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Successive Blurring Techniques for Image Processing[1]

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Abstract: *A new family of image processing algorithms, called STEC /Standard Time Equivalent Conversions/ is introduced in this paper . Their main advantage is their parallel design, which makes them ideal for implementation in parallel processing systems. The notion of parallelism in STEC is analogous to physical diffusion processes, which makes them suitable for wide area of tasks beside image recognition. Further in the paper a new concept called dynamic moments is examined. It is demonstrated how the latter can be used with contour and global methods for image recognition. Experimental results and graphics are provided*

Keywords: *Image processing algorithms, parallel design, image recognition.*

1. Introduction

In the last two decades calculating machines endured significant improvement. The speed for information transfer raised thousand times, mainboards and processors became 32 and 64-bit, hard disks have shrunk in size and their capacity may soon reach 1 TB. Computer processors are constantly increasing their frequency., there are already processors with velocity above 3 GHz.

There are however several physical limitations that impose upper limits on this development. When the size of magnetic layers or transistors becomes from the scale of the inter-atom distances the effects of electromagnetic interference become too significant. When processor frequency rises above 10 GHz the infrared radiation becomes too big and there are too many problems with processor cooling. Higher power consumption is needed, also electric switches; diodes and capacitors cannot

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work at such small time intervals. Even if PC processors become with frequency above 10 GHz, their difficult maintenance and high price will make them unsuitable for the common user. Besides, the upper limit for the mainboard frequency is even lower – below 1GHz, which further limits computer speed. Lately much attention has been paid to the idea of optical and quantum computers which could operate at 10 GHz frequency or above. However with this kind of computers there are even more serious problems, which is hardly to believe that would be resolved in the next twenty years.

It is clear that the computing power of the current sequential computers, or von Neuman computers, is insufficient for the resolving of the real problems in science. For example the calculations in molecular biology and physics are realized on parallel systems. There are already parallel supercomputers with more than 5000 processors [1].

There are however many parallel processes in nature, like gas diffusion, crystals lattice oscillations, electromagnetic waves propagations, quantum system vibrations etc., which in some cases can be used for parallel computing. The current paper examines the possibility of realizing the real parallel processes in the framework of natural physical phenomena. In particular a concrete example is presented demonstrating the use of diffusion processes for image processing and recognition. A successive linear blurring filter was used and direct analogy between this filter and heat diffusion is presented.

2. Dynamic moments

Several geometric moments are used for images description. The most commonly used are the *central inertia moments* [2]. Let us get an image, which contains N points with values a_i. The central inertia moments are defined for a gray-scale image from the equation

$$I^k = \frac{\sum_{i=1}^{N} a_i \, |r_i - r_0|^k}{M}, \tag{1}$$

where I^k is the inertia moment from order k, N is the number of the points in the image, a_i is the value at the i-th point, $|r_i - r_0|$ is the distance from the i-th point to the image *mass center*, which coordinates are given from

$$x_0 = \frac{\sum_{i=1}^{N} a_i x_i}{M}, \quad y_0 = \frac{\sum_{i=1}^{N} a_i y_i}{M}, \quad r_0 = (x_0, y_0)$$

and $M = \sum_{i=1}^{N} a_i$ is the *total mass* of the image.

In more general case the distances from the center can participate in the calculation of the geometrical moments with some weight, this gives the so-called *weighted inertia moments*:

(2)
$$I_w^k = \frac{\sum_{i=1}^{N} w(|r_i - r_0|) a_i |r_i - r_0|^k}{M},$$

where $w(|r_i - r_0|)$ is a certain weight function.

The inertia moments give some information of the point's distribution around mass or geometric center. They are rotational invariants and can be used for classification of one image. However when the figure has some important details on the figure ridge they cannot be caught from the central moments and it is better to use the so-called *distributed inertia moment*:

(3)
$$I_d^k = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} a_j a_i |r_i - r_j|^k}{M^2}.$$

The distributed inertia moment gives more precise information for the points distribution and does not depend on the mass center position. It is now possible from every figure to be extracted one unique identifier (UI) that could be used to classify this figure within a s set of similar figures. In this way all UIs can be ordered and fast binary search can be accomplish for every new image that enters the recognition system.

