On the Generalized Vehicle Routing Problem

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Outline

Generalized network design problems

- Many network design problems can be generalized in a natural way by considering a related problem on a clustered graph, where the original problem's feasibility constraints are expressed in terms of the clusters, i.e. node sets instead of individual nodes.
- Given an undirected weighted graph $G = (V, E)$ with node set V and edge set E. The nodes are partitioned into a given number of node sets called clusters and edges are defined between any two nodes belonging to different clusters and to each edge $e \in E$ we associate a nonnegative cost c_e.
- The goal of these problems is to find a subgraph $F = (S, T)$ of G where the subset of nodes $S = \{v_1, ..., v_m\} \subset V$ is containing exactly one node from each cluster with different requirements to be fulfilled by the subset of edges $T \subset E$ depending on the actual optimization problem.

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Generalized network design problems

- In this way, it is introduced the class of generalized network design problems (selective combinatorial optimization problems):
  - the generalized minimum spanning tree problem
  - the generalized traveling salesman problem
  - the railway traveling salesman problem
  - the generalized vehicle routing problem
  - the generalized fixed-charge network design problem
  - the generalized minimum edge-biconnected network problem
  - the selective graph coloring problem
  - ...
- Applications of the generalized combinatorial optimization problems: location problems, regional connection of local area networks (LAN), irrigation, telecommunications, designing networks, irrigation, energy distribution, logistics and distribution problems, scheduling, railway optimization, etc.

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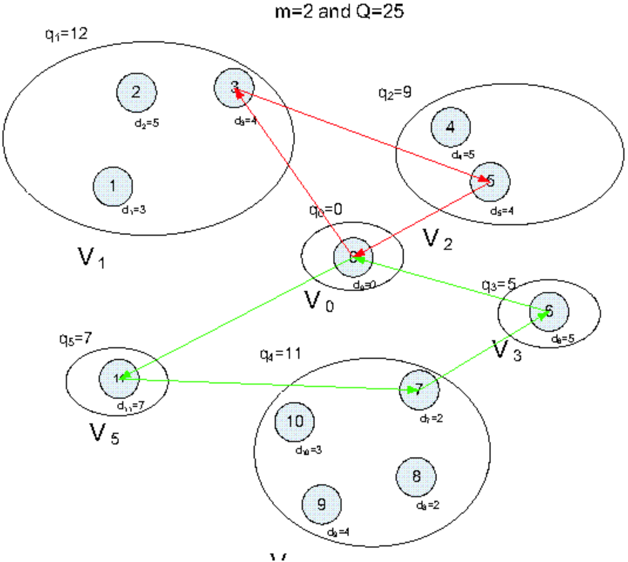
The Generalized Vehicle Routing Problem

- The generalized vehicle routing problem (GVRP) was introduced by Ghiani and Improta (2000).
- The goal of the problem is to design the optimal delivery or collection routes, subject to capacity restrictions, from a given depot to a number of predefined, mutually exclusive and exhaustive node-sets (clusters) with the condition that exactly one node is visited from each cluster.
- Applications: in the field of distribution and logistics.
- An illustrative scheme of the GVRP and a feasible tour is shown in the next figure.

The Generalized Vehicle Routing Problem (GVRP)



Definition and Complexity Aspects of the GVRP

- Let $G = (V, A)$ be a directed graph with $V = \{0, 1, 2, ..., n\}$ as the set of vertices and the set of arcs A with a cost $c_{ij} \geq 0$ associated with each arc $(i, j) \in A$. The set of vertices is partitioned into $k + 1$ mutually exclusive nonempty clusters $V_0, V_1, ..., V_k$.
- Each customer has a certain amount of demand and the total demand of each cluster can be satisfied via any of its nodes. There exists m identical vehicles, each with a capacity Q.
- The GVRP consists in finding the minimum total cost tours starting and ending at the depot, such that each cluster should be visited exactly once, the entering and leaving nodes of each cluster is the same and the sum of all the demands of any tour (route) does not exceed the capacity of the vehicle Q.
- The GVRP reduces to the classical VRP when all the clusters are singletons and to the GTSP when $m = 1$ and $Q = \infty$. The GVRP is NP-hard because it includes the GTSP as a special case when $m = 1$ and $Q = \infty$.

