A Numerical Study of the Upper Bound of the Throughput of a Crossbar Switch Utilizing MiMa-algorithm

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Abstract. In the present paper we propose a family of patterns for hotspot load traffic simulating. The results from computer simulations of the throughput of a crossbar packet switch with these patterns are presented. The necessary computations have been performed on the grid-cluster of IICT-BAS. Our simulations utilize the MiMa-algorithms for non-conflict schedule, specified by the apparatus of Generalized Nets. A numerical procedure for computation of the upper bound of the throughput is utilized. It is shown that the throughput of the MiMa-algorithm with the suggested family of patterns tend to 100%.

Keywords: performance analysis and design aids, generalized nets, switch node, modelling

1 Introduction

Crossbar switch node is a device which maximizes the speed of data transfer using parallel existing flows between the nodes of a communication network. In the ideal case the switch sends packets with a speed corresponding to the speed with which nodes produce these packets, without delay and without losses [1]. This is obtained by means of a non-conflict commutation schedule calculated by the control block of the switch node.

From a mathematical point of view the calculation of such a schedule is NP-complete [2]. The existing solutions partly solve the problem, using different formalisms [3]. Constantly increasing volumes of the information traffic requires new more effective algorithms, which have to be checked for efficiency. The efficiency of the switch performance is firstly evaluated by the throughput (THR) provided by the node. The next important characteristic is the average time for waiting (average cell delay), before the packet is send for commutation.

At the stages of design of switches, it is firstly assessed the THR of algorithms for non-conflict schedule. For a given algorithm, its THR will depend on the type of incoming traffic. The incoming traffic in real conditions is greatly variable. In order to evaluate the properties of the suggested algorithms, they should be compared by using strictly defined properties of the incoming traffic [3]. For a chosen traffic model, THR of a switch depends on the load intensity ρ of its input lines.

For a chosen algorithm, traffic model and load intensity ρ of the input lines, THR depends on the dimension of its commutation field $n \times n$ (n input lines, n output lines) and the dimension of the input buffer i. In our computer simulations of THR, we shall denote this dependence by a function f i.e.:

$$0 \leq THR(n,i) = f(n,i) \leq 1, \text{ where } n = 2,3,\ldots \;\; i = 1,2,\ldots$$

Here, THR with value 1 corresponds to 100% - normalized throughput with respect to the maximum throughput of the output lines of the switch.

During the simulations as well as in analytic investigations we shall look for an answer of the questions:

$$\lim_{\substack{i \to \infty, \\ n = const}} f(n,i) = ?, \qquad \lim_{\substack{i \to \infty, \\ n \to \infty}} f(n,i) = ?$$

where $i \to \infty$ means infinitely large input buffer and $n \to \infty$ means infinitely large commutation field.

In the present paper, a numerical procedure for computation of the upper bound of the THR [4] is utilized, which allows calculation of the first limit mentioned above. If it exists then the solution is unique. In this procedure we use the results from a computer simulation of the THR performed on the grid-structure BG01-IPP of the Institute of information and communication technologies IICT-BAS. Our modeling of the THR utilizes our MiMa-algorithm [5] and family of patterns for hotspot load traffic [6] with $\rho = 100\%$ load intensity of each input (i.i.d. Bernoulli). The obtained results give an upper bound of the THR for $n \in [3, 100]$ which enables us to estimate the limit of the THR for $n \to \infty$. This estimate is obtained to be 1 (100% THR).

2 Computation of the upper bound of throughput

We shall perform simulations for a specific algorithm for non-conflict schedule, a model for incoming traffic and a load intensity. We choose the interval for values of n and i, where i will define the increase in the size of the input buffer. As a result, we will have a set of curves for selected values of $n \in [n1, n2]$, and $i \in [1, 1000]$. Typical result is shown in Figure 1 (throughput [7] for PIM-algoritm [8] with hotspot load traffic [6]).