However geometric moments have two main drawbacks – they are not scale-invariant and their calculation is time-consuming. For example finding of the distributed geometric moment needs at least N^2 calculations and *N* scans of the image. It is not easy to be implemented even on parallel system.

The *dynamic moments*, introduced below, are substitutes for the geometric moments. Let us suppose that in some moment t_n a discrete image is given. The image can be further transformed by some kind of transformation law. The case when this transformation is a discrete linear digital filter, for example the average blurring filter, will be examined. It blurs the entire image according to the equation

(4)
$$a_{\text{new}} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} a_{ij}}{m^2},$$

where a_{ij} are the values in all neighboring pixels, and *m* is the neighboring pixels block width and height. This filter substitute the value in one pixel with the average of the pixels values in the block of pixels *m* x *m* with center of that pixel.

At the end of the blur a new transformed image is generated. If this operation is repeated many times an entire set of new images can be produced for every next moment of time. It will be supposed that this blurring process is implemented simultaneously. Only in this way the algorithms presented here are efficient.

This parallel blurring implementation however is similar to many real physical processes. Equation (2) can be written for the more general case of blurring using weights for the neighboring pixels for continuous time, for the interval dt in the following way:

$$a_{k,n} = a_{k,n-1} + dt\sum_{i=1}^{mxm}(p_i a_{i,n-1} - q_i a_{k,n-1}), \tag{5}$$

where dt is the time interval of the blur, $a_{k,n}$ is the new value of the pixel, $a_{i,n-1}$ are the values in neighboring pixels, p_i are *inflow coefficients* and q_i are *outflow coefficients* and *m* is the neighboring pixels block width and height. In another words the first member in the sum is the total quantity that flows in, and the second gives the total quantity that flows out from the pixel in the *n*-th moment. But if a_n expresses density of something, for example fluid density, and if we suppose that inflow and outflow coefficients are equal, (5) expresses nothing but a diffusion law. If we set flow coefficients equal to constant c, and re-order members in the brackets, (5) can be written in the following form:

$$a_{k,n} = a_{k,n-1} + cdt\sum_{j=1}^{D}(a^j_{i-1,n-1} - 2a^j_{k,n-1} + a^j_{i+1,n-1}), \tag{6}$$

where *D* is the number of the lines in which the magnitude of the flow is evaluated, $a_{k,n}$ is the value of one point *k* in the *n*-th moment and $a^j_{i-1,n-1}$, $a^j_{i+1,n-1}$ are values in the corresponding neighboring points in the *i-th* line. Fig. 1 illustrates the connection between (5) and (6) for 3x3 block, where *i* varies from 1 to 4 for 3x3 block, because there is 4 straight lines. For more details about numerical differencing and derivative approximations see [3, 4].

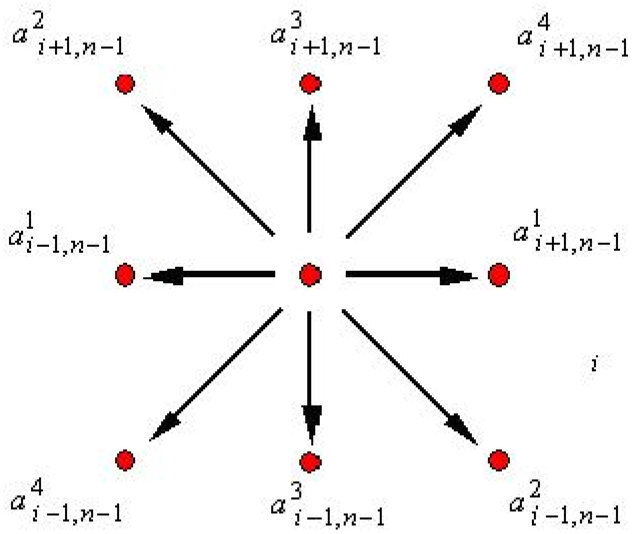

Fig. 1. Discrete approximation of diffusion in 3x3 box of the point *i*

The expression in the parenthesis in (6) is discrete approximation of second derivative. If time and the density are considered continuous, (6) becomes