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Several real-world situations can be modeled as a GVRP:

- the post-box collection problem described in Laporte et al. (1989) becomes an asymmetric GVRP if more than one vehicle is required;
- the distribution of goods by sea to a number of customers situated in an archipelago as in Philippines, New Zeeland, Indonesia, Italy, Greece and Croatia;
- the design of tandem configurations for automated guided vehicles described by Baldacci et al. (2010);
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An efficient transformation of the GVRP into the VRP

Some efficient transformations of the GCOPs into classical COPs have been developed:

- in the case of the GTSP, the first transformation into the TSP was introduced by Lien et al. (1993). Later, Dimitrijevic and Saric (1997) developed another transformation that decreased the size of the corresponding TSP. Behzad and Modarres (2002) provided an efficient transformation.
- in the case of the RTSP which is a practical extension of the GTSP considering a railway network and train schedules, Hu and Raidl (2008) provided two transformation schemes to reformulate the RTSP as either a classical asymmetric and symmetric TSP.
- in the case of the GVRP, Ghiani and Improta (2000) showed that the problem can be transformed into a capacitated arc routing problem (CARP) and Baldacci et al. (2010) proved that the reverse transformation is valid.

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Let denote by v_i^r the i-th node of the cluster V_r. Then we define the VRP on a directed graph G' associated to G as follows:

- The set of nodes of G and G' are identical.
- All nodes of each cluster of are connected by arcs into a cycle in G'. We denote by $v_{i(s)}^r$ the node that succeeds v_i^r in the cycle.
- The costs of the arcs of the transformed graph G' are defined as:

$$c'(v_i^r, v_{i(s)}^r) = 0$$

$$c'(v_i^r, v_j^t) = c(v_{i(s)}^r, v_j^t) + M, \ r \neq t$$

where M must be a sufficiently large number, for example $\sum_{(i,j)\in A} c(i,j)$.

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We can define now the one-to-one correspondence between tours in G' and generalized tours in G:

- Consider a tour in G' and connect the first nodes of its clusters paths together in the order of their corresponding clusters, then the result is a generalized tour in G.
- Consider a generalized tour in G that includes the following nodes $\cdots \rightarrow v_i^r \rightarrow v_j^t \rightarrow \cdots$, $r \neq t$. Replacing the node v_i^r with the V_r-th cluster path of starting with v_i^r and then connecting the last node of this path with to the next cluster path starting with v_j^t, we obtain a tour in G'.

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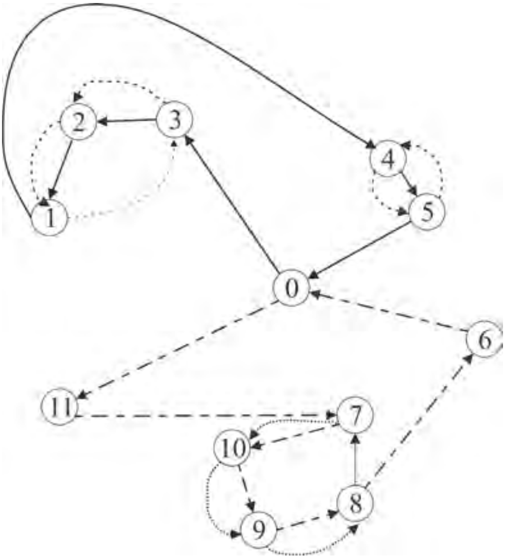

Integer programming formulations of the GVRP

- The first IP formulation for the GVRP was introduced by Kara and Bektas (2003)
- Four IP formulations of the GVRP: two based on multicommodity flow and two based on exponential sets of inequalities were described by Bektas et al. (2011)
- Two polynomial size IP formulations of the GVRP: a node based model and a flow based model have been introduced by Pop et al. (2012)
- These formulations have been extended also to the case in which the vertices of any cluster of each tour are contiguous, defined as the clustered generalized vehicle routing problem.

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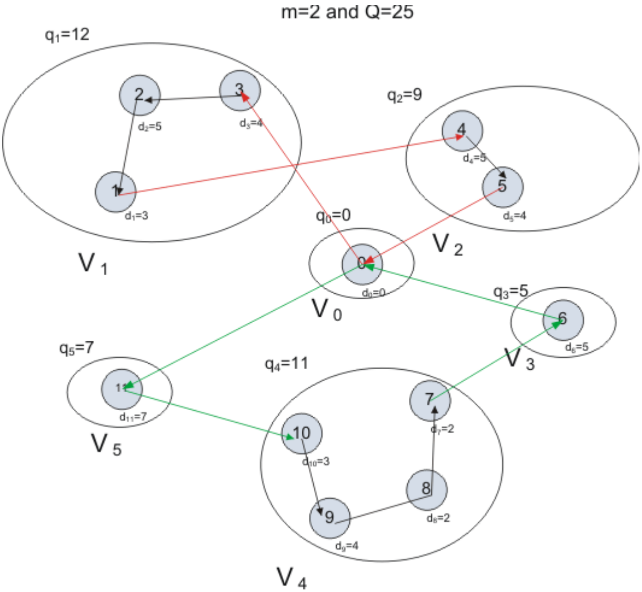

