNR, dB	Wavelet packet 96 channels		OFDM 2×48 channels	
	Number of channels BER > 10^{-3}	Number of channels BER ≤ 10^{-3}	Number of channels BER > 10^{-3}	Number of channels BER ≤ 10^{-3}
0	50	46	65	31
1	44	52	63	33
2	40	56	55	41
3	37	59	48	48
4	34	62	44	52
5	30	66	42	54
6	27	69	40	56
7	20	76	37	59
8	18	78	21	75
9	16	80	16	80
10	12	84	12	84
11	11	85	12	84
12	9	87	7	89
13	5	91	4	92
14	2	94	2	94
15	1	95	0	96
16	0	96	0	96
17	0	96	0	96
18	0	96	0	96

VII. Conclusion

The implemented experiments show similarity of the basic parameters for the systems based on frequency division of the type of OFDM and with a wavelet packet. In a particular aspect they may be considered as alternatives of each other.

Narrow band interferences may highly affect neighboring subchannels. A critical signal/noise ratio exists beyond which the BER is highly reduced. The investigation of the effect of narrow band interferences is not comprehensive enough and it must be extended.

A class of signals based on filter banks also exists [15], for which it is possible to obtain overlapped in frequency subchannels with preserved orthogonality. Fast algorithms for their generation, based on FFT exist. Since they are even as well localized in frequency it is natural to expect given narrow band interference to influence less neighboring subchannels.

References

1. M a l l a t, S. A Theory of Multiresolution Signal Decomposition: The Wavelet Representation. – In: IEEE Trans. Pattern Anal Machine Intell., **11**, July 1989, 674-693.
2. D a u b e c h i e s, I. Ten Lectures on Wavelets. Philadelphia, PA, SIAM, 1992.
3. D a u b e c h i e s, I. Orthonormal Bases of Compactly Supported Wavelets. – Commun. Pure Appl. Math., **41**, 1988, 906-996.

4. V e t e r l l i, M., C. H a r l e y. Wavelets and Filter Banks: Theory and Design. – In: IEEE Trans.Signal Processing, **40**, September 1992, No 9.
5. W o r n e l l, G., A. O p p e n h e i m. Wavelet-Based Representation for a Class of Self-Similar Signals with Application to Fractal Modulation. – In: IEEE Trans. Inform. Theory, **38**, March 1992, No 2.
6. G a n d h i, P. et al. Wavelets for Waveform Coding of Digital Symbols. – In: IEEE Trans. Signal Processing, **45**, September 1997, No 9.
7. L i v i n g s t o n, J., C. T u n g. Bandwidth Efficient PAM Signaling Using Wavelets. – In: IEEE Trans. Comm., **44**, December, 1996, No 12.
8. C o i f m a n, R., M. W i c k e r h a u s e r. Entropybased Algorithms for Best Basis Selection. – In: IEEE Trans. Inform. Theory, **38**, March 1992, No 2.
9. L i n d s e y, A. Wavelet Packet Modulation for Orthogonally Multiplexed Communication. – In: IEEE Trans. Signal Processing, **45**, May, 1997, No 5.
10. L e a r n e d, R., H. K r i m, B. C l a u s, A. W i l l s k y. Wavelet-Packet-Based Multiple Access Communication. – In: Proc. SPIE, October, 1994.
11. W o n g, K. et a l. Wavelet Packet Division Multiplexing and Wavelet Packet Design under Timing Error Effects. – In: IEEE Trans. Signal Processing, **45**, December, 1997, No 12.
12. W o n g, K. et a l. Performance of wavelet packet division multiplexing in impulsive and Gaussian noise. – In: IEEE Trans. Comm., **48**, July, 2000, No 7.
13. P a n a y i r c i, E. et a l. Design of Optimum Nyquist Signals Based on Generalized Sampling Theory for Data Communications. – In: IEEE Trans. Signal Processing, **47**, June, 1999, No 6.
14. P r o a k i s, J. Digital Communications. New York, McGraw-Hill, 4-th edition, 2001.
15. V a i d y a n a t h a n, P. Multirate Systems and Filter Banks. NJ, Prentice-Hall, Englewood Clifs, 1993.
16. Т и х о н о в, В. Статистическая радиотехника. Радио и связь, 1982.
17. N i k o l o v, Z., V. V a s s i l e v, L. N i k o l o v a. Frequency-Time Analysis of the Interferences in a 220 V Network. – In: IEEE ICC, Paris, 2004.