$$\frac{\partial a}{\partial t} = c\left(\frac{\partial^2 a}{\partial x^2} + \frac{\partial^2 a}{\partial y^2} + \frac{\partial^2 a}{\partial z^2}\right) = c\nabla^2 a \,, \tag{7}$$

where *x,y* and *z* are the spatial dimensions (they are only two for images because for simplicity we considered only the flow between the closest points and ignore diagonals, in other words we consider the flow between two diagonal points as two-step process).

In physics the differential law in (2) is well known, all diffusion equations are from this kind, for example in thermodynamics the heat transfer is given from the heat diffusion equation [5]

$$\frac{\partial T}{\partial t} = a\nabla^2 T + b \,, \tag{8}$$

where *T* is the temperature, and a and *b* are constants. In this example the temperature plays the role of "energy density", (6) and (7) are discrete analogues of this equation.

So diffusion processes and blurring are not so different things. Two kinds of dynamic moments from first order – *the average leaving time* and the *average resting time*, will be defined. Let's take an arbitrary binary image and start to blur it in such a way that over its contour there is absorption. Said in physical language, this means that there is boundary condition that the fluid density over the contour is zero:

$$a_{k,n} = 0, \quad \text{if} \quad k \in \Gamma \,,$$

where *Γ* is the multitude of all contour points as illustrated in Fig 2.

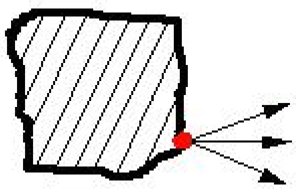

Fig. 2. Contour absorption. The quantity that reaches the contour does not return back

If the initial number of pixels in the image is assigned as *Q*, it is clear that during the blur all the quantity in the image will be absorbed on the contour if blurs are repeated infinity number of times

$$\sum_{n=0}^{\infty} S_n = Q \,,$$

where S_n is the total absorbed quantity in the *n*-th moment. The average leaving and rest times are given from the expressions:

$$\begin{aligned} t_{k,L} &= t_1 \frac{s_{k,1}}{S_k} + t_2 \frac{s_{k,2}}{S_k} + \ldots + t_n \frac{s_{k,n}}{S_k} + \ldots = \frac{dt}{S_k} \sum_{i=1}^{\infty} i s_{k,i} \,, \\ t_{k,R} &= t_1 \frac{a_{k,1}}{M_k} + t_2 \frac{a_{k,2}}{M_k} + \ldots + t_n \frac{a_{k,n}}{M_k} + \ldots = \frac{dt}{M_k} \sum_{i=1}^{\infty} i a_{k,i} \,, \end{aligned} \tag{9}$$

where $t_{k,L}$ is the average leaving time in the *k*-th pixel, $t_{k,R}$ – the average resting time in the *k*-th pixel, $a_{k,n}$ – pixel value in *k*-th pixel in the *n*-th moment, $s_{k,n}$ – total absorbed quantity in the *k*-th pixel in the *n*-th moment (or the quantity that leaves the system from that pixel), t_n is the time offset of the blur and is taken equal to ndt because moments of the blur is averagely distributed, S_k and M_k are defined as follows:

$$S_k = \sum_{n=0}^{\infty} s_{k,n}, \qquad M_k = \sum_{n=0}^{\infty} a_{k,n},$$

and represents dynamic moments from order zero. Similarly to inertial moments, dynamic moments from order r are defined as follows:

(10)
$$t^r_{k,L} = t^r_1 \frac{s_{k,1}}{S_k} + t^r_2 \frac{s_{k,2}}{S_k} + \ldots + t^r_n \frac{s_{k,n}}{S_k} + \ldots = \frac{dt}{S_k} \sum_{i=1}^{\infty} i^r s_{k,i},$$
$$t^r_{k,R} = t^r_1 \frac{a_{k,1}}{M_k} + t^r_2 \frac{a_{k,2}}{M_k} + \ldots + t^r_n \frac{a_{k,n}}{M_k} + \ldots = \frac{dt}{M_k} \sum_{i=1}^{\infty} i^r a_{k,i}.$$

If time is considered continuous (9) and (10) becomes:

(11)
$$t_{k,L} = \frac{1}{S_k} \int_{t=0}^{\infty} ts(t)dt, \qquad t_{k,R} = \frac{1}{S_k} \int_{t=0}^{\infty} ta(t)dt,$$
$$t^r_{k,L} = \frac{1}{M_k} \int_{t=0}^{\infty} t^r s(t)dt, \quad t^r_{k,R} = \frac{1}{M_k} \int_{t=0}^{\infty} t^r a(t)dt.$$

While the average resting time can be calculated for every pixel, the average leaving time can be calculated only for contour pixels, because only in these pixels there is irreversible flow out from the system. More generally, average leaving time is the average time or mathematical expectation a system to leaves its state and to pass to another state under some kind of conditions, and the average resting time is the average time that the system will remain in its current state. These two dynamic moments will be explained in more details. In the theory of Probability well known are the so-called ruin problems [6]. The classic ruin problem can be illustrated in the following way:

Lets suppose that two gamblers John and Pier are playing heads and tails. They start with some amount of money, say m and n dollars. When the coin falls on its heads side John wins one dollar, when the coin falls on its tails side Pier wins one dollar. The game continues until Pier or John money reach zero. The following variables had to be found:

1) q(m) and q(n) – probability John and Pier to win the game;

2) D – the average number of tosses until John or Pier wins (duration of the game).

This problem had been solved in detail in [6]. Written in the above assignments, the solutions are

$$q(m) = \frac{m}{m+n}, \quad q(n) = \frac{n}{m+n}, \qquad D = mn. \tag{12}$$

If for example John has 5 dollars, and Pier 3 from (12) follows that there is 62.5 % chance John to win, 37.5% Pier and that the average game duration would be 15 tosses.

The ruin problem example was used because the first order dynamic moments are analogical to the game average duration. The probability to win more generally can be named probability for leaving.

It can be shown that the dynamic moments as defined in (9) and (10) are convergent. Because of the constant discharge of the fluid through its contour, the quantity in the system decreases exponentially. This is so because the inflow and outflow coefficients in the used model do not depends from fluid density, which is typical for the diffusion processes. Then for every point from the contour is fulfilled

$$\frac{da_k}{a_k} = \mu_k,$$

where μ_k is a constant because the contour does not change during the blur. Then if we approximate the total leaved quantity through the k-th pixel as $a_k(t) = a_k(0)\exp(-\mu_k t)$, (11) becomes

$$t_{k,R} < a_k(0)\int_{t=0}^{\infty} t^r \exp(-\mu_k t)dt = a_k(0)\frac{1}{\mu_k^{r+1}}\int_{z=0}^{\infty}(z)^r \exp(-z)dz, \qquad z = \mu_k t,$$

which represents the well-known Euler's gamma function $\Gamma(r+1)$, multiplied by constant. Since $\Gamma(r+1)$ is equal to $r!$ [7], it is clear that $t_{k,R}$ is finite. Since the sum of finite numbers is also finite, total leaving and resting times are also convergent.