An integer programming formulation

$$\text{minimize} \quad \sum_{v\in M}\sum_{(i,j)\in A} c_{ij}x_{ij}^v$$

$$\text{subject to} \quad \sum_{i\in V_l} z_i = 1, \quad \text{for } l = 1, ..., k$$

$$\sum_{v\in M}\sum_{j\in V} x_{ij}^v = z_i, \ \forall i \in \{1, ..., n\}$$

$$\sum_{i\in V\setminus\{0\}} d_i \sum_{j\in V} x_{ij}^v \leq Q, \ \forall v \in M$$

$$\sum_{i\in V\setminus\{0\}} x_{0j}^v = 1, \ \forall v \in M$$

$$\sum_{i\in V} x_{ik}^v - \sum_{j\in V} x_{kj}^v = 0, \ \forall k \in V \setminus \{0\} \text{ and } \forall v \in M$$

$$x_{ij}^v, z_i \in \{0, 1\}, \ \forall i \in V \ \forall (i, j) \in A, v \in M$$

Solving the Generalized Vehicle Routing Problem

- an efficient transformation of the GVRP into a Capacitated Arc Routing Problem (CARP), Ghiani and Improta (2000);
- an ACS based algorithm described by Pop et al. (2008);
- an efficient transformation of the generalized vehicle routing problem into the vehicle routing problem, Pop (2011);
- an adaptive large neighborhood search proposed by Bektas et al. (2011);
- a memetic algorithm Pop et al. (2012);
- an incremental tabu search heuristic described by Moccia et al. (2012);
- an improved hybrid algorithm proposed by Pop et al.(2013).

A Variable Neighborhood Search Approach for Solving the GVRP

- VNS is quite a recent metaheuristic used for solving optimization problems based on a systematic change of the neighborhoods structures within the search in order to avoid local optima and to head for a global optimum.
- VNS is based on two simple facts:
  - **Fact 1:** A local minimum w.r.t. one neighborhood structure is not necessary so with another;
  - **Fact 2:** A global minimum is a local minimum w.r.t. all possible neighborhood structures.
- For more details on the VNS we refer to Hansen and Mladenovic [7, 8].

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Variable Neighborhood Search Framework for GVRP

Initialization. Select a set of neighborhoods structures $\mathcal{N}_l$, for $l = 1, ..., l_{max}$; find an initial solution x; choose a stopping criterion

Repeat the following sequence till the stopping criterion is met:

(1) Set $l = 1$;

(2) Repeat the following steps until $l = l_{max}$:

Step 1 (Shaking): Generate $x' \in \mathcal{N}_l$ at random;

Step 2 (Local Search): Apply a local search method starting with x' as initial solution and denote by x'' the obtained local optimum ;

Step 3 (Move or not): If the local optimum x'' is better than the incumbent x,

then move there ($x \leftarrow x''$) and continue the search with $\mathcal{N}_1$

otherwise set $l = l + 1$ (or if $l = l_{max}$ set ($l = 1$);

Go back to Step 1.

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Some questions concerning the selection of the neighborhood structures are:

- What properties of the neighborhoods are mandatory for the resulting scheme to be able to find a globally optimal or near-optimal solution?
- What properties of the neighborhoods will favor finding a near-optimal solution?
- Should neighborhoods be nested? Otherwise how should they be ordered?
- What are desirable properties of the sizes of neighborhoods?

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- To avoid being blocked in a valley the union of the neighborhoods around any feasible solution x should contain the whole feasible set:

$$X \subseteq \mathcal{N}_1(x) \cup \mathcal{N}_2(x) \cup ... \cup \mathcal{N}_{k_{max}}(x), \quad \forall\, x \in X$$

- These neighborhoods may cover X without necessarily partitioning it, which is easier to implement, e.g. when using nested neighborhoods, i.e.

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A Variable Neighborhood Search Approach for Solving the GVRP

- To avoid being blocked in a valley the union of the neighborhoods around any feasible solution x should contain the whole feasible set:

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A VNS Approach for Solving the GVRP

The local-global approach

- The local-global approach is a natural technique to tackle the generalized network design problems and it takes advantages between them and their classical variants.
- The approach aims at distinguishing between global connections (connections between clusters) and local connections (connections between nodes belonging to different clusters).
- Given a collection of r global routes of form $(V_0, V_{k_1}, ..., V_{k_p})$ in which the clusters are visited, we show that the best feasible route R^* (w.r.t cost minimization), i.e. a collection of r generalized routes visiting the clusters according to the given sequence can be done in polynomial time, by solving the following r shortest path problems.