Let us chose values for i :

$$i = 1, m_1, m_2, m_3, ..., m_p, \text{ where } 1 = m_0 < m_1 < m_2 < m_3 < \ldots < m_p \quad (1)$$

We shall perform $p+1$ simulations in order to obtain $p+1$ curves for THR. The obtained curves will be denoted as follows:

$$f_1(n,i) = f(n,m_0),\; f_2(n,i) = f(n,m_1),\; \ldots, f_{p+1}(n,i) = f(n,m_p) \qquad (2)$$

Denote the difference between two successive curves f_j and f_{j+1} by res_j:

$$res_1(n,i) = f_2(n,i) - f_1(n,i) = f(n,m_1) - f(n,m_0)$$
$$res_2(n,i) = f_3(n,i) - f_2(n,i) = f(n,m_2) - f(n,m_1)$$
$$\ldots$$
$$res_p(n,i) = f_{p+1}(n,i) - f_p(n,i) = f(n,m_p) - f(n,m_{p-1}) \qquad (3)$$

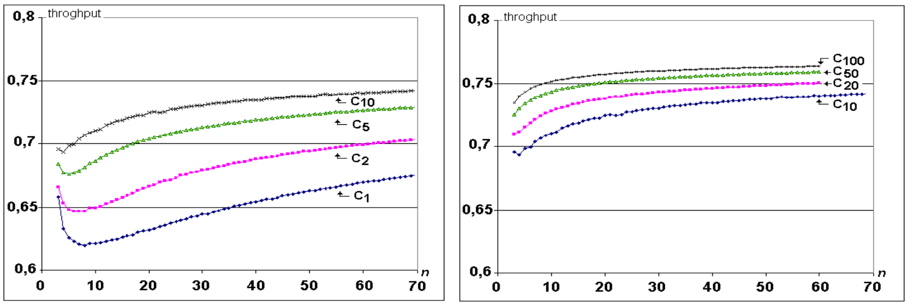

Fig. 1. Throughput for $Chao_1, \ldots, Chao_{100}$ hotspot traffic with PIM-algorithm

Denote the ratio of the values of two curves res_j and res_{j+1} through δ_j:

$$\delta_1(n,i) = \frac{res_2(n,i)}{res_1(n,i)} = \frac{f(n,m_2)-f(n,m_1)}{f(n,m_1)-f(n,m_0)}$$

$$\delta_2(n,i) = \frac{res_3(n,i)}{res_2(n,i)} = \frac{f(n,m_3)-f(n,m_2)}{f(n,m_2)-f(n,m_1)}$$

$$\ldots$$

$$\delta_{p-1}(n,i) = \frac{res_p(n,i)}{res_{p-1}(n,i)} = \frac{f(n,m_p)-f(n,m_{p-1})}{f(n,m_{p-1})-f(n,m_{p-2})} \quad (4)$$

Simulation data allow us to calculate $\delta_1, \delta_2, \ldots, \delta_{p-1}$. If we can find a dependency $\delta_{j+1} = \phi(\delta_j)$ for $\delta_1, \delta_2, \ldots, \delta_{p-1}$ in the case $j \to \infty$, then we can determine the expected upper bound.

From the last formula we obtain:

$$f(n,m_p) = f(n,m_{p-1}) + \delta_{p-1}(n,i).(f(n,m_{p-1}) - f(n,m_{p-2}))$$

or

$$f_{p+1}(n,i) = f(n,m_{p-1}) + \delta_{p-1}(n,i).(f(n,m_{p-1}) - f(n,m_{p-2}))$$

and for a known dependency $\delta_{j+1} = \phi(\delta_j)$, we can write

$$f_{p+2}(n,i) = f(n,m_{p-1}) + [1 + \phi(\delta_{p-1}(n,i))].\delta_{p-1}(n,i).(f(n,m_{p-1}) - f(n,m_{p-2}))$$

$$\ldots$$

$$f_{p+q}(n,i) = f(n,m_{p-1}) + [1 + \phi(\delta_{p-1}(n,i)) + \phi(\delta_{p-1}(n,i)).\phi(\delta_p(n,i)) + \ldots$$
$$\ldots + \phi(\delta_{p-1}(n,i)).\phi(\delta_p(n,i)).\ldots.\phi(\delta_{p+q-3}(n,i))].$$
$$.\delta_{p-1}(n,i).(f(n,m_{p-1}) - f(n,m_{p-2})) \quad (5)$$

When $q \to \infty$ then $f_{(p+q\to\infty)}(n,i)$ is the necessary bound $\lim_{\substack{i \to \infty, \\ n = const}} f(n,i)$.