The average leaving and resting time can be useful because from the entire set of blurred images every pixel can be taken with some weight, corresponding to the time offset from the original image. This is the simplest way for the calculation of the dynamic moments. It can be used when nothing is known for the image content *a priori*. If for example we know that in an image sequence, an explosion occurs in some moment we will calculate the dynamic moments in different way – the maximum weight will be taken in the moment of the biggest interest – the explosion.

While geometrical moments are influenced only by the spatial distribution of the points in the image, the dynamic moments are influenced by the change of the values of the points with time as blurring or some other transformation occurs. The use of the average leaving and resting time for image recognition is examined below.

3. Experimental results

Generally there are two methods using dynamic moments – contour and global methods. In the first case all moments on the contour of an image are used, and in the second, similarly to the recognition using geometric moments, one unique identifier is extracted for the whole image. Contour methods are broadly used for image recognition. The main problems that arise are contour discontinuity and overlapping. The main purpose of STEC algorithms is to solve these two problems. If we try to found a contour on the original image there would be and other problem. Because of the quantization the contour of one binary image, even if it were well preserved, can be very abrupt.

STEC contour methods are useful because of the smoother output curves and noise stability. The blur filters the small contour disruptions. In Fig. 3. is given the gradient of the average resting time over the contour of several images.

As it can be seen from Fig 3, to every edge of the image contour in the curves corresponds either minimum or maximum in the STEC output. Whenever there is convex edge, there is a minimum and whenever there is a concave edge – maximum.