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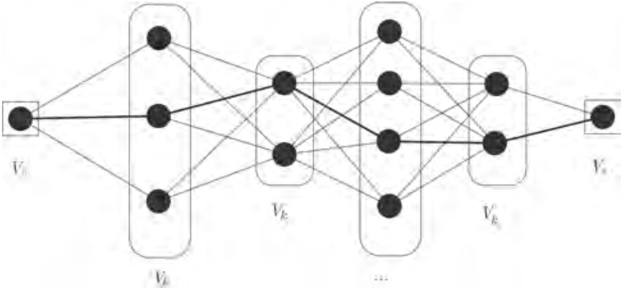

- We consider paths from 0 to $0'$, that visits exactly one node from each cluster $V_{k_1}, \dots, V_{k_p}$, hence it gives a feasible generalized route.
- Conversely, every generalized route visiting the clusters according to the sequence $(V_0, V_{k_1}, \dots, V_{k_p})$ corresponds to a path in the layered network from $0 \in V_0$ to $0' \in V_0$.

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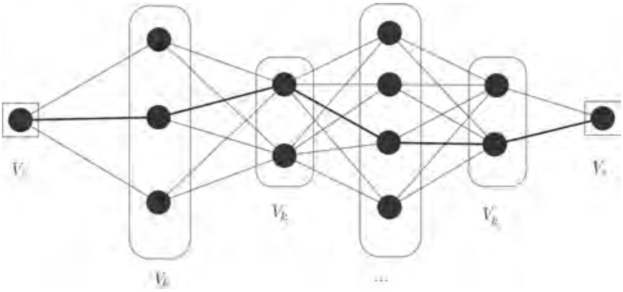

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A VNS Approach for Solving the GVRP

The adapted Clarke-Wright heuristic

- To compute a feasible solution for the GVRP problem, we replace all the nodes of a cluster V_i, $\forall i \in \{1, ..., k\}$ by a node denoted V_i^w and representing the weighted arithmetic mean of the nodes belonging to V_i. The cluster V_0 contains already one node.
- Next we use the Clarke-Wright heuristic in order to find a relatively good solution for the VRP defined on the weighted graph.
- This algorithm uses the concept of savings to rank merging operations between routes, where the savings is a measure of the cost reduction obtained by combining two small routes into one larger route.
- Having the sequences in which the clusters are visited, we use the local-global procedure in order to find the collection of best generalized routes, i.e. an initial feasible solution of the GVRP.

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A VNS Approach for Solving the GVRP

Neighborhoods

- Our VNS algorithm applies 8 types of neighborhoods, each of them focusing on different aspects and properties of the solutions to the GVRP.
- We divided these neighborhoods into two classes depending if they operate on a single route or if they consider more than one route simultaneously.
- All the considered neighborhoods are defined at the level of the global graph.
- The neighborhoods from the first class are obtained by moving one or more clusters from one position in the global route to another position in the same route and are called *intra-route neighborhoods*.
- We considered in our VNS three such neighborhoods: *Two-opt neighborhood*, *Three-opt neighborhood* and *Or-opt neighborhood*. The moves defined within the intra-route neighborhoods are used in order to reduce the overall distance.

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A VNS Approach for Solving the GVRP

Neighborhoods

- The other class, called *inter-route neighborhoods* work with two global routes.
- They are used in order to reduce the overall distance and in some cases they can reduce as well the number of vehicles.
- We considered in our VNS five such neighborhoods: *1-0 Exchange neighborhood, 1-1 Exchange neighborhood, 1-2 Exchange neighborhood, Relocate neighborhood* and *Cross-exchange neighborhood*.
- For each candidate solution provided by any of the mentioned neighborhoods, we apply the local-global procedure in order to find the best collection of routes (w.r.t. cost minimization) visiting the clusters according to the given sequences.

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A VNS Approach for Solving the GVRP

Two-opt neighborhood

- In the case of the GVRP, in a Two-opt neighborhood two global arcs corresponding to two arcs belonging to a single route are replaced by two other global arcs in order to improve the total cost of the route.
- The size of the Two-opt neighborhood is quadratic (w.r.t. the number of clusters) and there is only one proper move type.

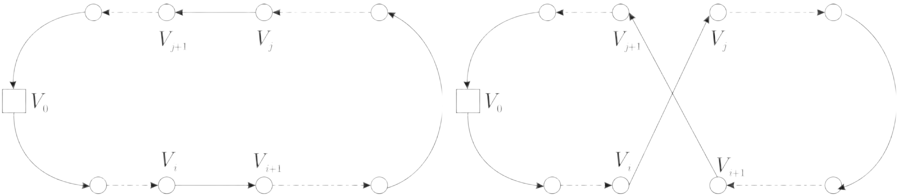

Figure : Example showing a two-opt exchange move

A VNS Approach for Solving the GVRP

Three-opt neighborhood

- The Three-opt neighborhood extends the Two-opt neighborhood and involves deleting three arcs in a route and reconnecting the three remaining paths in all other possible ways, and then evaluating each of the reconnecting methods in order to find the optimum one.
- The size of the Three-opt neighborhood is cubic and there are three proper move types.