If there is an upper bound of the throughput of a switch node, it is clear that the dependency $\delta_{j+1} = \phi(\delta_j)$ exists. Then the sum

$$[1 + \phi(\delta_{p-1}(n,i)) + \ldots + \phi(\delta_{p-1}(n,i)).\phi(\delta_p(n,i)).\ldots.\phi(\delta_{p+q-3}(n,i))]$$

for $q \to \infty$ is convergent and has a boundary.

3 Existence of the dependence $\delta_{j+1} = \phi(\delta_j)$.

We have found one such relation: for our model [7] of PIM-algorithm [8] (specified by means of Generalized nets [9]) with Chao-model for hotspot load traffic, for which we defined the family of patterns $Chao_i$ for traffic matrices [10]. For a simulation with this family of patterns (shown in Figure 2 - left $Chao_1$, right $Chao_i$) we have chosen the sequences for $i : i = 1, m^1, m^2, m^3, ..., m^p, \ldots$.

In this case the dependence $\delta_{j+1} = \phi(\delta_j)$ is a constant, i.e. $\delta_{j+1} = \delta_j = m^{-1/2}$ with an accuracy depending on the error of simulations. Thus, $\delta_j(n, i) = const$ when $i \in [1, \infty)$, $n \in [n1, n2]$, $m = const$, $m \in [2, 3, 4, \ldots)$ $(i = 1, m^1, \ldots, m^p, \ldots)$, with an accuracy within the error of simulations [4].

Here, we will test the validity of this assertion by simulations with $m = 2$. The utilized algorithm will be our MiMa-algorithm [5], working with the same model of load traffic (Chao-model).

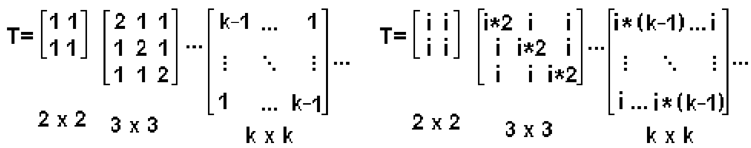

Fig. 2. Family of patterns for Chao-model of hotspot traffic

The algorithm MiMa can be described formally by the means of Generalized Nets (GN). The model is developed for switch node with n inputs and n outputs. Its graphic form is shown on Figure 3 [5]. The model has possibilities to provide information about the number of switching in crossbar matrix, as well as about the average number of packets transmitted by one switch. Analysis of the model proves receiving a non-conflict schedule. Calculation complexity of the solution depends on the power of three of the dimension n of the matrix T $(O(n^3))$. Numerical modeling should provide us with the answer to the question: do we have a better solution with this algorithm or not in comparison with existing ones (for example PIM)?

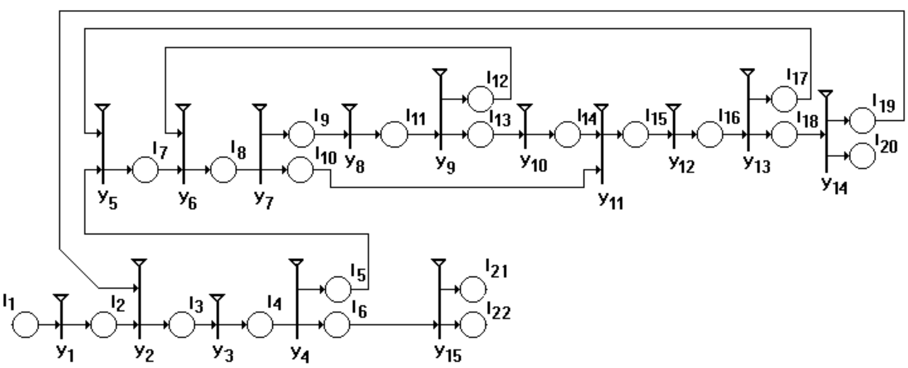

Fig. 3. GN-model for MiMa-algorithm

The transition from a GN-model to executive program is performed as in [11] using the program package VFort provided free of charge [12] by prof. Vabishchevich, Institute of Applied Mathematics, RAS. The source code has been compiled by means of the grid-structure BG01-IPP of the Institute of information and communication technologies - Bulgarian Academy of Sciences (http:// www.grid.bas.bg) and the resulting code is executed in the grid-structure.