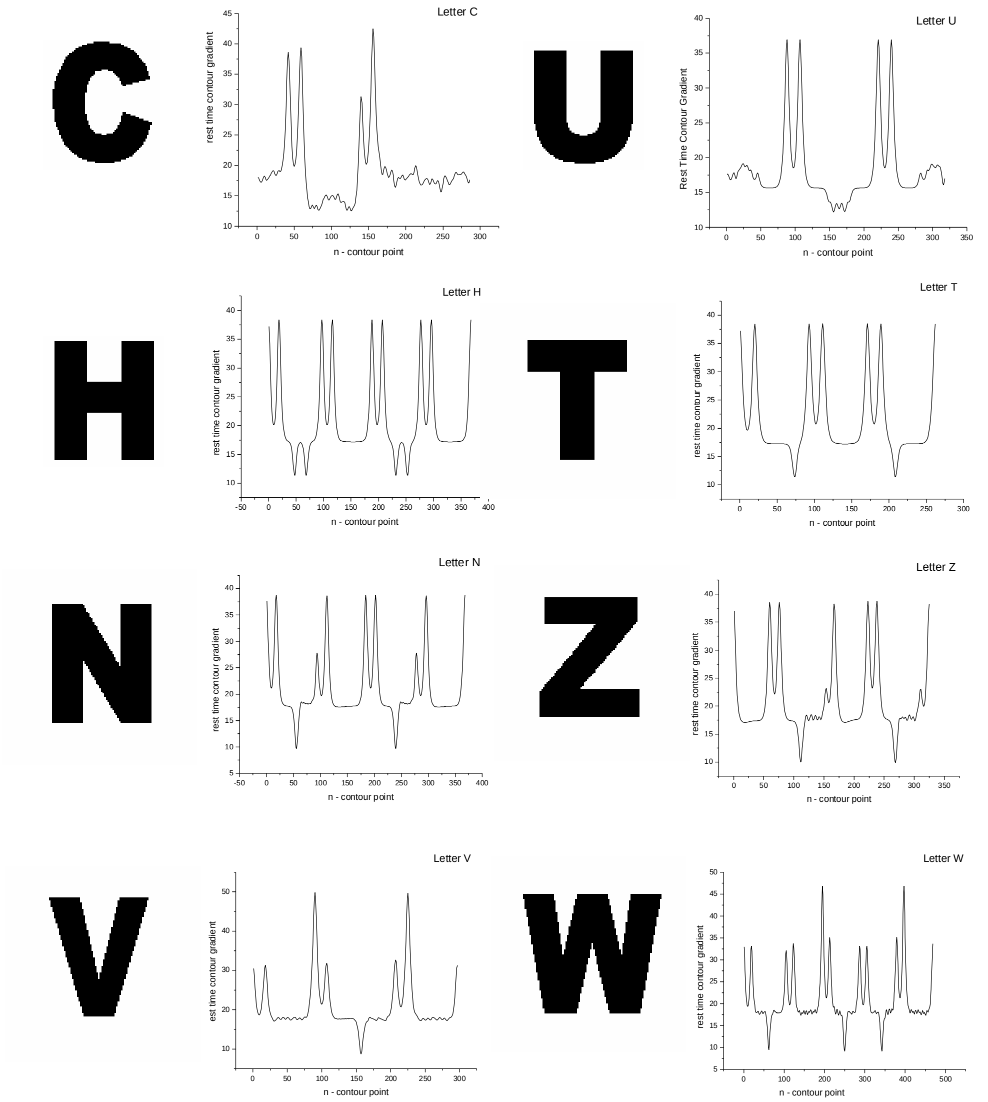

Fig. 3. Contour results for several images of letters. On the x-axis is given the contour points and on the y-axis is given the average rest time gradient values. The first point from the images is the lowest point on the left and the contour is passed in counter-clock direction

Every image can be recognized from the combination of minimums and maximums. This is how the problem of image recognition is reduced to determining extremums in one-dimensional signal. If an image is represented as min-max combination corresponding to the edges of its contour then the recognition will be possible after rotation, scaling and even stretch are applied.

As was noted, the dynamic moments are analogues to geometrical moments. They are even better then geometric moments when it comes to comparing different images. It is much more easy for two geometric moments of images of different classes to happens with close values than for the corresponding dynamic moments. It can be shown that dynamic moments are complex polynomial radicals, which depends from radius vectors of the contour, transition probabilities and space dimension. They are very sensitive to changes in the contour.

In Fig. 4. is given comparison between first geometric and dynamic moments for several figures. The dependency is between the first dynamic moments of the whole figure and the size of the figure (size is the initial number of pixels).

On the two upper figures are given the gradient of the resting moment and the average leaving time for the images of ten letters. On the two lower are given the corresponding central and distributed geometric moments. The moments are given as functions from image sizes. It is clear that geometric moments are much more convergent. The lines representing moment variation with image size overlaps and this makes images classification harder.