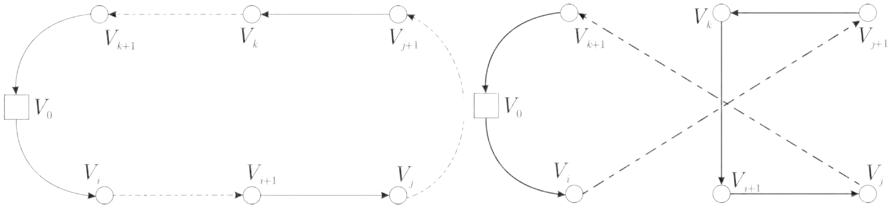

Figure : Example showing a three-opt exchange move

A VNS Approach for Solving the GVRP

Or-opt neighborhood

- In the case of the GVRP, in a Or-opt neighborhood a sequence of consecutive customers, usually one, two or three, are relocated within the route.
- The size of the Or-opt neighborhood is quadratic with the condition that the length of the sequence is bounded.

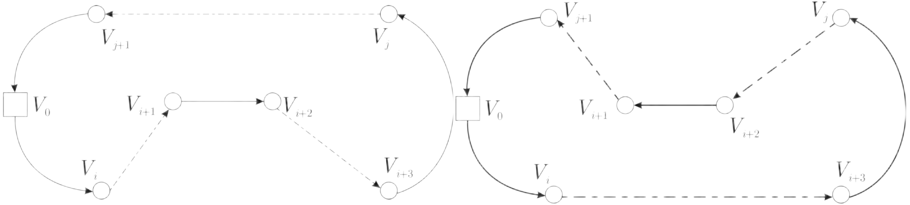

Figure : Example showing an Or-opt exchange move

A VNS Approach for Solving the GVRP

1-0 Exchange neighborhood

- Given a pair of global routes corresponding to a current solution of the GVRP, the 1-0 exchange neighborhood simply moves a cluster from one global route to the other, by replacing three global arcs.
- Then using the local-global procedure it is determined the corresponding best feasible solution of the GVRP w.r.t. the new collection of global routes.

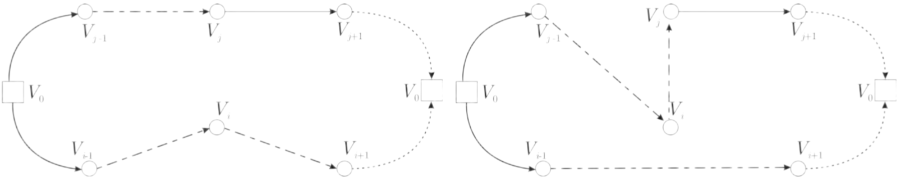

Figure : Example showing a 1-0 Exchange move

A VNS Approach for Solving the GVRP

1-1 Exchange neighborhood

- Given a pair of global routes corresponding to a current solution of the GVRP, the 1-1 exchange neighborhood swaps the positions of a cluster pair belonging to two different global routes, by removing four global arcs and creating four new ones.
- Then again using the local-global procedure it is determined the corresponding best feasible solution of the GVRP w.r.t. the new collection of global routes.

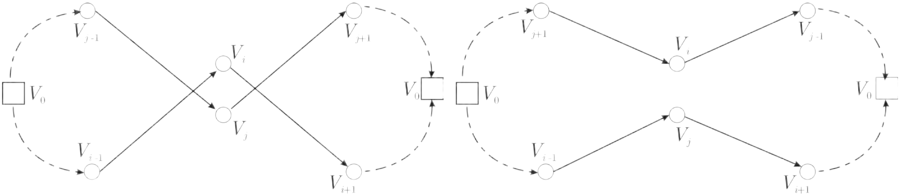

Figure : Example showing a 1-1 Exchange move

A VNS Approach for Solving the GVRP

1-2 Exchange neighborhood

- Given a pair of global routes corresponding to a current solution of the GVRP, the 1-2 exchange neighborhood swaps the positions of a cluster belonging to one global route with two consecutive clusters from the other global route, by removing four global arcs and creating four new ones.

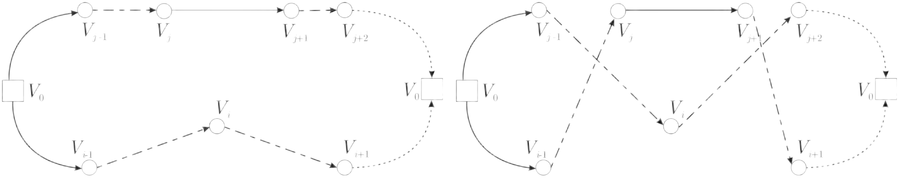

Figure : Example showing a 1-2 Exchange move

A VNS Approach for Solving the GVRP

Relocate neighborhood

- Given a pair of global routes corresponding to a current solution of the GVRP, the relocate neighborhood simply moves a sequence of 2,3 or 4 global arcs from one global route to another one.