4 Numerical procedure for calculation of the upper bound of throughput

The numerical procedure for computation of the upper bound of the THR whitch allows calculation of the first limit mentioned above is description in [4]. If the limit exists then the solution is unique.

We choose value $m = 2$. This is the minimal value of m in its definition area $m \in [2, 3, 4, \ldots)$. When $m = 2$, then $i = 1, 2, 4, 8, 16, 32, 64, \ldots, 2^p, \ldots$. The initial evaluation of the required number of curves for THR is at least 4 (from Pattern $Chao_1$). In our example, we have nine curves (patterns). In the figures below, $Chao_i$ is denoted as Ci for $i = 1, 2, \ldots$ We get results for $C1, C2, C4, C8, C16, C32, C64, C128, C256$ which are shown in Figure 4.

The dimension n varies from 3×3 to 100×100 and n simulations for each size $(n \times n)$ of pattern $Chao_i$ are executed. To achieve this goal we propose a modification of family of pastterns $Chao_i$, as it is shown an Figure 5.

Then we calculate the difference between throughput for neighboring patterns according to (3). The obtained curves for the differences are shown in Figure 6.

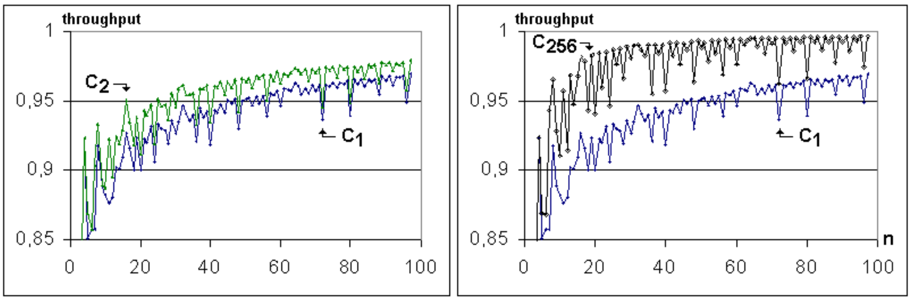

Fig. 4. Throughput for $Chao_1, \ldots, Chao_{256}$

Then we calculate the convergence parameter δ_j which is the ratio of the differences according to (4) and the obtained curves are shown in Figure 7. The values of δ_j tend to $(1, 5)^{-1}$.

From our simulations in the case $m = 2$, we have drawn the following conclusion:

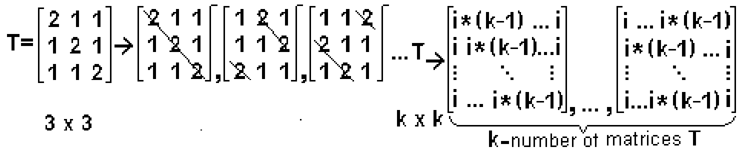

Fig. 5. Modification of Family of patterns for Chao-model

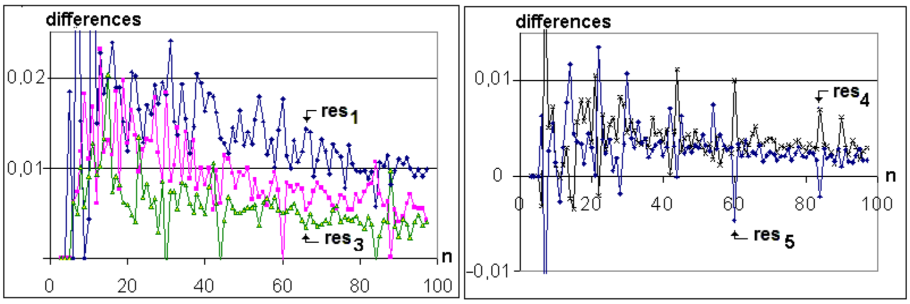

Fig. 6. Differences between throughput

Confirmed: The dependence $\delta_{j+1} = \phi(\delta_j)$ is a constant, i.e. $\delta_{j+1} = \delta_j = 2^{-1/2}$ with an accuracy depending on the error of simulations.