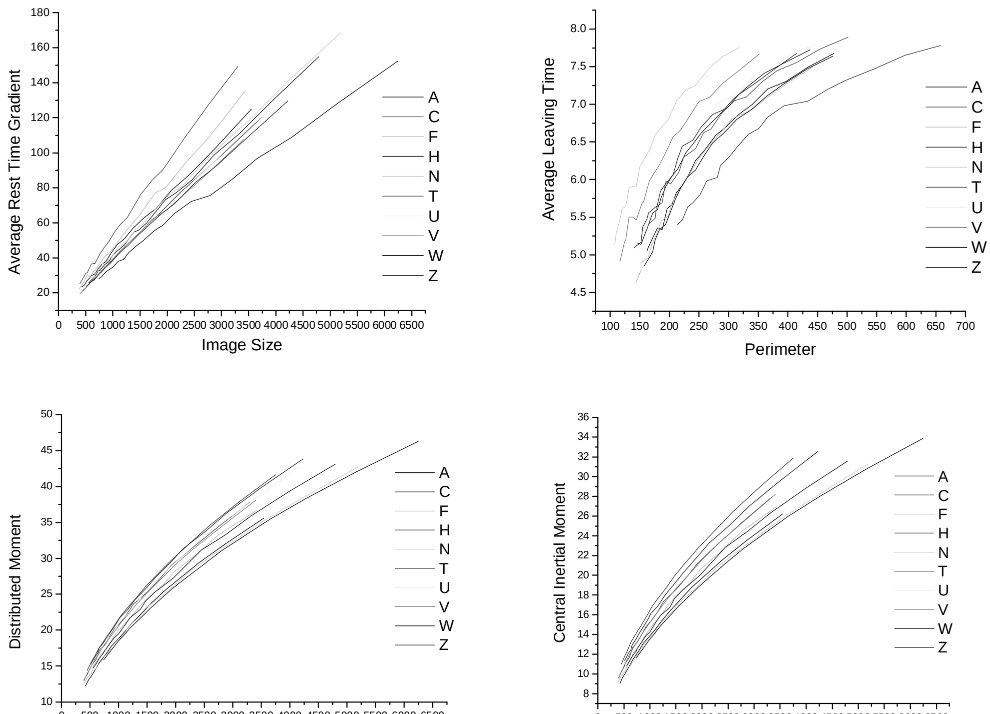

Fig. 4. Dynamic and inertia moments size-dependency comparison. Size-dependency is taken for 10 images scaled 20 times

If, from another hand, for the recognition are used dynamic moments, it is possible every image to be described with similar to above curves. If there are slight overlapping several dynamic moments can be used. Every dynamic moment can be referred as “image feature”. If we are able to extract these features from the image we can further classify this image and determine to which class it belongs. In this case dynamic moments are the unique identifiers of the image, this moments can describe the image more uniquely than geometric moments because they are formed in a more complex and image-dependent way.

Except both first order dynamic moments – the average leaving and resting times many other dynamic moments can be used. It can be chosen many time dependencies. For example the *exponential dynamic moment* is calculated as follows:

$$t_{\exp} = \sum_{i=0}^{\infty} a_i \exp(-\mu t) . \tag{13}$$

Actually this is an interesting moment and because of one another interpretation of its meaning. In signal processing well known are the so-called Laplace transformations [7, 8, 9, 10]. Equation (13) is discrete analog to these transformations. They are used when we know the input and the output signal of one unknown system and we need to find an analytical expression of the influence of this system. If μ is complex (13) becomes the well-known Fourier transformations.

If an image is treated as an unknown system, we can choose such a input signal that guaranties invariance of the output from the image rotation and scale. It can be averagely distributed over the image sinusoidal impulse, for example. If the output signal is traced, we can find the image influence on the input. Every image has unique influence and this can be used for its recognition. Fourier and Laplace transformations, in particular, uses coefficients a_i for the recognition, while the global methods proposed here use integrals like (13).

4. Conclusions

In this presentation was shown that the repeating blur of an image could be used for the calculation of the so-called dynamic moments. Then these dynamic moments, or their gradients, can be used for the image recognition. Since the blur is very simple for realization, and is similar to many diffusion processes in the nature, it is even possible to be realized on very low physical level. This very much simplifies the parallel realization of STEC algorithms. The only requirement is that on every blur pass the values over a contour or a grid must be known. These values are needed for the calculation of the dynamic moments.

As implies its abbreviation, STEC – the Standard Time Equivalent Conversions, is by ideology dynamic presentation of the geometric data. In this way many parallel-consequent and consequent-parallel transformations can be realized. It can be useful for fast parallel processing of the information and its consequent management. Thus it may be interesting these methods to be further examined.

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Обработване на изображения с техники за последователно изглаждане

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(Р е з ю м е)

В статията се разглежда нова фамилия алгоритми за обработка на изображения, наречени *STEC (Standard Time Equivalent Conversions)*. Основното им преимущество е паралелизмът, който ги прави удобни за приложение в системи за паралелна обработка. Идеята за паралелизма в STEC-алгоритмите е аналогична на физичните процеси на дифузия, което ги прави подходящи за широк кръг задачи извън разпознаването на образи. По-нататък в статията се разглежда нова концепция, наречена динамични моменти. Показано е как тя може да бъде използвана в контурните и глобалните методи за разпознаване на образи. Представени са експериментални резултати и графики.