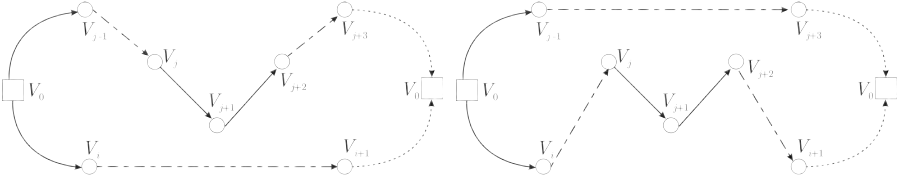

Figure : Example showing a Relocate move

A VNS Approach for Solving the GVRP

Cross-exchange neighborhood

- Given a pair of global routes corresponding to a current solution of the GVRP, the cross-exchange neighborhood involves the exchange between two sequences of arcs from the two global routes.
- Each sequence must contain the same number of required arcs, maximum three in our case.
- Then using the local-global procedure we determine the corresponding best feasible solution of the GVRP and check if we get an improvement of the solution.

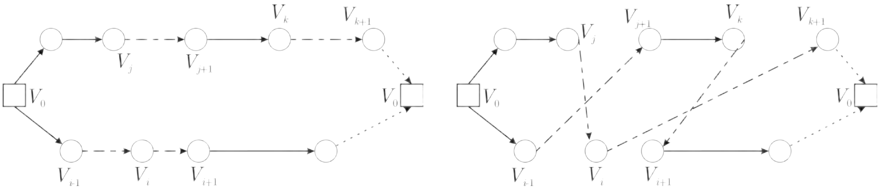

Figure : Example showing a cross-exchange for $k = 2$

A VNS Approach for Solving the GVRP

Our algorithm starts from an initial feasible solution x generated by a heuristic adapted from the Clarke-Wright heuristic and with the set of the following 8 nested neighborhood structures:

- 1-0 Exchange neighborhood ($\mathcal{N}_1$);
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A VNS Approach for Solving the GVRP

Then a point x' at random (in order to avoid cycling) is selected within the first neighborhood $\mathcal{N}_1(x)$ of x and a descent from x' is done with the local search routine. This will lead to a new local minimum x''. At this point, there exists three possibilities:

1) $x'' = x$, i.e. we are again at the bottom of the same valley and we continue the search using the next neighborhood $\mathcal{N}_l(x)$ with $l \geq 2$;
2) $x'' \neq x$ and $f(x'') \geq f(x)$, i.e. we found a new local optimum but which is worse than the previous incumbent solution. Also in this case, we will continue the search using the next neighborhood $\mathcal{N}_l(x)$ with $l \geq 2$;
3) $x'' \neq x$ and $f(x'') < f(x)$, i.e. we found a new local optimum but which is better than the previous incumbent solution. In this case, the search is re-centered around x'' and begins with the first neighborhood.

Test instances

We conducted computational experiments on two sets of instances.

- The first set of instances were generated in a similar manner to that of Fischetti *et al.* [5] who have derived the GTSP instances from the existing TSP instances. These problems were drawn from *TSPLIB* library test problems and contain between 51 and 101 customers (nodes), which are partitioned into a given number of clusters, and in addition the depot.
- The second set of instances used in our computational experiments were generated through an adaptation of the existing instances in the CVRP-library.

Computational results

- The testing machine was an Intel Core i7-3612QM and 8.00 GB RAM with Windows 8 as operating system.
- The VNS algorithm has been developed in Microsoft .NET Framework 4 using C #.

Table : Best values and computational times - *ACS*, *GA* and *VNS* algorithms for *GVRP*

Problem	*ACS*	*Time ACS*	*GA*	*Time GA*	*VNS*	*Time VNS*
11eil51	418.85	212	237.00	7	233.910	0.656
16eil76A	668.78	18	583.80	18	309.299	1.545
16eil76B	625.83	64	540.87	95	290.594	0.321
16eil76C	553.21	215.00	336.45	50	237.876	0.197
16eil76D	508.81	177.00	295.55	12	232.296	0.651

A VNS Approach for Solving the GVRP

Computational results

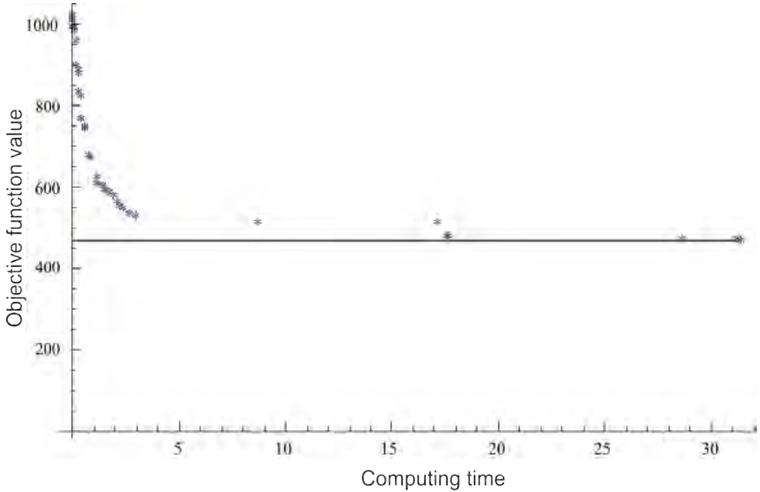
Table : Computational results on small and medium instances with $\theta = 3$

