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NEARFIELD ACOUSTIC HOLOGRAPHY

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Introduction

Near-field acoustic holography (NAH) is a technique that makes it possible to reconstruct the three-dimensional sound field between the source and the measurement plane.

Reconstructed parameters:

- Sound pressure
- Air particle velocity
- Sound intensity
### Measurement of the baffled plate movement

<table>
<thead>
<tr>
<th></th>
<th>2 kHz</th>
<th>3 kHz</th>
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</thead>
<tbody>
<tr>
<td><strong>True Intensity</strong></td>
<td><img src="image1" alt="2 kHz True Intensity" /></td>
<td><img src="image2" alt="3 kHz True Intensity" /></td>
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<tr>
<td><strong>WBH Intensity</strong></td>
<td><img src="image3" alt="2 kHz WBH Intensity" /></td>
<td><img src="image4" alt="3 kHz WBH Intensity" /></td>
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</table>
Statistically optimized holography (SONAH)

The sound pressure at any arbitrary position above the source is expressed as a weighted sum of sound pressures measured at N positions in the hologram plane

\[ p(r) = \sum_{n=1}^{N} c_n(r)p(r_{h,n}) = p^T(r_h)c(r), \]

The transfer vector depends only on positions, not on the pressure.

In the same way the infinite set of elementary waves

\[ \Phi_m(r) = e^{-j(k_{x,m}x + k_{y,m}y + k_{z,m}z)}, \quad m = 1, 2, \ldots, M, \quad M \to \infty, \]

is projected from the measurement plane to the prediction plane. In matrix form it is

\[ \alpha(r) = Ac(r), \]

where \( \alpha(r) \) is column vector with \( M \) elements, \( [\alpha(r)]_m = \Phi_m(r) \), \( A \) is \( M \) by \( N \) matrix, \( [A]_{mn} = \Phi_m(r_{h,n}) \). Since \( M > N \), this equation is overdetermined. The least squares solution is

\[ c(r) = (A^H A + \theta^2 I)^{-1} A^H \alpha(r), \]

Where \( I \) is the identity matrix, \( \theta \) is a regularization parameter, the superscript \( H \) indicates the Hermitian transpose. The sound pressure on the prediction plane is

\[ p(r) = p^T(r_h)(A^H A + \theta^2 I)^{-1} A^H \alpha(r). \]
**Equivalent Source Method (ESM)**

The ESM is quite a popular modern method of performing NAH that has simple implementation and comparable efficiency. It is based on the assumption that sound pressure at microphone position points can be represented as the weighted sum of amplitudes of some amount of virtual sources, located behind real source plane:

$$p_h(x) = \sum_{n=1}^{N} q_n G_{hv}(x, y_n)$$

or in the matrix form

$$p_h = G_{hv} q_v$$

Here the weighting matrix consists of so called free-space Green’s functions

$$G_{hv}(x, y) = \frac{e^{-jkr}}{4\pi r}$$

Estimates of source amplitudes are obtained from the inverse matrix equation

$$\hat{q}_v = (G_{hv})^+ p_h$$

At the final stage pressure on the actual surface is calculated from the next equation

$$p_s = G_{sv} \hat{q}_v$$
Wideband acoustic holography (WBH)

The invented by Bruel & Kjaer method of wideband holography works in wide frequency range and on big distances in comparison to conventional holography. The key idea is iterative solution of acoustic field propagation equation with consequent suppression of weak ghost sources.

Example of reconstruction results for 4 kHz signal on the distance 0.24 m. in the case of single source in front of measurement array.

Figures from Bruel & Kjaer
## Resolution comparison of ESM and SONAH holography methods

<table>
<thead>
<tr>
<th>Center frequency</th>
<th>Resolution (cm)</th>
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<tbody>
<tr>
<td></td>
<td>ESM</td>
<td>SONAH</td>
<td>ESM</td>
<td>SONAH</td>
<td></td>
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<tr>
<td></td>
<td>Distance 5 cm</td>
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</tbody>
</table>
Comparison of ESM and SONAH

ESM

10 cm, 1 kHz

SONAH

30 cm, 200 Hz
Acoustic field interpolation

Coordinates of virtual microphone between any pair of two real ones are defined from the condition that distance from the estimated source position to virtual microphone is a middle between distances to two other ones:

\[ l_v = \frac{l_1 + l_2}{2} \]

Therefore, signal in this point must have equal phase shift to each real microphone.

Having the relation for any sinewave signal

\[ s_i = \alpha s_{i-1} - s_{i-2}, \quad i = 2, N - 1, \quad \alpha = 2\cos(\omega \Delta t), \]

the virtual signal at virtual microphone can be written as

\[ s_v = \frac{s_1 + s_2}{2\cos(\omega \Delta t)}. \]

Time difference \( \Delta t \) is estimated from related microphone to source positions.

The important term: projection of the point \( s \) on the line \( m_1, m_2 \) must be outside the segment \( (m_1, m_2) \) in order to provide continuous grooving of phase between two selected microphones. Otherwise, there is the point with zero derivative by the phase and relation does not work.
Resolution improvement
SONAH (2.5 kHz, 10 cm, 0,0)
Before

After

SONAH (4.5 kHz, 10 cm, 0,0)
Before

After
Suppression of ghost sources
SONAH (6.5 kHz, 10 cm, 0,0)
Before After

ESM (2.5 kHz, 10 cm, 0,0)
Before After
Acoustic field interpolation via phase estimation

The idea is to estimate instantaneous phase and amplitude at each microphone from the next linearized likelihood equation system

\[
\begin{align*}
A_x \sum_{n=0}^{N-1} \sin^2(\omega \tau n) + A_y \sum_{n=0}^{N-1} \sin(\omega \tau n) \cos(\omega \tau n) &= \sum_{n=0}^{N-1} x_n \sin(\omega \tau n); \\
A_x \sum_{n=0}^{N-1} \cos(\omega \tau n) \sin(\omega \tau n) + A_y \sum_{n=0}^{N-1} \cos^2(\omega \tau n) &= \sum_{n=0}^{N-1} x_n \cos(\omega \tau n).
\end{align*}
\]

where final estimates of amplitude and initial phase are calculated as

\[
\rho_n^* = \sqrt{A_x^2 + A_y^2}, \quad \phi_n^* = \arctan(A_y/A_x),
\]

Having signal frequency, these estimations and approximate position of the source we can calculate signal value at points close to each real microphone.

At the moment the most effective found geometry consists of real microphones surrounded by four virtual microphones on the distance of half wavelength. Positions of microphones don’t depend on the source coordinates and this method can be used for arbitrary located source.
Improvement of localization precision
SONAH (2.5 kHz, 10 cm, 0.05, 0.05)
Before  After
SONAH (4.5 kHz, 10 cm, 0.15, 0)
Before  After
Conclusions

1. The brief introduction to the theory of near-field acoustic holography was given.
2. Two classic methods of performing NAH (built-in SONAH and ESM) were considered in relation to the present acoustic camera equipment. Basic results of performance comparison in some typical conditions were obtained. It was proven that these methods generally gives similar results with minor differences.
3. The new approach of improvement of NAH methods efficiency was proposed on the basis of acoustic field interpolation.
4. The proposed method allows increasing of spatial resolution and localization precision in the case of low amount of microphones and in the middle frequency range.