As a consequence, the upper boundary in case $m = const$ can be calculated according to (5) as:

$$f_{p+1}(n,i) = f(n,m^{p-1}) + \delta(m).(f(n,m^{p-1}) - f(n,m^{p-2}))$$
$$f_{p+2}(n,i) = f(n,m^{p-1}) + (\delta(m) + \delta^2(m)).(f(n,m^{p-1}) - f(n,m^{p-2}))$$
$$\dots$$
$$f_{p\to\infty}(n,i) = f(n,m^{p-1}) + [\delta(m) + \delta^2(m) + \dots + \delta^p(m) + \dots](f(n,m^{p-1}) - f(n,m^{p-2})) =$$
$$= f(n,m^{p-1}) + [m^{-1/2} + (m^{-1/2})^2 + \dots + (m^{-1/2})^p + \dots].(f(n,m^{p-1}) - f(n,m^{p-2})) =$$
$$= f(n,m^{p-1}) + [(m^{1/2} - 1)^{-1}].(f(n,m^{p-1}) - f(n,m^{p-2}))$$

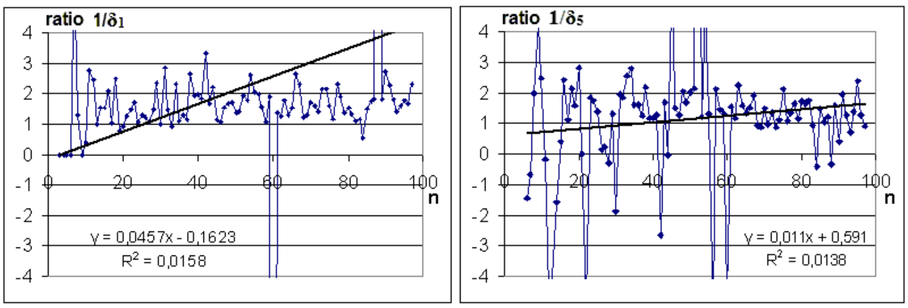

Fig. 7. Ratio $1/\delta_1$, $1/\delta_5$ between differences

In this simulation $m = 2$ and we calculate the boundary by

$$f_{p\to\infty}(n,i) = f(n,64) + [(2^{1/2} - 1)^{-1}].(f(n,64) - f(n,32))$$

This choise is for δ_5 - it has the least deviation from $m^{-1/2}$. The result is shown in Figure 8 (right). For comparison in Figure 8 (left) is shown a boundary which is calculated about δ_1 :

$$f_{p\to\infty}(n,i) = f(n,4) + [(2^{1/2} - 1)^{-1}].(f(n,4) - f(n,2))$$

Thus we conclude that $\lim_{\substack{i\to\infty,\\ n\to\infty}} f(n,i) = 1$.

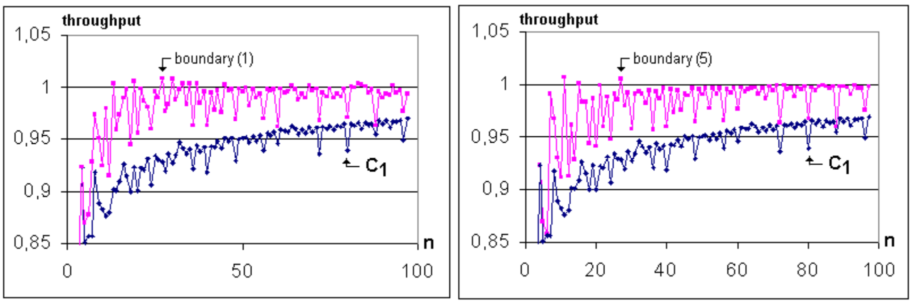

Fig. 8. Upper boundary of throughput

The differences between the values of δ_j obtained in the simulations and the value $\delta(m) = m^{-1/2}$ are a measure of the simulation accuracy. Therefore for calculation of the upper bound we chose these two successive curves f_j and f_{j+1} for which δ_j has the least deviation from $m^{-1/2}$

5 Conclusion.

Our computer simulation confirms applicability of the suggested procedure with modified pattern for load traffic. The obtained results give an upper bound of the THR for $n \in [3, 70]$ which enables us to estimate the limit of the THR of MiMa-algorithm for $n \to \infty$. This estimate is obtained to be 100%.

In a future study, the suggested modification will be tested using other models of the incoming traffic, for example unbalanced traffic models.

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