Instance	*LB*	*ALNS*	*ITS*	*HA*	*VNS*
A-n32-k5-C11-V2	386	386	386	386	385.911
A-n33-k5-C11-V2	315	318	315	315	315.762
A-n33-k6-C11-V2	370	370	370	370	370.658
A-n34-k5-C12-V2	419	419	419	419	419.102
A-n45-k6-C15-V3	474	474	474	474	474.193
A-n55-k9-C19-V3	473	473	473	473	472.193
B-n31-k5-C11-V2	356	356	356	356	355.422
B-n34-k5-C12-V2	369	369	369	369	369.315
B-n35-k5-C12-V2	501	501	501	501	501.032
B-n39-k5-C13-V2	280	280	280	280	279.580
B-n50-k7-C17-V3	393	393	393	393	393.715
P-n16-k8-C6-V4	170	170	170	170	169.96
P-n20-k2-C7-V1	117	117	117	117	117.306
P-n21-k2-C7-V1	117	117	117	117	117.070
P-n22-k2-C8-V1	111	111	111	111	111.194
P-n23-k8-C8-V3	174	174	174	174	175.019
P-n50-k7-C17-V3	261	261	261	261	261.101

A VNS Approach for Solving the GVRP

Computational results

Next figure shows the behavior of the VNS against the required time (in seconds) on instance A-n45-k6-C15-V3 when GVRP is solved.



A VNS Approach for Solving the GVRP

Computational results

- We can conclude that the convergence of VNS is very fast, after 2.986 seconds (in the first iteration) the solution is much improved from 1028.777 to 531.112 and than after 28.411 seconds an optimum solution of value 474.193 is reached.
- Overall the proposed VNS algorithm can be seen successful providing high-quality solutions in reasonable computational running times. The success of our VNS approach consists in the selection and properties of the neighborhoods that are covering the whole feasible set.
- Next we give an example of the progress of the objective function of the GVRP using our developed VNS algorithm starting with an initial solution and using shaking and local search strategies described in Algorithm 1. The instance used is B-n50-k7-C17-V3 and contains 50 nodes partitioned within 17 clusters which are visited by 3 vehicles.

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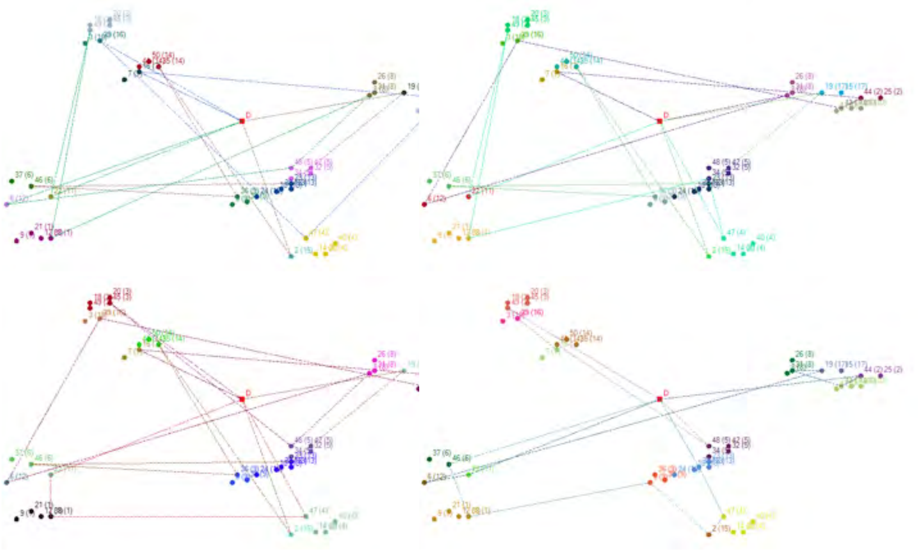

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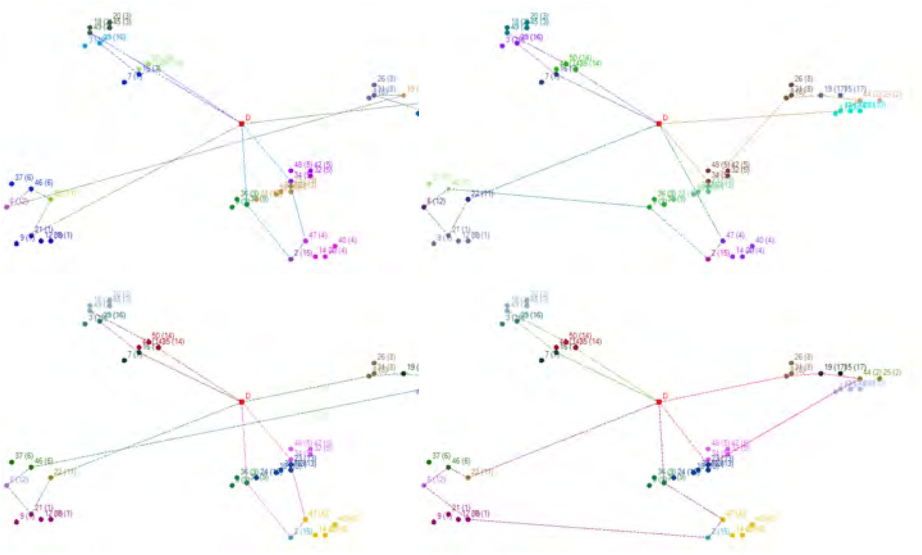

Conclusions and future work

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For Further Reading I

- R. Baldacci, E. Bartolini and G. Laporte, Some applications of the Generalized Vehicle Routing Problem, Le Cahiers du GERAD, G-2008-82, 2008.
- A. Behzad and M. Modarres, A new efficient transformation of the generalized traveling salesman problem into traveling salesman problem. In Proceedings of the 15th International Conference of Systems Engineering, pp. 68, 2002.
- T. Bektas, G. Erdogan and S. Ropke, Formulations and Branch-and-Cut Algorithms for the Generalized Vehicle Routing Problem, Transportation Science, Vol. 45, (3), pp. 299-316, 2011.
- V. Dimitrijevic and Z. Saric, An efficient transformation of the generalized traveling salesman problem into the traveling salesman problem on digraphs, Information Science, Vol. 102(1-4), pp. 105-110, 1997.

For Further Reading II

- Fischetti, M., Salazar, J.J., Toth, P.: A branch-and-cut algorithm for the symmetric generalized traveling salesman problem. Operations Research, **45**3, 378-394 (1997)
- G. Ghiani, G. Improta, An efficient transformation of the generalized vehicle routing problem, European Journal of Operational Research, Vol. 122, pp. 11-17, 2000.
- P. Hansen and N. Mladenovic, Variable neighborhood search: Principles and applications, European Journal of Operational Research, Vol. 130(3), pp. 449-467, 2001.
- N. Mladenovic and P. Hansen, Variable neighborhood search, Computers & Operations Research, Vol. 24(11), pp. 1097-1100, 1997.

For Further Reading III

- I. Kara, T. Bektaş, Integer linear programming formulation of the generalized vehicle routing problem, 5-th EURO/INFORMS Joint International Meeting, July 6-10, 2003, Istanbul, Turkey.
- Y. Lien, E. Ma and B.W. Wah, Transformation of the Generalized Traveling Salesman Problem into the Standard Traveling Salesman Problem, Information Sciences, vol. 74, pp. 177-189, 1993.
- L. Moccia, J-F. Cordeau and G. Laporte, An incremental neighborhood tabu search heuristic for the generalized vehicle routing problem with time windows, Technical Report, https://www.cirrelt.ca/DocumentsTravail/CIRRELT-2010-12.pdf, 2010.
- P.C. Pop, C.M. Pintea, I. Zelina and D. Dumitrescu, Solving the Generalized Vehicle Routing Problem with an ACS-based Algorithm, American Institute of Physics, Vol. 1117, pp. 157-162, 2009, (Conf. Proc. BICS Tg. Mures 5-7 November 2008).

For Further Reading IV

- P.C. Pop, C. Pop Sitar, I. Zelina, V. Lupse and C. Chira, Heuristic algorithms for solving the generalized vehicle routing problem, International Journal of Computers, Communications & Control, Vol. 6, No. 1, pp. 158-166, 2011.
- P.C. Pop and C. Pop Sitar, A new efficient transformation of the generalized vehicle routing problem into the classical vehicle routing problem, Yugoslav Journal of Operations Research, Vol. 21, No. 2, 2011.
- P.C. Pop, I. Kara and A. Horvat Marc, New Mathematical Models of the Generalized Vehicle Routing Problem and Extensions, Applied Mathematical Modelling, Vol. 36, pp, 97-107, 2012.
- T. Thomadsen and T. Stidsen, The generalized fixed-charge network design problem, Computers & Operations Research 34, pp. 997 - 1007, 2007.